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An Autoregressive Approach of the Arbitrage Pricing Model to the Portuguese Stock Market

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ABSTRACT

We use an autoregressive methodology in relation to the Arbitrage Pricing Model (APT) developed by Mei (1994), applying it, for the first time, to the Portuguese stock market. The results suggest that the number of risk factors explaining stock returns varies with time and that these were not the same, which seems to be strong evidence against the APT. Also, after testing the statistical significance of the factor risk premiums, it seems that we can conclude that the stock returns could be explained by more than one factor and that those factors are priced.

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Key words: APT; Capital asset pricing models; Arbitrage; Risk factors; Autoregressive models.

I. INTRODUCTION

By emphasising the possibility of existing several risk factors, the APT, at least intuitively, seems to be superior to the CAPM. Although there are different approaches for testing the APT, we can identify two major ones: one that uses factor analysis for identifying the factors and the other that defines *a priori* the risk factors using variables with economic meaning.

The recent methodology developed by Mei (1994), does not follow any of those methodologies in order to identify the risk factors, in the context of the APT. The intuition behind this approach is that today's returns can, partially, be explained by yesterday's returns.

This paper is divided into five sections. After this introduction, in a second section, we briefly present the APT. A synthesis of the methodology developed by Mei (1994), our sample and the tests are presented in the third one. The results are presented in the fourth section and the conclusions in the fifth.

II. THE APT MODEL

According to the APT, in equilibrium, the stock returns, or the returns of a group of stocks, in a period t, can be explained by:

$$R_{i,t} = R(E_{0,t}) + (\lambda_{k,t} + f_{k,t})B_{i,k} + \varepsilon_{i,t} \tag{1}$$

where: $R_{i,t}$ is the return on a stock or group of stocks;

$R(E_{0,t})$ is the expected return on a risk free asset;

$\lambda_{k,t}$ are the risk premium factors;

$\beta_{i,k}$ represents the sensitivity of stock i (or group of stocks) to the movements of K factors;

$f_{k,t}$ is a group of random factors with mean equal to zero;

$\varepsilon_{i,t}$ is the specific random variable, also with zero mean, usually known as noise variable.

Some major assumptions are that, at the beginning of the period, the factors are independent; also that the factors and the noise variable are not correlated, as well as the different stocks (i's) (or groups of stocks).

III. METHODOLOGY

3.1 The model

It is assumed that the APT model explains the returns of a stock or groups of stocks. If we can obtain information about β_i (stock sensitivities to the k factors changes), it is possible to estimate (1) using a cross sectional regression. Mei's (1994) contribution is a new methodology for obtaining β_i. So, assuming that the historical returns contain information about β_i and that some parameters are constant across firms, we can use (1) and apply an autoregressive approach.

If:

$$S_t^{'} = \left(S_{1,t},\ldots,S_{k,t}\right) = \left(f_{1,t},\ldots,f_{k,t}\right) + \left(\lambda_{1,t},\ldots,\lambda_{k,t}\right)$$

and

$$B_i^{'} = \left(B_{i,1},\ldots,B_{i,k}\right)$$

then

$$R_{i,t} = \lambda_{0,t} + S_t^{'} B_i^{'} + \varepsilon_{i,t}\,, \quad i=1,\ldots N \quad t=1,\ldots T \qquad (2)$$

In order to obtain information about β_i, we use historical data and equation (1) for each stock i, from t-1 until t-k, and being:

$$S_t = \begin{bmatrix} S_{1,t-1} & \ldots & S_{k,t-1} \\ \ldots & \ldots & \ldots \\ S_{1,t-k} & \ldots & S_{k,t-k} \end{bmatrix}$$

and

$$\pi_{i,t} = \begin{bmatrix} R_{i,t-1} - \lambda_{0,t-1} - \varepsilon_{i,t-1} \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ R_{i,t-k} - \lambda_{0,t-k} - \varepsilon_{i,t-k} \end{bmatrix}$$

