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Efficiently Estimated Mean and Volatility Characteristics for the Nordic Spot Electric Power Market

P.B. Solibakke
Department of Business Administration and Economics,
Molde College, P.O.Box 308, N-6401 Molde, Norway

ABSTRACT

This paper investigates the conditional mean and volatility characteristics of the Nordic Spot electric power market. The investigation is motivated by the fact that the system price is the underlying instrument for several derivatives in the electric power market, the variance-covariance matrix may be applied for value at risk calculations and consumption patterns may suggest extensive predictability in mean and volatility. An adjustment procedure shows that the raw data series show strong day, month and scaling effects. The applied BIC efficient ARMA-GARCH-in-Mean specifications for the adjusted time show close to zero drift and an autocorrelation pattern for the conditional mean, suggesting consumption patterns. As expected, the in-Mean parameter is redundant and clearly significant ARCH and GARCH effects in the conditional volatility process. All specification tests reject data dependence in the residuals. Our results therefore suggest that a BIC efficient ARMA-GARCH lag specification seems to model the market dynamics adequately.

JEL: C22, C52

Keywords: *Electric power market; System price; Consumption pattern; Conditional heteroscedasticity; Supply and demand curves*

I. INTRODUCTION

This paper studies the characteristics of the conditional mean and volatility of daily price changes of the so-called System Price for the Nordic spot electric power market. The spot market is a Nordic contract market where electric power is traded on a daily basis for delivery the following day, with full obligation to pay. The prices are fixed on the basis of all participants' collected purchase and sale requests. The System Price is the balance price for the aggregated supply and demand graphs; i.e. the price is fixed at the market equilibrium. Hence, our investigation is an empirical investigation of the dynamics of the so-called system price series.

The motivation is based on the fact that the price series is the underlying instrument for several derivatives in the electric power market. A first glance inspection of the series suggests a consumption pattern in the mean and a very high, changing and mean reverting volatility. Consequently, the electric power market may benefit strongly for a higher understanding of both mean and volatility characteristics. The consumption pattern may show serial correlation in the mean, which may suggest predictability in the mean process. Moreover, when valuing derivatives in financial markets, forecasts of volatilities and correlations over the whole life of the derivative are usually required. The system price change series seems to show several extreme observations. The distribution may therefore show signs of heavy tails suggesting leptokurtosis in the time series. The deviation from the normal distribution may be an important factor to account for when valuing options especially. Finally, as the mean-variance analysis we perform can calculate the whole variance-covariance matrices, value at risk calculations may be employed. We employ elaborate specification test statistics for model misspecifications. Insignificant test statistics suggest appropriate specifications and consequently an appropriate model for exogenous variable analyses in both the mean and the latent volatility series.

The empirical investigation is performed allowing serial correlation and changing volatility models. Specifically, we apply an ARMA-GARCH-in-Mean lag specification[1], where the ARMA lag specification models the mean and the GARCH lag specification models the latent volatility process. The in-Mean specification models total (residual) risk mean effects. Then lags are Bayes Information Criterion (BIC) (Schwartz, 1978) preferred in both mean and volatility. Hence, the ARMA-GARCH specification pertains to model the first observed series characteristics. Consequently, the changing volatility model seems therefore to be an obvious candidate as the stochastic volatility specification fails to find a BIC preferred model. The model class is a univariate time series investigation where simple transformation of squared returns serves as the driving force behind volatility. Consequently, the applied ARMA-GARCH lag specification may give new and interesting information regarding the characteristics of the mean and volatility processes in the electric power market. We specify the ARMA-GARCH models to report serial correlation, total (residual) risk, and leptokurtic distribution symptoms in the conditional mean and shocks, serial correlation, persistence, asymmetry and mean reversion to long-run average levels for the conditional volatility. Our main objective is therefore to analyse mean and volatility dynamics in a univariate estimation context that controls for consumption patterns and volatility clustering.

We believe that the contribution of this paper is a higher understanding of the workings of return and volatility processes in the electric power market. Firstly, the specifications seek consistent and significant coefficients in the conditional mean. Consumption patterns may therefore suggest predictability in the time series. Secondly, consistent and significant shocks, serial correlation and persistence effects and mean reversion to long-run average volatility may show new and unfamiliar volatility characteristics. Thirdly, as the ARMA-GARCH model applies a student-t distributed log-likelihood function, the estimated degree of freedom parameter may measure the degree of leptokurtosis and therefore non-normality in model residuals. Fourthly, we follow an expansion path starting from adjusted raw returns and eventually specify both ARMA (mean) and GARCH (volatility) lag specifications for the price change series applying the Schwarz BIC (Schwarz, 1978) information criterion for the lag specifications. These lag structures may suggest predictability in both the mean and volatility series. Fifthly and finally, to obtain measures for model misspecifications, we employ several elaborate test statistics focusing on both the mean and the volatility. A joint bias test measures signs of biases in conditional volatility prediction embodied in the GARCH specification.

The remainder of this paper is therefore organised as follows. Section 2 gives a brief overview of the Nordic power exchange, Nord Pool and the so-called system price. Section 3 gives a literature overview of ARMA-GARCH models. Section 4 defines the data and describes the general adjustment procedure for systematic location and scale effects obtaining a stationary time series. The adjustments and effects are reported. Section 5 reports the empirical results of the ARMA-GARCH-in-Mean estimations. Section 6 reports our findings and discusses any policy implications based on the investigations. Finally, Section 7 summarises and concludes.

