Mean-Reversion Across MENA Stock Markets: Implications for Derivative Pricing

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ABSTRACT

This paper investigates the mean reversion patterns in MENA stock markets. Following Fama and French (1988) and Lo and Mackinlay (1988) and using recent stock market data between 1995 and 2000 on Egypt, Jordan, Morocco, and Turkey we find evidence of mean reversion in Turkey with mixed results for the remaining MENA stock markets. The paper then introduces a non-parametric model to estimate the reverting mean and speed of reversion. The highest reverting mean is noted for Turkey, followed by Egypt, then Morocco, with Jordan ranking last. Monte Carlo simulations show consistent increases in the volatility of MENA stock returns as the speed of reversion slows. Our results have an important bearing on the pricing of equity derivatives in MENA and are useful for investors employing tactical asset allocation strategies.

\textit{JEL: G100, G120, G130}

\textit{Keywords: Middle East Finance; African Finance; Derivatives; Option Pricing}
I. INTRODUCTION

The last decade has witnessed an increasing interest in the debate over mean reversion in stock market returns. Starting from the seminal work of Poterba and Summers (1988) and Fama and French (1988), who documented mean reversion in stock market returns during a time horizon greater than one year, analysts have began to investigate the implications of their findings on the efficient market hypothesis. A debate has emerged between 2 tracks. One line of reasoning (DeLong et. al. [1990]) contend that if stock prices have a significant predictable component, this suggests the existence of irrational market participants where prices exhibit long but ultimately temporary swings away from fundamental values. Another camp argues that the same stock price behavior could be the outcome of equilibrium expected returns that are time varying in an efficient market. Regardless of the efficiency debate, if stock prices returns tend to revisit a long-term average, the mean reversion property has significant implications for optimal asset allocations.

Recently, a number of studies have examined the implication of mean reversion on investment decisions. Barberis (1997) compares two investment strategies: ‘buy-and-hold’ vs. dynamic re-balancing when stock market returns have a predictable component. He concludes that a risk-averse investor will allocate a larger proportion to equities, the longer the horizon, even when parameter uncertainty about the predictor variable exists. A similar comparison was conducted by Richards (1997). And recently, Balvers et. al. (2000) find strong evidence of mean reversion in the relative stock-index prices of 16 OECD countries plus Hong Kong and Singapore.

We propose a study to investigate in an intuitive setting the mean reversion patterns of the stock markets in the Middle East and North Africa (MENA). Mean reversion in a stock market suggests that bad returns are likely to be followed by periods of good returns. By contrast, in a random walk setting, the future is a flip of a coin, regardless of the return outcomes in earlier periods. An important implication to our findings is that because MENA stock returns exhibit mean reversion, the volatility of returns would be lower than that implied by a random walk model. Two important consequences emerge from our results. Our findings have significant bearing on (1) the portfolio allocation of mutual funds, corporate and private investors in MENA stock markets; and (2) the pricing of equity based derivatives in the region.

This paper complements the existing finance literature on the MENA region (for example El Loughani 1995, Dahel and Laabas 1998, and Kumar 1995, Darrat & Hakim 2000, Hakim & Neaime 2000) by analyzing mean-reversion and its implications on investment asset decisions and the pricing of derivative securities. While the existing literature on the MENA region has analyzed markets interdependence, spillovers, and efficiency, it has never touched on mean reversion issues as well as issues related to market volatility in relation to the speed of adjustment.

The paper is organized as follows. In the next section we introduce the data and analyze the descriptive statistics. Section III explains the estimation methodology of the mean reversion parameters. The empirical results are discussed in section IV where we demonstrate how the return volatility falls (rises) as the speed of mean reversion accelerates (slows). Section V summarizes the findings and concludes the paper.
II. DATA AND SAMPLE

Our data consists of weekly and daily closing price series for the stock indices of the 3 prominent equity markets in Middle East and North Africa. They are Amman, Cairo (Egypt), and Casablanca (Morocco). While not geographically located in MENA, the study will also include Istanbul’s stock market. The data period is recent, covering 5 years, starting in April 1995 and ending in May 2000. The data was acquired from Morgan Stanley. Compounded week-to-week returns are calculated as the natural log differences in prices: \( \log \left( \frac{P_t}{P_{t-1}} \right) \).