Assuming S as non-singular:

$$B_i = S_t^{-1} \pi_{i,t}$$

Substituting in equation (2) for period t:

$$R_{i,t} = \lambda_{0,t} + S_t^{'} S_t^{-1} \pi_{i,t} + \varepsilon_{i,t}\,, \quad i=1,\ldots N \quad t=1,\ldots T \qquad (3)$$

Being

$$\psi_t^{'} = \left(\psi_{1,t} \dots \psi_{k,t}\right) = S_t^{'} S_t^{-1}$$

then

$$R_{i,t} = \psi_{0,t} + \sum_{j=1}^{k} \psi_{j,t} R_{i,t-j} + \eta_{i,t}\ ,\ \ i=1,...N \quad t=k+1,...T \tag{4}$$

where

$$\psi_{0,t} = \lambda_{0,t} - \sum_{j=1}^{k} \psi_{j,t} \lambda_{0,t-j}$$

and

$$\eta_{i,t} = \varepsilon_{i,t} - \sum_{j=1}^{k} \psi_{j,t} \varepsilon_{i,t-j}\ ,\ \ t=k+1,...T$$

For estimating the autoregression (4) we need to use panel data sets. From an econometric point of view, Mei's (1994) methodology is based upon the construction of an autoregressive vector - VAR and upon the model developed by Holtz-Eakin and Newey and Rosen (1988).

The autoregressive vectors have been used mainly in macroeconomic studies, where the time series consisted of a large number of observations [Greene (1993)]. In the microeconomic literature, the studies using VAR are very recent, the first being that from Chamberlain (1983), quoted by [Greene (1993)], which developed a technique allowing the estimates of VAR's using panel data.

Holtz-Eakin and Newey and Rosen (1988), following Chamberlain (1983) and using panel data sets, estimate the coefficients of an autoregressive vector. The developed model also allows the existence of non-stationary individual effects, these authors are concerned with the determination of the number of lag variables, and with the construction of statistical tests.

As already pointed out, for estimating (4) we need panel data sets. Usually the system of equations is formed by a set of time series regressions for different individuals, but here the system is a group of cross section regressions for different time periods.

As can be easily seen in (4), the autoregressive variables are correlated with the transformed error term. This correlation does not enable us to use the normal regression techniques (OLS).

The usual solution implies using instrumental variables and, consequently, the regression methodology 2SLS [Gujarati (1995)]. The Z instrumental variables are not correlated with the error term but are highly correlated with the autoregressors, which means that they are a good proxy of the autoregressive variables [Ramanathan (1995)]. The problem of using the instrumental variables technique is in the identification of those variables, in other words, the variables that will proxy the autoregressors. According to Holtz-Eakin and Newey and Rosen (1988), the variables that should be used as instruments of the autoregressors are the lag variables of those autoregressors.

A necessary condition for the identification of this system of equations is that there are, at least, as many instrumental variables as autoregressive variables.

Another question to bear in mind, when estimating (4), is the heteroscedasticity and the serial correlation in the error term that will cause inefficient estimates of the coefficients.

Following Holtz-Eakin and Newey and Rosen (1988) notation, the model presented in equation (4) will have the following variables:

$$Y_t = \left[R_{1,t}, R_{2,t}, \ldots R_{N,t}\right]',$$

dependent variables forming a vector with NX1 dimensions in a t time period.

$$W_t = \left[e, Y_{t-1}, \ldots, Y_{t-k}\right],$$

independent variables, with e being a vector of ones with dimension Nx1.

$$V_t = \left[\eta_{1,t}, \eta_{2,t}, \ldots, \eta_{N,t}\right]',$$

the transformed error terms.

$$B_t = \left[\psi_{0,t}, \psi_{1,t}, \ldots, \psi_{k,t}\right]',$$

the coefficients that will be estimated, and:

$$Z_t = \left[e, Y_{t-k-1}, \ldots, Y_1\right],$$

the instrumental variables that vary with t.
The equation (4) can be rewritten as:

$$Y = WB + V \tag{4}$$

To estimate B, we need to multiply equation (4) with the transpose of the instrumental variables:

$$Z'Y = Z'WB + Z'V \tag{5}$$

It is now possible to obtain a consistent estimate if we apply the generalised least squares to equation (5). To do so, we need to estimate the covariance matrix, Ω_t, of the transformed errors, $Z'V$.

