INTERNATIONAL JOURNAL OF BUSINESS, 2(1), 1997 ISSN: 1083-4346

Exchange Rate Dynamics with Chartists, Fundamentalists, and Rational Speculators in the Foreign Exchange Market

Michael Frenkel

This paper extends previous work on speculative dynamics in the foreign exchange market. The analysis shows how the behavior of chartists, fundamentalists, and rational speculators, together with uncertainty about the long-run equilibrium exchange rate level, can result in fluctuations of the spot rate triggered by a random shock to the market. Although the exact exchange rate path depends on the extent of the shock and on specific values of model parameters, the heterogeneity of expectations can explain several characteristics of short term exchange rate developments, which have often been emphasized in empirical studies on exchange rate dynamics.

I. INTRODUCTION

According to a famous saying in economics, no theory has ever been overthrown by empirical studies. Several approaches in exchange rate economics seem to be examples of this observation. In fact, the variety of approaches trying to explain exchange rate movements is much higher than in most areas of economic theory. While traditional approaches emphasize the importance of the influence of macroeconomic fundamentals on exchange rates and use the rational expectation hypothesis, many alternative views have been presented in the literature since the mid-1980s. Triggered by the poor prediction quality of

Michael Frenkel, Department of Economics, WHU Koblenz, Otto Beisheim Graduate School of Management, Burgplatz 2, 56179 Valendar, Germany.

traditional exchange rate approaches and the frequently observed bias of forward exchange rates, a number of more recent contributions question the view that participants in the foreign exchange market base their decisions on rational expectations about the development of fundamentals.[1] For example, studies by Chinn und Frankel [1], Frankel and Chinn [6], Frankel and Froot [7,8,9], Froot and Ito [11], Goodhart [14], Ito [16], Liu and Maddala [21], Taylor [25], and Taylor and Allen [25] on the formation of expectations of foreign exchange market participants reject the rational expectation hypothesis. These studies often suggest that speculators base their expectations on chart analyses of exchange rate movements. This has led to some models that explain exchange rate dynamics, at least for shorter time horizons, on the basis of the behavior of chartists or noise-traders (Frankel and Froot [8] and Cutler et al. [2]).

The emphasis of chartists in explaining exchange rate movements has not been without criticism. A number of authors reject the view that irrationality plays a role in foreign exchange markets. They see the disappointing performance of models that are based on fundamentals in the presence of uncertainty about the development of fundamentals and in changing risks of assets. They show that observed characteristics of exchange rate dynamics can be explained by phenomena such as the peso-problem, news, a learning process, rational bubbles, or changing risk premia. Often-cited papers of this category of approaches are, for example, those of Edwards [5], Garber [12,13], Hartley [15], Krasker [18], and Lewis [19,20]. The common feature of all these approaches is the use of the rational expectations hypothesis.

The purpose of this paper is to develop a simple model that integrates the described opposite views, i.e., that participants in the foreign exchange market have rational expectations or base their expectations on the development of fundamentals and that speculators are chartists. By combining different types of speculators, this model allows us to take into account heterogeneous expectations, a feature emphasized, in particular, by Ito [16] and Takagi [24]. The model extends previous work by Cutler et al. [2] and allows us to generate

some dynamic features, which have often been observed as characteristics of the foreign exchange market. The rest of the paper is structured as follows. Section 2 lays out the structure of the model. Section 3 examines the dynamics of the spot rate and the forward rate under the assumption that all speculators are chartists. Section 4 extends the model by incorporating speculators with regressive expectations and uncertainty among these speculators. Section 5 adds speculators with rational expectations, and Section 6 summarizes the results and indicates some extensions of the analysis.

II. THE MODEL

The basis of the model developed in this section is the following four observations. First, some actors in the foreign exchange markets base their expectations on technical analyses of past exchange rate movements. Second, some speculators are convinced that fundamentals drive the exchange rate in the long run. Third, speculation in the foreign exchange market is conducted in the spot market. Fourth, from time to time, random factors influence the exchange rate, for example, due to wrongly perceived changes in fundamentals.

The model applies a simple asset market approach and, thus, assumes that the spot exchange rate at time t is determined by the stock of domestic assets relative to foreign assets and the relative demand for these assets. Denoting the logarithm of relative demand for foreign assets at time t by d_t and relative supply of domestic assets by m_t, the logarithm of the spot exchange rate (s_t), expressed as the price of one unit of foreign currency in domestic currency units, can be written as

$$s_t = m_t + d_t + u_t , \qquad (1)$$

where u_t represents a random influence. Both an increase in the stock of domestic assets and a shift of asset demand towards foreign assets lead to a rise in the exchange rate, i.e., to a depreciation of the domestic currency.[2] To focus on the interaction of different types of speculators in the foreign exchange market, which is reflected in the desired changes in

the structure of their portfolio, the supply of assets is assumed to be constant. Thus, only demand and random factors affect the exchange rate.