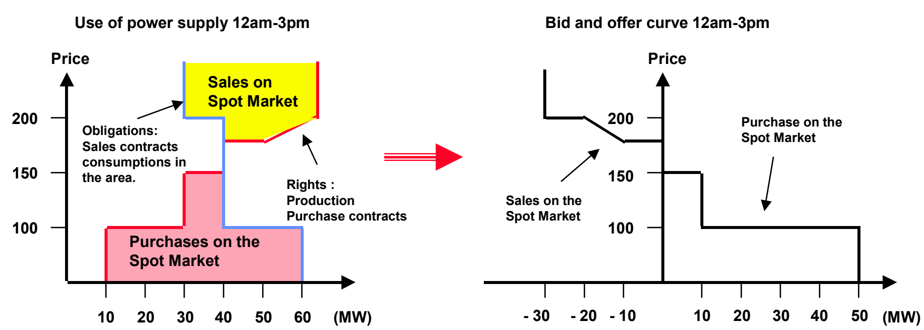
II. THE NORD POOL POWER EXCHANGE

Every week Nord Pool distributes information to all market participants on which bidding areas that apply for the following week based on data from the system operator in Norway, Statnett SF. The participants will set up a plan for how their own production, contractual rights and obligations area are to be dealt with for all hours the next day. Spot sales and purchases will form an integral part of this plan. Based on the plan, a price-differentiated bid and offer for each bidding area and each hour will be set up. The bid or offer will show purchase/sale quantities at different prices. Participants will not know the price or their own trades before all participants have sent in their bids and the price has been calculated. Once the price has been fixed, however, each participant will receive an exchange quantity that will always correspond to that participant's price-differentiated bid or offer.

In the example[2] shown in Figure 1, it is assumed that a participant has price dependent and non-price dependent obligations for a total of 60 MegaWatt (MW). Of the price dependent obligations 20 MW has a disconnection price of 100 Norwegian

Kroner (NOK) per MegaWatt hour (MWh) and 10 MW a disconnection price of 200 NOK/MWh. The non-price dependent obligations are 30 MW. For prices below 100 NOK/MWh, the participant chooses to use 10 MW through power from bilateral contracts or its own production, and to purchase the remaining 50 MW on the Spot Market. At prices between 100 and 200 NOK/MWh, the participant will reduce its purchasing and use steadily more of its own production capacity and purchasing contracts. At prices over 175 NOK/MWh, the participant will sell. Sales reach their peak at 30 MW at prices over 200 NOK/MWh. The participant sets up a corresponding bid for all the hours he wishes to report a bid. The bid or offer can be made valid for the entire week or for several single days. New bids or offers may be sent in daily. The latest bid or offer received for each hour is the one used in calculating the price and exchange quantity. The prices in the bid and offer form are treated as breakpoints on a continuous bid and offer curve, with linear interpolation between the points. The bids are sent to Nord Pool via fax or electronically in standard exchange format (Electronic data communication, EDK, or as an Internet-application called Elweb).

Figure 1
Bid and ask for electric power



Aktør : Aker Energiverk
Week : 10/95
Area : B

Bid/offer valid for : (cross off)

Mon.	Tues.	Wed.	Thurs.	Fri	Sat.	Sun.	Whole week
			x				

Time From - To	*0*	*100*	*101*	*150*	*151*	*175*	*176*	*200*	*201*	*900*
12-15	50	50	10	10	0	0	-10	-20	--30	-30

The participants' overall bids and offers are grouped together on an offer graph (sale) and a demand graph (purchase). The price is set as the balance price at the intersection of offers and demands (the equilibrium point). The system price (P_s) is calculated first, without constraints in the national grid. The so-called system price is our major interest in this study.

III. THE ARMA-GARCH-IN-MEAN METHODOLOGY

Non-linear stochastic models will in our study imply conditional models and so-called ARMA-GARCH methodology. Autoregressive and moving average (ARMA) are terms applied to the structure of the conditional mean, while generalized autoregressive conditional heteroscedasticity (GARCH) is a term applied to the structure of the conditional volatility. Note that in financial markets the ARMA usually models non-synchronous trading and the GARCH models conditional heteroscedasticity or volatility clustering[3].

ARMA models can be studied in details in for example Mills (1990), while ARCH specifications were first studied by Engle (1982), and moved further on by Bollerslev (1986) who specified the Generalized ARCH or GARCH. The development to GARCH from ARCH was done preliminary owing to the number of lags in the ARCH specification[4]. ARCH/GARCH specifies the volatility as a function of historic price changes and volatility. In the international literature of finance quite a number of studies have shown the use of results from these pioneer works. See for example Bollerslev et al. (1987,1992), Engle et al. (1986, 1993), Nelson (1991), Weiss (1986) and deLima (1995a, 1995b). For a comprehensive introduction to ARCH models and applications in finance see Gouriéroux (1997).

In univariate and simplest possible form we specify ARMA (p,q)-GARCH(m,n)-in-Mean models through the following equations (1)-(4):

$$y_t = \phi_0 + \sum_{i=1}^{p} \phi_i \cdot y_{t-i} + \rho_1 \cdot \sqrt{h_t} + \upsilon_t \quad (1)$$

$$\nu_t = \varepsilon_t + \sum_{j=1}^{q} \theta_j \cdot \varepsilon_{t-j} \quad (2)$$

$$\varepsilon_t \sim N(0, h_t) \text{ and } D(0, h_t, \omega) \quad (3)$$

$$h_t = \alpha_0 + \sum_{i=1}^{m} \alpha_i \cdot \varepsilon_{t-i}^2 + \sum_{j=1}^{n} \beta_j \cdot h_{t-j} \quad (4)$$

Equation (1) is the structural mean specification (for both linear and non-linear models), where y is price change and y_{t-i} is lagged price changes, Equation (2) defines moving average which is modeled by measuring lagged residuals effect on price changes, Equation (3) defines the distribution of the residuals (ε_t); that is a normal distribution *N0)* or a student-t distribution *D0)* with ω degrees of freedom and Equation (4) specifies the structural form of the conditional volatility (h_t). ϕ is the vector for lagged price changes (the AR-process), θ is the vector for lagged residuals (the MA-process), α_0 is a parameter for the weight to the long-run average volatility, α_t is the vector for the weights of the lagged and squared residuals ε^2_{t-i} (the ARCH-process) and β is the weights for the lagged conditional volatility h_{t-j} (the GARCH-process).