So, being $\widetilde{B}$ the consistent estimator of B, obtained by doing the 2SLS regression to equation (5), for each time periods and using the correct list of instrumental variables, we get:

$$\widetilde{B}_t = \left[W_t^{'} Z_t \left(Z_t^{'} Z_t \right)^{-1} Z_t^{'} W_t \right]^{-1} W_t^{'} Z_t \left(Z_t^{'} Z_t \right)^{-1} Z_t^{'} Y_t \tag{6}$$

We can obtain the error vector, V_t using the estimates from (6). The consistent estimate of $\left(\Omega / N \right)$ will be given by:

$$\left(\widetilde{\Omega} / N \right)_{r,s} = \sum_{i=1}^{N} \left(\widetilde{v}_{i,r} \widetilde{v}_{i,s} Z_{i,r}^{'} Z_{i,s} \right) \tag{7}$$

where $Z_{i,t}$ is the i[th] line of Z_t and $\widetilde{v}_{i,t}$ (t=r, s) is the i[th] element of $\widetilde{v}_t$ .The matrix $\widetilde{\Omega}$ can be obtained by concatenating the different sub-matrices.

Finally, applying the generalised least square regression method to all observations jointly, we can estimate the coefficients matrix, $\hat{B}$:

$$\hat{B} = \left[W' Z \left(\widetilde{\Omega} \right)^{-1} Z' W \right]^{-1} W' Z \left(\widetilde{\Omega} \right)^{-1} Z' Y \tag{8}$$

A consistent and efficient estimate of B can be obtained following the steps below:

1. Using the 2SLS method and the correct list of instrumental variables, estimate the appropriate equation for each time period.
2. Using the error matrix resulting from step 1, estimate the matrix of the weights $\widetilde{\Omega}$. This matrix is referred by the authors as being a White (1980, 1982) type of matrix, with its diagonal composed by the squares of the error terms.
3. A more efficient estimate of (4) is obtained by calculating the parameters jointly using the generalised least squares technique.

According to Holtz-Eakin and Newey and Rosen (1988), the estimate of the coefficients matrix will be consistent and efficient despite the existence of serial correlation and heteroscedasticity.

3.2 The number of factors to use

The autoregressive approach developed by Holtz-Eakin and Newey and Rosen (1988), can be used to determine the number of factors needed to explain the stock returns [Mei (1994)].

Several empirical studies based upon the APT have been concerned with the determination of the number of factors and some of these studies concluded that it varies with time.

Holtz-Eakin and Newey and Rosen (1988) suggested a test to determinate the number of factors that is quite similar to the Wald's test, quoted by Greene (1993), which is, probably, the usual test for analysing jointly the significance of several coefficients.

The test proposed by Holtz-Eakin and Newey and Rosen (1988) is based upon the residual some of squares - RSS that can be obtained by the following formula:

$$\mathrm{RSS} = \left(Y - W\hat{B}\right)' Z\left(\tilde{\Omega}\right)^{-1} Z' \left(Y - W\hat{B}\right) / N \tag{9}$$

By analogy with the F statistic of the Wald's test, the correct test will be:

$$L = \mathrm{RSS}_{\mathrm{restrict}} - \mathrm{RSS}_{\mathrm{nonrestrict}} \tag{10}$$

where $\mathrm{RSS}_{\mathrm{restrict}}$ is the residual sum of squares of the restricted model and $\mathrm{RSS}_{\mathrm{nonrestrict}}$ is the residual sum of squares of the non-restricted model. L follows a χ^2 distribution with degrees of freedom equal to the degrees of freedom of the $\mathrm{RSS}_{\mathrm{nonrestrict}}$ minus the degrees of freedom of the $\mathrm{RSS}_{\mathrm{restrict}}$. The degrees of freedom of the RSS can be obtained by estimating the difference between the number of instrumental variables and the number of parameters [Holtz-Eakin and Newey and Rosen (1988)]. If L is higher then the χ^2 critical value, then we reject the null hypothesis.