We begin by examining the statistical properties of each equity market by observing the plot of its histogram. We also compare the weekly performance of each market between May 95 and May 2000. With an average weekly return of 0.30%, we find Cairo Stock Exchange (CSE) to be significantly higher than the emerging market index and second only to Istanbul in the MENA region. This represents an average over 5 years and corresponds to 15% in annual terms. For an investor, what matters of course is the return per unit of risk, a measure similar to a Sharp Ratio. Using this measure, we find the CSE to be second only to Casa but with considerable more liquidity. This should provide a degree of comfort to foreign investors seeking higher risk-adjusted returns with a sufficient market capitalization and liquidity.

Examining the distribution of MENA stock markets, we find they have skewed returns with significant variability in kurtosis. For a normal distribution, \( S \) and \( K \) are respectively 0 and 3. Clearly, most markets exhibit substantial departures from normality. We formally tested for normality of the return distributions using the Jarque Bera Statistic (JB). Under the null hypothesis of normality, JB is distributed \( \chi^2 \) with 2 degrees of freedom. JB is defined as

\[
JB = \frac{T}{6} \left[ S^2 + \frac{1}{4} (K - 3)^2 \right],
\]

where \( S \) and \( K \) represent the Skewness and Kurtosis. The null hypothesis of normal returns is rejected for all MENA stock markets. The positive skewness in the Cairo, Casablanca and Amman stock markets returns can be attributed to (a) the recent privatization schemes which were implemented in these financial markets; (b) the extensive sale of government assets to private firms; and (c) the considerable efforts devoted during that period towards enhancing the efficiency, depth, and liquidity of the three stock markets, especially in Egypt and Morocco. These positive factors resulted in record market capitalization growth rates. Specifically, market capitalization in Egypt grew from USD 5.78 billion in 1995 to USD 33.7 billion in 2000. In Morocco and Jordan, over the same period it grew from USD 2.9 to USD 14.1 and from USD 3.56 to 5.82 billion respectively (see Arab Monetary Fund, annual report 2000). International and domestic investors perceived these favorable structural changes positively and started buying heavily in those markets driving up stock returns during the period under consideration. To test for heteroskedasticity, we provide the results of an ARCH LM
test statistic computed from a secondary regression. To conduct the test, we run the regression

\[ u_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \alpha_2 u_{t-2}^2 + \xi_t, \]

where \( u_t \) represents the residuals of the return equation. The equation represents a regression of the squared return residuals on a constant and lagged squared residuals of order two. Table 1 shows the results of Engle’s LM test statistic, which is essentially a modified F-statistics of joint significance of the two omitted lagged squared residuals. The results indicate the presence of heteroskedasticity only for Egypt and Morocco. The results were consistent for higher order lag terms.