From these specifications we obtain three features for our models. Firstly, conditional homoscedasticity[5] for the residuals are explicitly modeled. Secondly, the important serial correlation in the mean is modeled. Thirdly, as ARMA-GARCH applies a maximum likelihood algorithm we are able to model the kurtosis and skew (leptokurtosis[6]) by applying student-t distributed likelihood functions[7] with the degree of freedom estimated by the model.

Ding et al. (1993) extends the symmetric GARCH model into asymmetric GARCH. Asymmetric GARCH (AGARCH) models the volatility as Equation (5):

$$h_t = \alpha_0 + \sum_{i=1} \alpha_i \cdot (|\varepsilon_{t-i}| - \gamma_i \cdot \varepsilon_{t-i})^{\delta} + \sum_{j=1} \beta_j \cdot h_{t-j} \tag{5}$$

where α_t is the vector for the weights of the lagged residuals ε^{δ} (the ARCH-process). For the classical asymmetric model we define $\delta=2$, while in "power" AGARCH model we also estimate δ. It is γ_i that measures asymmetry in the volatility. Especially one model has been applied many times in the international finance literature. The truncated GARCH (GJR) (Glosten et al, 1993) specifies the volatility as in Equations (6)-(7):

$$\lambda_{it} = \gamma_i \quad \text{if and only if} \quad \varepsilon_{t-i} < 0 \tag{6}$$

$$h_t = \alpha_0 + \sum_{i=1} (\alpha_i + \lambda_{it}) \cdot \varepsilon^2_{t-i} + \sum_{j=1} \beta_j \cdot h_{t-j} \tag{7}$$

If $\lambda_{it} > 0$, the GJR specification will generate higher values for h_t when $\varepsilon_t < 0$ then when $\varepsilon_t > 0$; otherwise equal in absolute size. The Exponential GARCH model (EGARCH) (Nelson, 1991) specifies the volatility by using the natural logarithm. The EGARCH model specifies the volatility as in Equations (8)-(9):

$$\varepsilon_t \sim \sqrt{h_t} \cdot \upsilon_t \tag{8}$$

$$\ln h_t = \beta_0 + \sum_{j=1} \beta_j \cdot \ln h_{t-j} + \sum_{i=1} \gamma_i \cdot (\theta_0 \cdot \upsilon_{t-i} + \gamma_0 \cdot \{|\upsilon_{t-i}| - E|\upsilon_t|\}) \tag{9}$$

Equation (8) shows the distribution of the residuals[8] and Equation (9) shows the structure in the conditional volatility. θ_0 in (8) defines the asymmetric volatility and υ

measures the thickness of tails in the distribution. Note that for all the GARCH specifications we require that $\sum_{i=1}^{m}\alpha_i + \sum_{j=1}^{n}\beta_j < 1$, with the exception of EGARCH. EGARCH requires that $\beta_j < 1$.

M (aximum) L (ikelihood) estimates of the GARCH-in-Mean model can be obtained by maximising the likelihood function using the BHHH[9] algorithm. Note however, that the information matrix is no longer block diagonal, so that all the parameters must be estimated simultaneously. This requires an iterative solution technique[10], also known as non-linear optimisation.

IV. EMPIRICAL DATA AND ADJUSTMENT PROCEDURES

The study uses daily price changes of the so-called system price in the Nordic spot market for electric power spanning the period from October 1992 to January 2000. The daily prices are the average prices for 24 hours system prices.

We adjust for systematic location and scale effects (Gallant, Rossi and Tauchen, 1992) in all time series. The log first difference of the price index is adjusted. Let ϖ denote the variable to be adjusted. Initially, the regression to the mean equation $\varpi = x \cdot \beta + u$ is fitted, where *x* consists of calendar variables, which are most convenient for the time series and contain parameters for trends, week dummies, calendar-day separation variable, month and sub-periods. To the residuals, $\hat{u}$, the variance equation model $ln(\hat{u}^2) = x \cdot \gamma + \varepsilon$ is estimated. Next $\frac{\hat{u}}{\sqrt{e^{x \cdot \hat{\gamma}}}}$ is formed, leaving a series with mean zero and (approximately) unit variance given *x*. Lastly, the series $\hat{\varpi} = a + b \cdot (\frac{\hat{u}}{\sqrt{e^{x\hat{\gamma}}}})$ is taken as the adjusted series, where *a* and *b* are chosen so that $\frac{1}{T} \cdot \sum_{i=1}^{T}\hat{\varpi}_i = \frac{1}{T} \cdot \sum_{i=1}^{T}\varpi_i$ and $\frac{1}{T-1} \cdot \sum_{i=1}^{T}(\hat{\varpi}_i - \overline{\varpi})^2 = \frac{1}{T-1} \cdot \sum_{i=1}^{T}(\hat{u}_i - \overline{u})^2$. The purpose of the final location and scale transformation is to aid interpretation. In particular, the unit of measurement of the adjusted series is the same as that of the original series. We report[11] the result of these raw data series adjustments in Table 1.

The adjustments are made for both the mean (column 2 and 3) and the volatility (column 4 and 5). The factors we adjust for are described in the lower part of Table 1. The results show clear price change patterns over the week and especially during the weekend. The price change is markedly positive on Mondays and strongly negative on Saturdays. This may be explained by highly electric power intensive industry production from Monday to Friday. We find no monthly patterns in the series. The volatility series show patterns for both days and months. As expected from the price change the volatility increases strongly Mondays and Saturdays. Moreover, the volatility is especially strong during the May-July period. Hence, production planning to obtain highest possible income will in this period probably be strongly appreciated.

Finally, we find a negative volatility trend. This may show sign of a market that matures. However, to obtain stationary series the trend is taken out of the series.