3.3 Identification of significant changes between the coefficients of two groups of stocks

One of the assumptions of APT is the fact that the factors, which explain the stock returns, must be common among those stocks. So, if we divide the whole sample of returns into two sub-samples and estimate (4), we expect to find no significant differences between their estimated coefficients. Then, if

$$R_{i,t} = \psi_{0,t} + \textstyle\sum_{j=1}^{k} \psi_{j,t} R_{i,t-j} + \eta_{i,t} \quad \text{with i=1,...N and t=k+1,...T}$$

explains the return on a group of stocks in a period t, and

$$R_{i,t} = \omega_{0,t} + \textstyle\sum_{j=1}^{k} \omega_{j,t} R_{i,t-j} + \eta_{i,t} \quad \text{with i=1,...N and t=k+1,...T}$$

explains the returns on another group of stocks for the same time period, we expect not to reject the hypothesis:

$$H_0 : \omega_{j,t} = \psi_{j,t} .$$

To test H_0 we will use the Wald's test. Let ψ_1 be the vector of coefficients resulting from estimating (4) for the first panel data set and ω_2 the vector of coefficients for the second panel data set. The variance - covariance matrix of each vector of coefficients will be given by:

$$\Theta = \mathrm{Var}\left(\hat{B}\right) = \left[W' Z \left(\tilde{\Omega}\right)^{-1} Z' W \right]^{-1} \tag{11}$$

The variance-covariance matrix of each B's vectors, ψ_1 and ω_2 will be, respectively, Θ_1 and Θ_2. The total variance-covariance matrix will then be:

$$\Theta_{total} = \Theta_1 + \Theta_2 \tag{12}$$

If (4) is well specified, then, there will be no correlation between ψ_1 and ω_2, since the error terms are not correlated [Mei (1994)]. The Wald's statistic will then be:

$$W = \left(\psi_1 - \omega_2\right)' \Theta_{total}^{-1} \left(\psi_1 - \omega_2\right) \tag{13}$$

where W has a χ^2 distribution with degrees of freedom equal to the product between the number of estimated coefficients and the number of time periods contained on the panel data set.

3.4 The statistical significance of the coefficients

Some authors [e.g., Shanken (1982)] believe that, although some studies provide empirical evidence that the stock returns can be explained by more than one risk factor, those risk factors are not priced, which means that they are not economically significant. This argument is based upon the idea that, in the economy, if an investor prefers risk, by taking risky decisions, then he demands a risk premium. So, if any factor is not priced, then it is not fundamental in explaining the return on a stock and should be excluded. According to this argument, although some studies concluded that there exist several risk factors, which explain the stock returns, eventually, those factors are not significant.

Following this idea, the risk premiums, $\lambda_{j,t}$, from equation (1) should be different from zero, at least for some j's and some t's.

We will test the hypothesis $H_0 : \lambda_{j,t} = 0$ for all j's and all t's. If the null hypothesis is true, then the non-conditional average of the historical coefficients $\psi_{j,t}$ of equation (4) will equal zero.

Using lagged variables as instruments to determine $\psi_{j,t}$, we will need to widen the estimating interval of the $\psi_{j,t}$'s, so that we can assure that the $\psi_{j,t}$'s are independent which, in turn, will allow us to use the t test.

We will determine the sample average and the standard error of each $\psi_{j,t}$, and then will calculate the t statistic with degrees of freedom equal to N-1 with N being the number of panel data sets.

3.5 The sample

Our sample consists of monthly returns, adjusted to dividends, of around 100[2] traded stocks on the Portuguese stock market, between January 1985 and December 1996.