### Table 1
Weekly return comparison with other stock markets (in USD)
April 1995 – May 2000

<table>
<thead>
<tr>
<th></th>
<th>Cairo</th>
<th>Casa</th>
<th>Amman</th>
<th>Istanbul</th>
<th>Tel Aviv</th>
<th>Emerging Markets</th>
<th>London</th>
<th>New York</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean (%)</td>
<td>0.30</td>
<td>0.20</td>
<td>-0.15</td>
<td>0.37</td>
<td>0.28</td>
<td>0.04</td>
<td>0.24</td>
<td>0.42</td>
</tr>
<tr>
<td>Median (%)</td>
<td>0.02</td>
<td>0.16</td>
<td>-0.34</td>
<td>0.38</td>
<td>0.59</td>
<td>0.31</td>
<td>0.29</td>
<td>0.59</td>
</tr>
<tr>
<td>Maximum (%)</td>
<td>11.78</td>
<td>6.29</td>
<td>6.90</td>
<td>19.39</td>
<td>7.68</td>
<td>6.72</td>
<td>5.67</td>
<td>6.43</td>
</tr>
<tr>
<td>Std. Dev. (%)</td>
<td>2.75</td>
<td>1.51</td>
<td>1.70</td>
<td>6.08</td>
<td>2.92</td>
<td>2.45</td>
<td>1.66</td>
<td>1.81</td>
</tr>
<tr>
<td>Skewness</td>
<td>1.01</td>
<td>0.28</td>
<td>0.89</td>
<td>-0.08</td>
<td>-0.70</td>
<td>-1.00</td>
<td>-0.13</td>
<td>-0.58</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>5.63</td>
<td>5.17</td>
<td>4.84</td>
<td>3.66</td>
<td>4.95</td>
<td>6.72</td>
<td>3.22</td>
<td>5.20</td>
</tr>
<tr>
<td>Sharp Ratio (%)</td>
<td>11.00</td>
<td>14.00</td>
<td>-9.00</td>
<td>6.00</td>
<td>10.00</td>
<td>2.00</td>
<td>15.00</td>
<td>23.00</td>
</tr>
<tr>
<td>Jarque-Bera</td>
<td>119.50</td>
<td>54.65</td>
<td>71.70</td>
<td>5.06</td>
<td>62.53</td>
<td>194.17</td>
<td>1.20</td>
<td>67.36</td>
</tr>
<tr>
<td>Probability</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.08</td>
<td>0.00</td>
<td>0.00</td>
<td>0.55</td>
<td>0.00</td>
</tr>
<tr>
<td>LM test</td>
<td>0.00</td>
<td>0.00</td>
<td>0.88</td>
<td>0.40</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
</tbody>
</table>

### III. METHODOLOGY

Our study proposes to conduct various tests suggested by Fama and French (1988), and Lo and Mackinlay (1988) to investigate the evidence of mean reversion in the stock markets of Egypt, Jordan, Morocco and Turkey. One test for mean reversion is the variance ratio (VR) popularized by Lo and Mackinlay. For a k-year return, the test is

\[ VR(k) = \frac{\text{Var}(R_t(k))}{k\text{Var}(R_t)}, \]
where $R_t(k) = \sum_{i=1}^{k} R_{t-i+1}$ represent the continuously compounded weekly returns.

If returns are mean reverting, then $VR(k) < 1$ for $k = 1, 2, 3, \ldots$ We implement the variance ratio test for $k = 5$ trading days. Once mean reversion is established, it is important to estimate the parameters of this property. More importantly, to the extent that stock market prices can be modeled as a stochastic process, one needs to estimate the parameters of their dynamics. To that end, we will assume that the return dynamics of each market are governed by a general Ito process of the form

$$dR = A(R,t)dt + B(R,t)dz,$$

where $dz$ is a Wiener process, and $A$ and $B$ are functionals which determine the behavior of the instantaneous return of each individual market. The stochastic differential equation in (4) encompasses many examples, more notably the following:

(a) Vasicek (1977): $dR = \kappa(\mu - R)dt + \sigma dz,$

(b) Cox Ingersoll and Ross (1980): $dR = \kappa(\mu - R)dt + \sigma \sqrt{R}dz,$

(c) Brennan and Schwartz (1979): $dR = \kappa(\mu - R)dt + \sigma Rdz,$

where returns revert to the level $\mu$ at a rate $\kappa$. Under mean reversion, we have $0 < \kappa < 1$. Models (a)-(c) and variations of them are used to price interest and exchange rate derivatives. Fama and French (1988) and Chan et al. (1992) use a similar approach to test for mean reversion. The inverse of the mean reversion rate ($1/\kappa$) can be interpreted as the number of periods elapsed between reversions, in effect, it measures the speed of reversion. The reverting mean, $\mu$, represents the level which long-term returns revisit more often than others after wandering off. It is assumed that while equity returns wander randomly, over time they get pulled to the level $\mu$ at a speed $1/\kappa$. A similar framework is used by Chan et al. (1992) in the context of interest rate dynamics, and Samuelson (1969, and 1994) for equities. Specifically, Samuelson (1969, 1994) has shown that if an investor's relative risk aversion is greater than unity, then the asset allocation choice is independent of the investment horizon provided that the risky asset returns follow a random walk. This is a refutation of the popular time diversification argument. Samuelson qualifies the above result by showing theoretically that in the presence of mean reversion, the optimal proportion allocated to equities increases as the investment horizon lengthens. This has been interpreted particularly in the practitioner literature (see Reichenstein and Dorsett [1995]) as redemption for the time diversification position.