Table 1
Data adjustment coefficients for system price changes

	Raw Price Change		Ln (residual²)	
	Coefficient	t-value	Coefficient	t-value
INTR	-0.6449	{0.3447}	2.0537	{5.4978}
MOND	12.4631	{13.3225}	2.0619	{11.8131}
TUES	2.1781	{2.3286}	0.7673	{4.3963}
THUR	-0.3865	{0.4132}	-0.0189	{0.1080}
FRID	-1.9503	{2.0848}	0.2216	{1.2696}
SAT	-8.0122	{8.5589}	1.6187	{9.2674}
SUN	-2.7133	{2.8984}	0.3838	{2.1976}
JAN_1	1.4734	{0.5694}	0.2159	{0.4470}
JAN_2	-0.5180	{0.2002}	-0.1728	{0.3577}
JAN_3	0.2211	{0.0854}	0.1172	{0.2425}
JAN_4	1.0791	{0.4550}	-0.1839	{0.4153}
FEBR	0.1625	{0.0811}	-0.4073	{1.0886}
MAR	-0.1886	{0.0951}	-0.2634	{0.7114}
APR	-0.2465	{0.1239}	0.3488	{0.9390}
MAY	-0.6726	{0.3433}	1.1702	{3.1993}
JUN	0.9972	{0.5080}	0.8537	{2.3296}
JUL	-0.5007	{0.2558}	0.9887	{2.7066}
AUG	1.5847	{0.8097}	0.4267	{1.1683}
SEPT	1.2395	{0.6314}	0.0408	{0.1114}
OCT	1.0111	{0.5166}	-0.0079	{0.0216}
NOV	0.8183	{0.4169}	-0.6162	{1.6822}
DEC_1	0.7187	{0.2875}	-0.2715	{0.5821}
DEC_2	0.6006	{0.2403}	-0.1705	{0.3655}
DEC_4	0.0560	{0.0243}	-0.3413	{0.7937}
TRD	----	----	-2.1468	{3.2915}
TRD2	----	----	1.1500	{1.8207}

Intr=Constant; Mond=Monday; Tues=Tuesday; Thur=Thursday; Fri=Friday; Sat=Saturday; Sun=Sunday; Jan_1=01-07 January; Jan_2=08-15 January; Jan_3=16-23 January; Jan_4=24-31 January; Febr=February; Mar=March; Apr=April; May=May; Jun=June; Jul=July; Aug=August; Sept=September; Oct=October; Nov=November; Dec_1=01-07 December; Dec_2=08-15 December; Dec_4=24-31 December; Trd=Trend; Trd2=Squared Trend.

The characteristics of the change in the so-called system price as a raw series and as an adjusted series are reported in Table 2. The following immediate observations can be extracted from Table 2. The mean and standard deviation are equal as advocated. The mean is positive but close to zero. The standard deviation seems high relative to other raw material markets. The numbers for daily maximum and minimum are moved in the negative direction. The distance between maximum and minimum shows a small increase from the raw to the adjusted series. The kurtosis has increased and the skewness has become more negative from the raw to adjusted series. Hence, the adjusted series is far from normally distributed which is confirmed by the Kolmogorov-Smirnov Z-test statistic[12] (K-S Z-test). The adjusted series show a strong decline in autocorrelation in both ordinary (Q (6)) and squared series (Q^2 (6))[13], but both are still clearly significant. The ARCH (12) test statistic (Engle, 1982) suggests conditional heteroscedasticity and the RESET (12; 6) (Ramsey, 1969) suggests non-linearity in the mean in both series. Finally, the BDS test statistic (Brock and Deckert, 1988 and Scheinkman, 1990) suggests general non-linear dependence in both series. We report the system price and the raw and adjusted change in the system price in Figure 2 and 3, respectively.

Table 2

Raw and adjusted system price characteristics

	Mean/ Std. Dev.	Maximum/ Minimum	Kurtosis/ Skew	Q (6)/ Q^2 (6)	K-S Z-test	ARCH (12)	RESET (12; 6)	BDS m=2; ε=1
Raw	0.0275	250.402	101.902	350.994	9.7668	883.381	219.343	19.0783
Series	14.410	-283.685	-0.7052	868.940	{0.0000}	{0.0000}	{0.0000}	{0.0000}
Adjusted	0.0275	186.089	184.454	96.423	7.1639	212.639	113.973	17.4056
Series	14.410	-376.018	-5.7833	99.106	{0.0000}	{0.0000}	{0.0000}	{0.0000}

Skew is a measure of heavy tails and asymmetry of a distribution (normal) and kurtosis is measure of too many observations around the mean for a distribution (normal). K-S Z-test: Used to test the hypothesis that a sample comes from a normal distribution. The value of the Kolmogorov-Smirnov Z-test is based on the largest absolute difference between the observed and the theoretical cumulative distributions. Q (6) and Q^2 (6) is a test of serial correlation up to lag 6 for ordinary and squared series, respectively. ARCH (6) is a test for conditional heteroscedasticity in returns. Low {·} indicates significant values. We employ the OLS-regression $y^2 = a_0 + a_1 \cdot y^2_{t-1} + \ldots + a_{12} \cdot y^2_{t-12}$. $T \cdot R^2$ is $\square^2$ distributed with 12 degrees of freedom. T is the number of observations, *y* is returns and R^2 is the explained over total variation. $a_0, a_1 \ldots a_{12}$ are parameters. RESET (12,6): A sensitivity test for mainly linearity in the mean equation. 12 is number of lags and 6 is the number of moments that is chosen in our implementation of the test statistic. $T \cdot R^2$ is χ^2 distributed with 12 degrees of freedom. BDS (m=2, ε=1): A test statistic for general non-linearity in a time series. The test statistic $BDS = T^{1/2} \cdot [C_m(\sigma \cdot \varepsilon) - C_1(\sigma \cdot \varepsilon)^m]$, where *C* is based on the correlation-integral, m is the dimension and ε is the number of standard deviations. Under the null hypothesis of identically and independently distributed (i.i.d.) series, the BDS-test statistic is asymptotic normally distributed with a zero mean and with a known but complicated variance.