We assumed that the sufficient number of factors that can explain the stock returns would be eight plus the constant. We did that because Mei (1994) limits this number to seven and concludes that, for some periods, at least seven factors were necessary. The ideal will be to start with a higher number of factors, but as we increase this number and, consequently, the number of instrumental variables needed to estimate (4), the error covariance matrix grows proportionally - the number of instruments times the number of periods - which means that it grows quite rapidly, causing problems in inverting $\widetilde{\Omega}$.

Due to the fact that we used eight historical returns as independent variables, so that we need to use, at least, the same number of instruments plus the constant [Holtz-Eakin and Newey and Rosen (1988)], we decided to use nine instrumental variables plus the constant.

Following Mei's (1994) methodology, we will estimate (4) for each panel data set. So, keeping in mind what was assumed above, we can only include a stock in a panel if it was traded continuously over the year, which will be studied as well as in the seventeen previous months.

After the construction of the panel data sets, our sample was reduced to 82 stocks.

IV. RESULTS

4.1 Analysis of the autoregression results

We obtained the coefficient estimates plus the constant for the years 1987 until 1996 after applying equation (4) for all the panel data sets.

In each panel we run two regressions: one for the first six months, using the preceding seventeen monthly returns plus the constants as independent and instrumental variables and another, for the last six months of each panel, using the same methodology for choosing the variables.

With the objective of estimating the errors that will be used for the construction of the error covariance matrix, we used 2SLS for each month independently. After obtaining the errors and constructing $\tilde{\Omega}$, we obtained the final estimates using the GLS methodology.

With the objective of analysing the statistical significance of the coefficients we performed a t test and concluded that the coefficients were statistically significant.

4.2 Determination of the number of autoregressive variables needed to explain the stock returns

As we mentioned immediately before, the methodology proposed by Holtz-Eakin and Newey and Rosen (1988), for determining the number of autoregressive variables used to explain the stock returns is similar to the Wald's test.

After obtaining the results from applying equation (4), for all the specifications of the number of variables to use, which meant, running one hundred and sixty regressions, keeping unchanged the error covariance matrix as well as the instrumental variables matrix, we determined the residual sum of squares for each specification. Then, we calculated the difference between the RSS of a given K specification and the RSS of the K-1 specification. The obtained value, L, is quite similar to the F statistic of the Wald's test [Holtz-Eakin; Newey and Rosen (1988)].

The results for 1996 are shown in Table 1. Several specifications of K are presented in the first column of Table 1. The non-restricted model, K=8, is tested against the alternative hypothesis, K=7, if the hypothesis is not rejected, then K=7 will be tested against K=6 and so on, until we reject the hypothesis.

The rejection will occur when L is higher than the χ^2 critical value (L follows a χ^2 distribution) [Holtz-Eakin and Newey and Rosen (1988)]. The RSS degrees of freedom are obtained by the difference between the number of instrumental variables, including the constant, and the number of estimated coefficients, multiplied by the number of time periods in the panel. For K=8, we estimated nine coefficients and we used ten instrumental variables. As in each panel we estimated six months, the number of degrees of freedom will be six [(10-9) X6]; the same principle was applied to all remaining specifications of K.

The L statistic was estimated as the difference between the RSS of the restricted model and the unrestricted one and its degrees of freedom by the difference between the degrees of freedom of the restricted model and the unrestricted one.

Table 1
Results for the year 1996

1996 - 1st Half				
Nº of Independent Variables	RSS	Degrees of Freedom	L	Degrees of Freedom
K=8	211,9230	6		6
K=7	215,3846	12	3,4615	6
K=6	226,1538	18	10,7692	6
K=5	267,8846	24	41,7307	6
K=4	269,8076	30	1,92307	6
K=3	269,8076	36	0	6
K=2	296,5384	42	26,7307	6
K=1	443,8461	48	147,3076	6
1996 - 2nd Half				
Nº of Independent Variables	RSS	Degrees of Freedom	L	Degrees of Freedom
K=8	253,2692	6		6
K=7	316,9230	12	63,6538	6
K=6	317,3076	18	0,3846	6
K=5	353,6538	24	36,3461	6
K=4	353,6538	30	0	6
K=3	411,3461	36	57,6923	6
K=2	420,5769	42	9,2307	6
K=1	423,8461	48	3,2692	6