Because we wish to leave the drift and volatility terms unspecified, we estimate the discrete form of this model using a Generalized Method of Moments (GMM) technique initially suggested by Tauchen (1986) and, recently re-examined by Butler (2000).

The choice of a GMM estimator is predicated on the fact that it does not require knowledge of the distribution of MENA stock market returns. This assumption allows a
great deal of flexibility to choose among many models each making a specific assumption about the distribution of returns over time. Moreover, the GMM estimator is consistent when the errors are conditionally heteroskedastic. Our methodology takes into account the findings of Bekaert and Harvey (1997), who argue that when studying emerging market returns, one should be wary of the significant leptokurtosis and skewness their stock market returns manifest.

We let $\theta$ represent a vector with elements the reverting mean $\mu$ and speed of reversion $\kappa$. We Define

$$H(R, \theta) = dR - \kappa(\mu - R)dt.$$  

(5)

Our moments conditions can be written as: $E(H(\theta)) = 0$. The GMM estimator is defined by replacing the moment condition above by its sample counterpart

$$\frac{1}{N} \sum_{t=1}^{N} H(R, \hat{\theta}) = 0.$$  

(6)

The GMM estimator is obtained by minimizing

$$H(\theta)'\Sigma H(\theta),$$  

(7)

where $\Sigma$ is a weighting matrix.

**IV. EMPIRICAL RESULTS**

The variance ratio tests, reported in Table 2, provide evidence of mean reversion for Turkey (0.89) and Jordan (0.97). The test statistic for Egypt and Morocco hovers around 1, suggesting equity returns are independently and identically distributed, and therefore, $Var[r(k)] = Var[r]$. Earlier results by Darrat and Hakim (1997 and 2000), and Hakim and Neaime (2000) showed that stock prices in Turkey and Morocco followed a random walk and can be modeled as a unit root process.

The estimated return dynamics of the MENA stock markets are provided in Table 3. The highest reverting mean is noted for Turkey, followed by Egypt, then Morocco, with Jordan ranking last. The results for Jordan are not statistically significant, probably a reflection of the fact that Jordanian equities have been drifting consistently downward during the last 4 years, a factor reflected in the only negative reverting mean in our series. Here, one needs to analyze the Jordanian market over a longer time series before any conclusive observations are drawn.

Evidently, one cannot rank these markets based solely on reverting means without consideration to their individual risks. At 1.1 weeks, Turkey also has the highest speed of reversion. This suggests that after an initial shock, returns of Turkish equities are likely to revert the quickest to their equilibrium level. In terms of speed, Morocco ranks second (1.4 weeks), followed by Egypt (2.2 weeks), and Jordan (10.5 weeks).
Table 2
Heteroskedasticity variance ratio tests
MENA stock market returns
for \( i = \text{Egypt, Jordan, Morocco, Turkey}, t = \text{April 95 – May 2000} \)

\[
VR(k) = \frac{\text{Var}[r(k)]}{k \cdot \text{Var}[r]}, \quad R_t(k) = \sum_{i=1}^{k} R_{t-i+1} \text{ continuously compounded weekly returns}
\]

<table>
<thead>
<tr>
<th>MENA Country</th>
<th>Egypt</th>
<th>Jordan</th>
<th>Turkey</th>
<th>Morocco</th>
<th>Emerging Market Index</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k = 5 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>VR(k)</td>
<td>0.99</td>
<td>0.97</td>
<td>0.89</td>
<td>1.07</td>
<td>1.08</td>
</tr>
</tbody>
</table>