Figure 2
System price Nordic electric power market

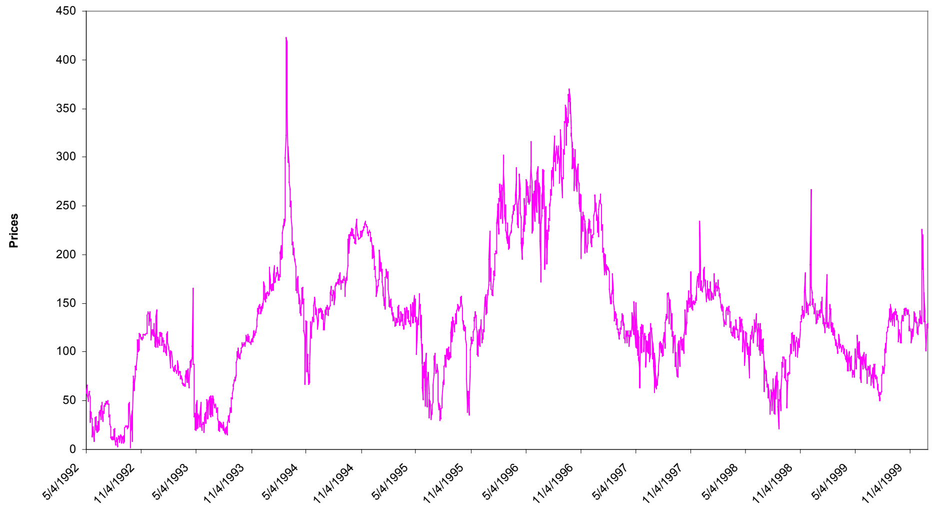
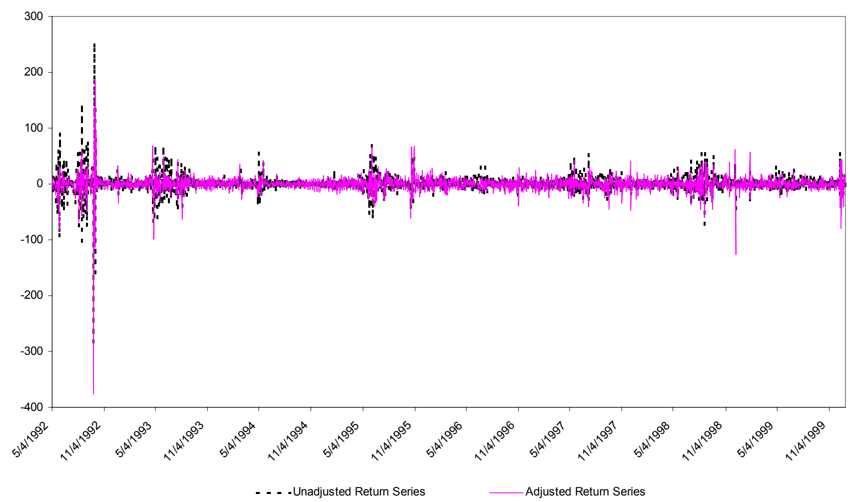


Figure 3
Raw and adjusted system price changes



V. EMPIRICAL RESULTS

Maximum likelihood estimates[14] of the parameters for the ARMA-GARCH lag specifications on the adjusted time series are reported in Table 3. The GJR and AGARCH show a non-significant drift (α_0) coefficient, while the EXP specification shows significant positive drift. The BIC (Schwarz, 1978) preferred ARMA lag specification defines a serial correlation mean structure. All lags are significant and suggest dependence in price changes up to 14 days. We find that especially lag 1, 7 and 14 show strong positive autocorrelation[15]. We may therefore pertain that the applied adjustments described in Section 4 may therefore not fully adjust for all mean dependence. However, in contrast to financial markets serial correlation, which may stem from non-synchronous trading, the electric power market serial correlation may stem from a regular consumption pattern during days of the week, which is fully absorbed by the market participants. Finally, the in-Mean parameter ρ seems redundant for all three specifications. Hence, the volatility does not suggest mean directions over time suggesting a rejection of the mean-variance total risk model.

Table 3

ARMA (p, q)-GARCH (m, n)-in-mean model for the system price

	Mean Equation							
	Φ_0	Φ_1	Φ_4	Φ_5	Φ_6	Φ_{12}	Φ_{14}	ρ_1
GJR	0.13253	0.10490	-0.04811	-0.05728	0.16393	-0.03353	0.10020	0.06593
	{0.9758}	{5.3808}	-{2.7218}	-{3.2435}	{9.2663}	-{2.2127}	{6.6694}	{1.2423}
EXP	0.25696	0.13177	-0.04858	-0.05770	0.16470	-0.04642	0.09447	0.00000
	{2.2531}	{7.8010}	-{3.0202}	-{3.7611}	{11.4379}	-{3.5824}	{7.5703}	{0.0000}
AGARCH	0.13253	0.10490	-0.04811	-0.05728	0.16393	-0.03353	0.10020	0.07297
	{0.9814}	{5.3878}	-{2.7215}	-{3.2654}	{9.3959}	-{2.2135}	{6.6949}	{1.2445}
	Volatility Equation						Log-likelihood Function	
	a_0	a_1 / γ_0	b_1	$\Sigma(a_1, b_1)$	γ_1 / θ_0	υ		
GJR	9.02388	0.27379	0.68181	0.95560	0.08070	4.04690	-9960.14	
	{5.4236}	{7.5768}	{19.4217}		{1.6276}	{12.0947}		
EXP	0.30379	0.41238	0.93189	0.93189	-0.02610	1.05610	-9990.77	
	{5.5798}	{14.2270}	{77.2670}		-{1.3827}	{52.9125}		
AGARCH	9.02384	0.27379	0.68181	0.95560	-0.46044	4.04693	-9960.14	
	{8.0366}	{7.7582}	{25.6240}		-{0.7792}	{12.4213}		