The model assumes that three types of foreign exchange market actors influence the exchange rate. We call them chartists, fundamentalists, and rational speculators.[3] This is similar to Cutler et al. [2] who distinguish chartists, who have extrapolative expectations, and fundamentalists, who have regressive expectations. However, we use more specific assumptions about the behavior of actors in the foreign exchange market, which allows us to analyze in more detail the effects of shocks on exchange rate dynamics. Furthermore, we incorporate uncertainty of market participants as an additional characteristic of the foreign exchange market.

We assume that chartists' relative demand for foreign assets (d_t^c) is based on the following function

$$d_t^c = \theta_1(s_t - s_{t-1}) + \theta_2(s_{t-1} - s_{t-2}). \quad (2)$$

For simplicity, we will refer to the relative demand simply as the demand for foreign exchange. The demand of chartists as expressed in equation (2) is based on a trend. Chartists compare the current change in the exchange rate with the change in the previous period. The parameters θ_1 and θ_2 are the demand elasticities of chartists with respect to the changes in the exchange rate.

A second group of speculators are fundamentalists with regressive expectations. They assume that the exchange rate will eventually move towards a level s_t^f , which reflects a perceived equilibrium value based on macroeconomic developments relevant for exchange rate determination. The demand of fundamentalists for foreign assets (d^f) is

$$d_t^f = \gamma(s_t^f - s_t). \quad (3)$$

The higher the long-term expected exchange rate is compared to the actual exchange rate, the higher is the demand of fundamentalists. The parameter γ denotes the demand elasticity. We will later take into account that fundamentalists may not exert any demand unless the difference in parentheses

on the right-hand side of equation (3) exceeds a certain threshold. In addition, fundamentalists may require some time to form an opinion about the long-term exchange rate. This may be caused by uncertainties about the true stance of economic policies.

A third group of speculators is assumed to have rational expectations. We refer to them as rational speculators. They know the dynamics of the exchange rate triggered by the behavior of chartists and fundamentalists. Their demand for foreign assets and, thus, for foreign exchange (d^r) is a function of the expected difference between the exchange rate expected for the next period and the current exchange rate. This yields

$$d_t^r = \mu E_t(s_{t+1} - s_t). \tag{4}$$

Here, μ and E denote the elasticity of rational speculators' demand and the conditional rational expectation value, respectively.

Combining the demand of the three groups of speculators and denoting the shares of chartists and fundamentalists by α and β yields

$$d_t = \alpha d_t^c + \beta d_t^f + (1-\alpha-\beta) d_t^r. \tag{5}$$

The smaller α and β are, the more is the foreign exchange market dominated by rational speculators.

Equations (1) through (5) describe the model. The dynamics of the model, following a random shock in the foreign exchange market, depend on specific assumptions about the elasticities of speculators' demand and the share of the different speculator groups in the foreign exchange market. The next sections discuss different variations of the model and examine whether the resulting dynamics following a shock are consistent with observations in the foreign exchange market.

III. THE FOREIGN EXCHANGE MARKET WITH CHARTISTS ONLY

A first variation of the model described in the previous section assumes that speculators can simply be described as chartists, i.e., that $\alpha=1$ and $\beta=0$. In this case, combining equations (1) and (2) gives

$$s_t = \theta_1(s_t - s_{t-1}) + \theta_2(s_{t-1} - s_{t-2}) + m_t + u_t . \tag{6}$$

Using this expression together with the assumptions about the shares in

equation (5) yields

$$s_t = [-(\theta_1 - \theta_2) s_{t-1} - \theta_2 s_{t-2} + m_t + u_t] / (1 - \theta_1). \qquad (7)$$

Solving for s_t gives the differential equation

$$s_t + \frac{\theta_1 - \theta_2}{1 - \theta_1} s_{t-1} + \frac{\theta_2}{1 - \theta_1} s_{t-2} = \frac{m_t + u_t}{1 - \theta_1}. \qquad (8)$$

We initially abstract from the random element u_t and use the following definitions of parameters:

$$a_1 = (\theta_1 - \theta_2) / (1 - \theta_1),$$

$$a_2 = \theta_2 / (1 - \theta_1), \qquad (9)$$

$$b = m_t / (1 - \theta_1).$$

This reduces equation (8) to

$$s_t + a_1 s_{t-1} + a_2 s_{t-2} = b. \qquad (10)$$

Assuming constant asset stocks over time, no time subscript needs to be taken into account on the right-hand side of this equation. With respect to the solution of equation (10), three cases can be distinguished:

case I:	$a_1^2 - 4a_2 > 0$;	here, two real and different roots result from the corresponding characteristic equation;
case II:	$a_1^2 - 4a_2 = 0$;	here, there are two identical real roots of equation (10);
case III:	$a_1^2 - 4a_2 < 0$;	here, two conjugate complex roots result.