Table 3
Generalized method of moments
Estimation of return dynamics
Stochastic differential equation
\[
dR_t(t) = \kappa (\mu - R_t(t))dt + \sigma_i (R_t, t)dz_i
\]
Estimated Vector \( \theta \equiv [\mu, \kappa] \)
\[
H(R, \theta) = dR - \kappa (\mu - R)dt
\]
for \( i = \text{Egypt, Jordan, Morocco, Turkey}, t = \text{April 95 – May 2000 (259 weeks)} \)

<table>
<thead>
<tr>
<th>MENA Country</th>
<th>Egypt</th>
<th>Jordan</th>
<th>Turkey</th>
<th>Morocco</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reverting Mean Return ( \mu )</td>
<td>Coefficient</td>
<td>Std. Error</td>
<td>Coefficient</td>
<td>Std. Error</td>
</tr>
<tr>
<td>Speed of Reversion in weeks ( \kappa )</td>
<td>15.6%</td>
<td>0.001</td>
<td>-10.7%</td>
<td>0.003</td>
</tr>
<tr>
<td>2.2*</td>
<td>0.262</td>
<td>10.5</td>
<td>1.649</td>
<td>1.1*</td>
</tr>
</tbody>
</table>

* Significant at 10%.

Based on these findings, we examine an investment, which exploits the dampened volatility in stock returns under mean reversion. Specifically, we investigate the important question whether investors can ‘trade’ on the mean-reversion property to reduce their portfolio risk exposure, more than under a pure random walk.

To that end, we simulate weekly returns using the reverting mean and speed based on a popular mean reverting model in option pricing by Vasicek (1977):

\[
dR = \kappa (\mu - R)dt + \sigma dz. \tag{8}
\]
Monte Carlo simulations of return dynamics
Vasicek Mean Reverting Model
\[ \text{d}R_i = \kappa_i(\mu_i - R_i)\text{d}t + \sigma_i\text{d}z_i \]

for \( i = \text{Egypt, Jordan, Morocco, Turkey}, \) 10,000 Simulations

<table>
<thead>
<tr>
<th>MENA Country</th>
<th>Egypt</th>
<th>Jordan</th>
<th>Turkey</th>
<th>Morocco</th>
</tr>
</thead>
<tbody>
<tr>
<td>Speed Multiplier 100% - original</td>
<td>Speed</td>
<td>Vol.</td>
<td>Speed</td>
<td>Vol.</td>
</tr>
<tr>
<td>Speed Multiplier 50%</td>
<td>Speed</td>
<td>Vol.</td>
<td>Speed</td>
<td>Vol.</td>
</tr>
<tr>
<td>Speed Multiplier</td>
<td>2.2</td>
<td>23.0%</td>
<td>0.84</td>
<td>16.8%</td>
</tr>
<tr>
<td>Speed Multiplier</td>
<td>4.4</td>
<td>30.3%</td>
<td>1.68</td>
<td>18.2%</td>
</tr>
</tbody>
</table>

A speed multiplier of 50% reduces the speed of reversion in half, and therefore a larger \( \kappa_i \).

For each country, we simulate around its reverting mean \( \mu \) using the volatility \( \sigma \) obtained from the residuals of the GMM regression model in (4). Here, it is important to distinguish between the estimated volatility obtained from model (4) and the standard return volatility computed directly from \( \log \left( \frac{p_t}{p_{t-1}} \right) \). The volatility from model (4) is generally lower because it is computed after mean reversion is taken into account, and therefore, measures the true remaining risk on equities. Ten thousand Monte Carlo simulations are generated which allow us to demonstrate how a stock market risk evolves with the speed of reversion. The simulation is computer intensive. We apply it to all countries. The results are reported in Table 4. For example, using the initial speed of reversion of 2.2 weeks, Egypt’s simulated return volatility is 23%, which corresponds to the actual volatility obtained from the residuals of model (4). At half the initial speed of reversion (speed multiplier 50%), Egypt’s return volatility jumps to 30% as the price cycle drifts longer and away from the reverting mean. Similar results are noted for Morocco and Turkey which all show consistent increases in volatility as the speed of reversion slows. At the same time, the volatility ranking across markets is not affected by the change in the speed of reversion. Turkey remains the most volatile market followed by Egypt and Morocco, regardless of the speed multiplier (100% or 50%). Jordan’s estimated coefficients were not significant and therefore its results are not reliable. Figure 1 of the Appendix shows a histogram of return distribution at the initial speed of reversion. Figure 2 is identical to Figure 1 except for a slower speed, producing a flatter distribution (higher standard deviation). It is clear, therefore, that a lower speed of reversion raises volatility and would have an impact on portfolio allocation and investment decisions.