The estimated conditional variance coefficients are all strongly significant. Conditional homoscedasticity and constant volatility is therefore strongly rejected for the time series. The α_1 parameter reports the shock effects from the previous period and the b_1 parameter reports autocorrelation in the conditional volatility. All three models report strong and highly significant influence from passed shocks (ARCH-effects). In contrast, the past volatility (GARCH-effects) is rather low, which most likely, stems from high past shock effects suggesting that it moves around randomly. We find insignificant asymmetric volatility coefficient (γ_i) for all specifications in the spot market suggesting equal reaction patterns to positive and negative shocks. Finally, the degree of freedom coefficient (υ) reports the thickness of the return distribution tails. All the coefficients are strongly significant. Our result indicates "long tails" in the distribution, which suggests some extreme price movements in the series. Table 3 reports the log-likelihood function values for all three models. It seems like the function values report preference for the GJR and AGARCH versions of the ARMA-GARCH lag specification models. Applying a likelihood ratio test, we find that the GJR and AGARCH versions report significant improvements from the exponential-GARCH model[16].

To investigate for ARMA-GARCH model misspecification we employ several elaborate test statistics. As a first specification test of the model, we calculate the sixth order Ljung and Box (1978) statistic for the standardised residuals (Q) and squared standardised residuals (Q^2) in Table 4. We find no significant evidence of serial correlation in the residuals (*Q (6)*) and squared residuals up to lag 6 (Q^2 *(6)*). The numbers for kurtosis and skews for the standardised residuals are lower in absolute value for all models. The result suggests more normal residuals. The ARCH (12) test statistic rejects conditional heteroscedasticity and the RESET (12; 6) test statistic rejects non-linearity in the mean. The BDS test statistic for standardised residuals cannot reject i.i.d. from the ARMA-GARCH model at any dimension. Finally, the joint bias test (Engle and Ng, 1993) reports no significant biases in standardised residuals. Hence, all our ARMA-GARCH specifications survive the specification tests and we cannot report model misspecification. All three models are therefore candidates for the changing volatility model employed for the electric power market and the so-called system price.

Table 4
Advanced specification tests*

	Model-specifications:					
	GJR		EXP		AGARCH	
Q (6)	4.6550	{0.5890}	2.3190	{0.8880}	2.8970	{0.8220}
Q² (6)	1.1320	{0.9800}	0.6670	{0.9950}	0.9060	{0.9890}
K-S Z-test	3.6224	{0.0000}	3.5662	{0.0000}	3.5954	{0.0000}
ARCH (12)	2.1639	{0.9991}	1.0095	{1.0000}	1.7895	{0.9997}
Reset (6; 12)	16.6178	{0.1645}	16.3140	{0.1773}	16.7083	{0.1609}
BDS (m=2, ε=1)	1.0657	{0.2261}	1.8853	{0.0675}	1.3706	{0.1560}
BDS (m=3, ε=1)	0.6983	{0.3126}	1.3058	{0.1701}	0.7626	{0.2983}
BDS (m=4, ε=1)	0.4548	{0.3597}	0.9356	{0.2575}	0.3762	{0.3717}
Joint BIAS-test	8.4251	{0.0380}	7.5153	{0.0572}	8.5513	{0.0359}

*See Table 2 for a description of the specification test statistics. The joint BIAS-test statistic tests the relation $\varepsilon^2_t = a + a_1 \cdot S_{t-1} + a_2 \cdot \varepsilon^2_{t-1} \cdot S_{t-1} + a_3 \cdot \varepsilon^2_{t-1} \cdot (1-S_{t-1})$. The statistic tests whether all the a-parameters are significantly different from zero. $T \cdot R^2$ is χ^2 distributed with 3 degrees of freedom.

VI. ELECTRIC POWER MARKET FINDINGS

The specification test statistics show that the ARMA-GARCH-in-Mean models are appropriate models for mean-variance analyses in the Nordic electric power market. Consequently, the mean and volatility characteristics in the electric spot market may therefore be applied and integrated with other financial markets. Covariance and relevant risk measures may therefore be of outmost interest for international investors[17]. The electric power market may therefore contain interesting investment instruments for investors seeking mean-variance optimal tangency portfolios. Moreover, as the in-Mean parameter seems redundant a total (residual) risk model is rejected.

The consumption pattern in the mean process may signal predictability in the mean process. As especially lag 1, 7 and 14 show strong positive serial correlation this information shows clear price change patterns over a weekly interval. Especially Mondays and Tuesdays report an upward adjustment while Saturday and Sunday report a downward adjustment. These adjustments are obviously based on consumption patterns in corporate Norway. Consequently, we may suggest a predictability of price changes during a weekly interval for the system price.

Mean reversion and volatility clustering suggest a volatility moving around randomly, but over time it tends to get pulled back toward some long-run average level. The weights in the GARCH (1,1) lag specification must sum to unity. Consequently, the long-term variance rate is $\frac{a_0}{1-a_1-b_1} = \frac{9.0238881}{1-0.273789-0.681811} = 203.240563$ for the GJR model and 203.225953 for the AGARCH model, or 14.2562% and 14.2557% per day, respectively. Figure 4 shows the way the volatility for the System Price has changed over the period covered by the data. Most of the time the volatility was between 5 and 15% per day, but volatilities over 25% were experienced during some periods. If we assume that the current variance rate is 100% per day, corresponding to a volatility of 10% per day, the expected variance rate in 10 days for the GJR models is $203.240563+0.9556^{10}\cdot(100-203.240563)=137.684664$. The expected volatility per day is therefore 11.734%. However, the expected volatility in 100 days is 14.218% per day, very close to the long-term volatility. We can now use the GARCH model to estimate a volatility term structure. Table 5 shows the System Price volatility term structure per annum predicted from GARCH(1,1) models[18] from low and high current volatility for GJR and AGARCH models, respectively. Moreover, based on the volatility term structure analyses we can calculate impact of volatility changes on options of varying maturities.