For a 5% significance level and six degrees of freedom, the χ^2 critical value was 12.542, when testing the specification K=7 against K=8, for the first half of 1996 L equals 3.462. So, the hypothesis K=7 is not rejected since L is lower than the χ^2 critical value. The same happens when K=6 is tested against K=7, but when K=6 is tested against K=5, the hypothesis is rejected, showing that, in that period at least six factors are required to explain the stock returns.

The specification K=7 is rejected against K=8, for the second half of 1996, which implies that, in that period, at least eight factors are required to explain the stock returns.

In Table 2 we present a summary of our results for all panels. Analysing Appendix 1 it seems that we can conclude that the number of factors varies with time, which in accordance with Mei's (1994) study. This variability occurs also within the same year; for example, in the first half of 1987, at least eight factors are required and in the second three or more.

At least 7 factors seem to be required for most of the years. This conclusion seems to contradict some studies, like the one of Roll and Ross (1980), who concluded that five factors were sufficient to explain the stock returns.

Table 2
Number of factors per data panel

	K=1	K=2	K=3	K=4	K=5	K=6	K=7	K=8[3]
1st Half 1987							R	A
2nd Half 1987		R	A	A	A	A	A	A
1st Half 1988	A	A	A	A	A	A	A	A
2nd Half 1988	A	A	A	A	A	A	A	A
1st Half 1989				R	A	A	A	A
2nd Half 1989						R	A	A
1st Half 1990						R	A	A
2nd Half 1990							R	A
1st Half 1991							R	A
2nd Half 1991						R	A	A
1st Half 1992							R	A
2nd Half 1992							R	A
1st Half 1993						R	A	A
2nd Half 1993					R	A	A	A
1st Half 1994							R	A
2nd Half 1994						R	A	A
1st Half 1995						R	A	A
2nd Half 1995							R	A
1st Half 1996					R	A	A	A
2nd Half 1996							R	A

4.3 Comparing two groups of stocks within the same time period

One of the assumptions of the APT is that the factors explaining the stock returns are common. So, if we split our sample up into two sub-samples, at least in theory, we expect to find no significant differences between the two in terms of estimated coefficients.

We performed this test for all K specifications for the 1996 data. The results obtained after running the regression in relation to equation (4) for the two sub-samples, A and B, for the first half of 1996 and for six, seven and eight factors, can be seen in Table 3.

Table 3
Results in relation to the 1st half of 1996

K=6	ψ_0	ψ_1	ψ_2	ψ_3	ψ_4	ψ_5	ψ_6
Sample - A	0.04732	-0.05368	-0.8088	-0.2298	0.07179	0.1031	0.5548
Sample - B	0.02769	-0.3829	0.05071	0.5086	0.3789	0.7613	-0.3155
Wald's Statistic				520.5			

K=7	ψ_0	ψ_1	ψ_2	ψ_3	ψ_4	ψ_5	ψ_6	ψ_7
Sample - A	0.04042	-0.2353	-0.9079	-0.3866	0.321	0.004588	0.7473	-0.7196
Sample - B	0.04836	-0.4252	-0.1408	0.4405	0.4562	0.771	-0.2022	0.3361
Wald's Statistic				582.1				

K=8	ψ_0	ψ_1	ψ_2	ψ_3	ψ_4	ψ_5	ψ_6	ψ_7	ψ_8
Sample-A	0.00589	0.3372	-0.4472	0.1204	0.3277	0.3101	-0.2945	-0.7418	-0.7691
Sample-B	0.0724	-0.6273	-0.1073	0.3021	0.5254	0.8391	-0.04973	0.3421	0.476
Wald's Statistic					420.1				

The Wald's statistic has a χ^2 distribution with degrees of freedom equal to the number of coefficients multiplied by the number of time periods [Greene (1993)]. For six factors it has forty-two degrees of freedom. Considering a 5% significance level, the χ^2 critical value is 52.124; as the Wald's statistic has a value of 520.5, the null hypothesis of equality between the coefficients is strongly rejected. When we repeated the test for seven and eight factors the results were identical.