V. CONCLUSION

This paper analyzed the mean reversion phenomenon in security returns in the MENA region and the implications for asset investment decisions. We use the latest and up-to-
date data from Morgan Stanley between 1995 and 2000 on security prices for four MENA countries Egypt, Jordan, Morocco, and Turkey over a 5-year horizon.

Observing the distribution of stock returns, we find significant and consistent departures from normality in terms of skewness and excess kurtosis which we tested more formally using the Jarque Bera test. Based on the Variance Ratio test, we found evidence for mean reversion in Turkey with mixed results in Egypt, Morocco, and Jordan. Specifically, in Egypt and Morocco, stock returns seem independently and identically distributed (unit roots). The results however are sensitive to the choice of the granularity of return computation. We selected 5 trading days (one week). A more formal test of unit roots is beyond the scope of this study, as the focus of this paper is not on market efficiency but the pattern of equity returns and its risk impact for portfolio investments. Specifically, we argued that mean reversion implies that stock return volatility is lower than what is predicted by a random walk model. To that end, we estimated the return volatility, net of mean reversion, using a variety of popular models widely used in the theory of option pricing. We used a generalized method of moment technique, which has the advantage of being non-parametric, allowing a great deal of flexibility in modeling the pattern of return dynamics over time. The econometric model also provided estimates of the reverting mean and speed of reversion, which we used to generate new returns using Monte Carlo simulations. From the simulated returns, we demonstrated how the volatility of stock returns picks up with a slower speed, as returns on prices have a tendency to wander a longer period away from the reverting mean.

We suggest that that it is important to construct an investment strategy that exploits the dampened volatility in risky asset returns that is brought about by a mean reversion cycle. For example, if one postulates a certain investment horizon with an asset allocation change permitted frequently, it is possible to show that in a standard utility maximization framework, an investor with constant relative risk aversion can generate returns, which outperform a buy-and-hold strategy. Obviously, there would be no particular advantage to changing the initial proportions of a portfolio if prices followed a random walk.

Two immediate benefits emerge from the results of this study. First, our results have an important bearing on the pricing of equity derivatives in MENA. While options in MENA are still limited to over-the-counter trading, the market appetite for the risk-return profile of MENA-based derivatives is strong.

As trading in MENA equity derivatives strengthens, it is important that the pricing models used include an assumption about mean reversion. Without this factor, equity-based derivatives may overstate their true risk. Second, and aside from derivatives trading, our results would be useful for investors employing tactical asset allocation strategies and may demonstrate that, despite their infancy, MENA equity prices exhibit characteristics akin with more mature markets.

NOTES

1. For example Kandel and Stambaugh (1996).
2. The plots are not included here for space consideration but are available from the authors on request.
3. The returns weekly mean and standard deviation are annualized by multiplying each with 52 and square root of 52 respectively.
4. We experimented with \( k=10 \) (biweekly) and \( 22 \) (one month) with similar results.
5. For example, the Vasicek model assumes that the stock market returns are normally distributed, whereas the Cox Ingersoll and Ross model assumes they derive from a non-central chi-square distribution.
6. The heteroskedasticity consistent variance ratio tests are almost identical to the homoskedastic tests except for Egypt. The corresponding Z statistics are respectively 1.01 and 0.99.

REFERENCES


APPENDIX

Figure 1

Distribution of Egyptian Equity Returns
10,000 Monte Carlo Simulations
Mean = 15.6%, Volatility = 23%

Figure 2

Distribution of Egyptian Equity Returns
10,000 Monte Carlo Simulations
Mean = 15.6%, Volatility = 30%