The persistence in the volatility process can be calculated by the half-life of a shock to the process, that is, the time it takes for half of the shock to have dissipated. The GJR and the AGARCH models can calculate this persistence. Some algebra shows that the half-life in trading days for portfolio i may be calculated as[19] $\text{Half-life}_i = \ln(0.5) / \ln(a_{i,1} + b_{i,1})$ and for calendar days as $(252 \cdot \text{Half-life}_i) / 365 = \ln(0.5) / \ln(a_{i,1} + b_{i,1})$. Hence, $\text{Half-life}_i = (\ln(0.5) / \ln(a_{i,1} + b_{i,1})) \cdot (365/252)$. The persistence is approximately 15 trading days for both the GJR and AGARCH models. The information regarding both the shock- and persistence-effects may be useful for investors building volatility strategies in an option market for electric power. Moreover, investors should be aware of the increased volatilities following shocks in the market.

Figure 4
Daily volatility of the system price changes

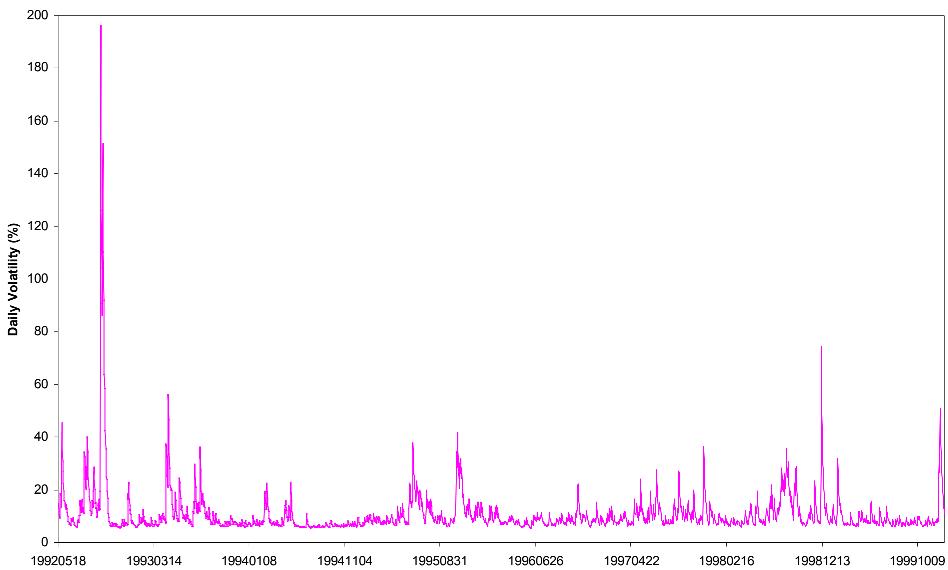


Table 5
System price volatility term structure predicted from GARCH (1, 1)

Option life (Days)			10	30	50	100	500
Option Volatility per annum (%)							
Model	GJR	Low	186.270	211.082	220.297	225.698	226.311
		High	258.263	239.707	231.805	226.884	266.311
	AGARCH	Low	187.612	212.469	221.161	225.853	266.303
		High	257.346	238.545	231.019	226.727	266.303

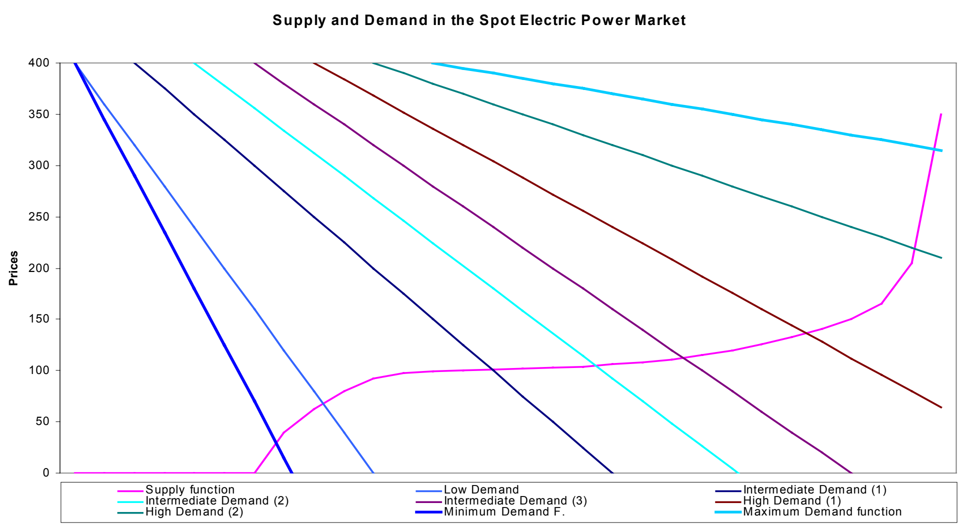
The average and standard deviation for the conditional variance process per day is 216.6 and 1392.4, respectively. The high Variance-Mean Ratio of 6.43 suggests major challenges in purchase/sales decisions for spot electric power for especially high volatility regime periods. Hence, production companies may have strong incentives for advanced risk management and optimal production planning tools.

For every model that is developed to track variances there is a corresponding model that can be developed to track covariances. The univariate GARCH procedure can therefore, be used to update the complete variance-covariance matrix (multivariate GARCH) used in value at risk calculations.