In Table 4, we present the results for the second half of 1996. As in the previous section, we concluded that at least, eight factors are required for explaining the stock returns so that we only performed the test for that specification.

Table 4
Results in relation to the 2nd half of 1996

K=8	ψ_0	ψ_1	ψ_2	ψ_3	ψ_4	ψ_5	ψ_6	ψ_7	ψ_8
Sample A	0.006276	-0.8119	-0.02677	-0.1757	0.125	0.2338	-0.1608	-0.01248	-0.1369
Sample B	0.02104	0.8959	-0.7277	0.9515	-0.612	0.3017	-0.8143	-0.7065	-0.09622
Wald's Statistic					607.9				

The χ^2 critical value, considering fifty-four degrees of freedom, is 72.153, so once again we rejected the hypothesis that the vectors of coefficients are equal. This means that the factors explaining the stock returns, in 1996, seem to be different between the two halves.

4.4 Statistical evaluation of the risk premiums

We estimated if the risk premiums in equation (1), $\lambda_{j,t}$, are different from zero, at least for some stocks and for some time periods. With this in mind, we tested the hypothesis: $H_0 : \lambda_{j,t} = 0$, for all j's and all t's.

We only studied four panel data sets in order to assure that the data were not overlapped, so that the coefficients, $\psi_{j,t}$, were independent, allowing the use of the t test. We performed the regression for the years 1990, 1992, 1994 and 1996, firstly for the first half using the seventeen earlier months as independent and instrumental variables and then, for the second half of each year, using the same methodology.

The only K specification that is common between the panels is K=8, so we decided to work only with that specification. Then, we obtained seventy-two coefficients (including the constants).

To determine the t statistic, we calculated the sample mean for each coefficient,

$$\psi_{mean} = \frac{\psi_{k,t}}{N},$$

where $\psi_{k,t}$ is the K coefficient for the tth half, N the number of panel data sets and the sample variance [Gujarati (1995)],

$$Var_{mean} = \frac{1}{N^2} \sum_{k=1}^{K} Var_k$$

So we obtained eighteen coefficients and average variances (including the constants).

The critical value of t, with a 5% significance level and three degrees of freedom, is 3.182, which implies the rejection of the null hypothesis for all the mean coefficients, except for the third of the second semester.

The data strongly rejects the hypothesis that the risk premiums are equal to zero. This conclusion is different from that of Mei (1994) whose results did not reject that hypothesis so strongly.

V. CONCLUSIONS

We applied the methodology proposed by Mei (1994) to a sample of Portuguese stocks, covering the period between August 1985 and December 1996.

Our data was divided into twenty panel data sets. The analysis of each panel allowed us to investigate questions concerning the variability of the number of factors explaining the stock returns. We concluded that the number of factors seems to vary with time, with usually K=8.

When we divided the sample into two sub-samples, in a certain time period, in order to compare their significant factors, we concluded that they were different, which seems to mean that, unlike what would be expected in theoretical terms, the factors were not the same (common).

When we tested the risk premiums, we concluded that they were significantly different from zero, which is in opposition with some of the authors who argue that, although the stock returns can be explained by more than one factor, those factors are not priced and so are not important.

As far as future work is concerned we think that the introduction of other instrumental variables could be interesting, since we only used historical variables. Probably, the explanation of the stock returns could be improved with the introduction of variables like the P/E.

NOTES

1. The authors would like to acknowledge Professor Dean Paxson from Manchester Business School for useful comments.
2. This includes all the stocks on the BVL "official market" which were continuously traded for the time period.
3. R means that the hypothesis has been rejected and A means that the K's specification has been accepted.

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