The system price seems to follow a process similar to comparable financial markets like for example the equity market in London and energy market at New York Mercantile Exchange. However, while the volatility clustering is comparable, the serial correlation in the mean seems to suggest predictable consumption pattern not found in other equity and energy markets. A major puzzle in our investigation is the high daily volatility (12-15%), which is considerably lower in equity and other energy markets. As the system price is an equilibrium price between an inelastic demand curve and an inelastic supply curve at low and high quantities and an elastic supply curve at intermediate quantities, the high volatility is explainable from high and low price regime periods[20]. Hence, we may find periods of low volatility and periods with very high volatility. During the summer months where the prices are low, the relative changes are large, which turn out to be very high volatility periods. During cold winter periods where prices are high, the absolute changes are high but the relative changes are smaller relative to changes in low price level periods implying high volatility period but clearly lower than the summer months due to higher levels (Jensen's inequality). Hence, to reduce spot market volatility (assumed desirable) in-line with other equity and energy markets, it will be important to find better matches between supply and demand during especially low price level regimes. As the demand curve is inelastic and consumption level dependent on temperature and industry production, to obtain a lower volatility in the electric power market in line with other financial markets, a more elastic supply curve should be induced from market participants during especially low price level regimes. Moreover, possibly surprising, is the fact that from empirical data we find low to no volatility increases at high price levels. May we have exaggerated the problems of capacity on the supply side of the electric power market? From this discussion we may induce the following price–quantity relationship illustrated in Figure 5, in the spot electric power market.

The demand curves become more inelastic as the price level decreases and more elastic as the price level increases. The demand curves may show shifts to left and right as temperature and industry production changes. Hence, we show seven demand curves in Figure 5. In contrast to the demand curve, the supply curve will show lower degrees of shift. Any shifts may be caused by accidents or maintenance at especially nuclear power stations. However, this is not shown in Figure 5. Below a certain price level the supply becomes strongly inelastic. The production and production planning at these price levels seem to produce small production planning changes. Hence, small changes in demand may show very high price changes[21]. Surprisingly, the same features are not empirically found in our series. However, the high price regimes are rare and therefore present in the market but not empirically found. When demand and supply crosses at high price levels the supply curves reach their capacity level and become inelastic while demand becomes elastic. Hence, the situation may occur for short time periods but will probably not last for long time periods. This may explain our low volatility findings in high price regimes.

Figure 5
Supply and shifting demand curves in the Nordic electric power market



VII. SUMMARIES AND CONCLUSIONS

We have modelled and estimated an ARMA-GARCH-in-Mean lag specification for the conditional mean and volatility for the spot Nordic electric power market for the period October 1992 to January 2000. The time series are adjusted for systematic season, trend and scale effects and all the estimated ARMA-GARCH specifications are BIC preferred. Our model captures the serial correlation structure (consumption patterns) in the return series, the effect of “thick distribution tails” (leptokurtosis) and residual risk in the conditional mean. The conditional variance equation captures shocks, serial correlation, persistence, and asymmetric volatility and mean reversion to a long run average volatility. The specification test statistics cannot reject our BIC preferred ARMA-GARCH-in-Mean lag specification models. Hence, the residuals report no serial correlation, conditional homoscedasticity and no data dependence.

We summarize our findings as follows. The drift is close to zero. Two specifications suggest non-significant drift while the exponential specification reports a marginal significant positive drift. We find serial correlation structures up to 14 days after an adjustment procedure that accounts for seasonal, trend and scale effects. The serial correlation structure suggests price predictability and change patterns in the market. Employing the log-likelihood function the volatility equation seems to prefer the GJR or the AGARCH version of the univariate ARMA-GARCH lag specification model. We find high past shock effects (ARCH) and rather low past volatility effects (GARCH), suggesting a persistence of shocks of approximately fifteen trading days. We find a significant weight to the long-term variance rate and we forecast the expected volatility per day for several range of days, which constitute the volatility term structure. The results can easily be extended to impact of volatility changes and value at risk calculations. The low and strongly significant degree of freedom parameter suggests leptokurtosis in the time series inducing extreme price changes for electric power. Finally, for especially low price level regimes (summer periods) the volatility is very high and suggests major challenges to derivative pricing and optimal production planning. Moreover, couture of supply and demand profiles may be sketched for the spot electric power market. Empirically, high price regimes close to the capacity limit seem not to induce higher volatility. Hence, a minimum system price may have more effect on volatility than a maximum system price. However, empirically we find a limited number of extremely high system prices, which may underestimate high price volatility.

NOTES

1. We also employed a stochastic volatility specification applying SNP and EMM (Gallant, Rossi and Tauchen, 1992) methodologies for the time series but the models failed to find an optimal SNP specification.
2. The example is from Nord Pool: EL-Spot (www.nordpool.no); Nordpool (1998).
3. See Solibakke (2001a and 2001b).
4. Gallant & Tauchen (1997) find 18 (!) ARCH-lags for time series retrieved from the U.S. financial market.
5. See Morgan, 1976.
6. Departure from normally distributed price changes.
7. Se Gouriéroux, C., 1997 og Campbell et al., 1997.
8. In the exponential GARCH model we assume a General Error Distribution, which contains the normal distribution as a special case ($\upsilon = 2$).
9. The BHHH algorithm is described in: Berndt, Hall, Hall, Hausman (1974)
10. The technique is available in GAUSS ver 3.2.1.
11. The time series adjustment procedure is implemented in Gauss.
12. The K-S Z test statistic is a procedure to test the null that a sample comes from a population in which the variable is distributed according to a normal distribution.
13. See Box & Jenkins, 1976 and Ljund and Box, 1978.

14. Based on Likelihood Ratio Test statistics (LRT) the student-t log-likelihood function s strongly preferred to a normal likelihood function.
15. The serial correlation may stem from the strong day effects found in the adjustment procedure. Our procedure seems not to remove all systematic seasonal effects.
16. The likelihood ratio test is a general test for testing restrictions imposed on a model. The LRT statistic is evaluated as LRT = 2(LRT1- LRT2). Under the null hypothesis, the LRT is distributed as χ^2 with number of restrictions as the degree of freedom. The above calculations report a test statistic: LRT = 61.2534, which is significant at 1%.
17. Total risk and variance is not relevant in a portfolio setting.
18. Note that it is possible to forecasts the effect of volatility changes on options of varying maturities.
19. See Taylor, 1986/00 for details.
20. Adding a dummy variable for price levels below NOK 100 in the volatility equation shows a highly significant parameter, suggesting a strongly steeper supply curve below this price level.
21. Just calculate the price changes between the demand and supply crosses at minimum and low price level relative to two intermediate demand curves in Figure 5.

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