We now consider all three solutions in turn. In the first case, the solution is

$$s_t = C_1 \lambda_1^t + C_2 \lambda_2^t + m, \qquad (11)$$

where C_1 and C_2 are constants, which are determined by initial conditions. In addition, the variable m, which denotes the foreign asset stock relative to the domestic asset stock, is the particular integral of the solution of equation (10). The roots λ_1 and λ_2 are the solutions of the characteristic equation with

$$\lambda_1 = -\frac{a_1}{2}+\sqrt{\frac{a_1^2}{4}-a_2}; \qquad \lambda_2 = -\frac{a_1}{2}-\sqrt{\frac{a_1^2}{4}-a_2}. \tag{12}$$

Replacing a_1 and a_2 by the definitions (9), it can be shown that case I materializes if the parameter θ_1 is relatively large compared to θ_2. If the absolute value of the root is smaller than one, a disturbance of the equilibrium is followed by a monotonous adjustment to the long run equilibrium. Thus, even with $u_t \neq 0$ and with chartists dominating completely the foreign exchange market, stability of exchange rates prevails. However, if the greater root of the characteristic equation is greater than one in absolute terms, a random shock is followed by explosive exchange rate movements.

In case II, the general solution is

$$s_t = C_1(-\frac{a_1}{2})^t + C_2 t(-\frac{a_1}{2})^t. \tag{13}$$

C_1 and C_2 are again determined by initial conditions. This solution can generate either a converging or an explosive exchange rate movement. For convergence, the condition $|-a_1/2|<1$ has to be met.

Case III seems to be particularly interesting for the issue discussed in this paper. In this case, a random shock triggers fluctuations around the level that can be explained by fundamentals. The general solution of this case is

$$s_t = C_1 R^t \cos(\varphi t) + C_2 R^t \sin(\varphi t) + m, \tag{14}$$

where R and φ are determined by the relations

$$R\cos\varphi = -\frac{1}{2}a_1 \quad und \quad R\sin\varphi = \sqrt{a_2-\frac{a_1^2}{4}} \tag{14a}$$

and C_1 and C_2 are again parameters determined by initial conditions.[4]

Figure 1
Cyclical exchange rate movements after a random shock in the foreign exchange rate market with chartists only (assumptions: $s_0=1$; $s_1=1.05$)

If the equilibrium exchange rate is initially at the level determined by fundamentals and a random shock occurs, then, chartists cause fluctuations around this initial equilibrium. Depending on the values of the parameters θ_1 and θ_2, these fluctuations can be converging, constant, or diverging. Figure 1 illustrates these possibilities by using alternative assumptions about the values of θ_1 and θ_2. While the upper diagrams show converging fluctuations, which are generated whenever $a_2<1$. The latter, in turn, requires that $\theta_1+\theta_2<1$. If this sum is equal to one, fluctuations with constant amplitudes result (lower-left diagram in Figure 1). The unstable situation, which follows from $\theta_1+\theta_2>1$, is shown in the lower-right diagram.

In case of converging fluctuations, the speed with which the fluctuations converge depends on the elasticities θ_1 and θ_2. For example, if both parameters have the same value, the fluctuations converge faster the smaller the sum $\theta_1+\theta_2$ is. By contrast, given the value of the sum $\theta_1+\theta_2$, the fluctations are the greater, the greater is θ_2. This is the case because a greater value of θ_2 implies an effect of exchange rate changes on the demand of chartists over a longer period of time and, thus, causes the flucutations to persist longer.

The version of the model that includes chartists as the only type of speculators is interesting because it already reveals some typical characteristics of exchange rate movements. For example, a single considerable shock to the foreign exchange market equilibrium leads to longer fluctuations around the value that can be explained by fundamentals. The exchange rate paths can easily look like smaller bubbles which are not reversed in one period but over a few periods. In addition, the model implies positive serial correlation of exchange rates for relatively short periods and negative serial correlation for longer periods. Goodhart [14], Ito und Roley [17], as well as Cutler et al. [2] emphasize that this is characteristic for the foreign exchange markets. However, unlike in the model of Frankel und Froot [8], where fluctuations can be as long as 5 to 10 years, the case studied here implies very short term fluctuations in the foreign exchange markets.

So far, the analysis focused on the dynamics of the spot exchange rate. In order to analyze the dynamics of the forward exchange rate and expectational errors, we assume that the right-hand side of the equation (2) can also be used to express exchange rate expectations of chartists. Given the fact that the parameters θ_1 and θ_2 describe the combined effect of exchange rate expectations and the elasticity of demand for foreign exchange with respect to expected exchange rate changes, our assumption implies that the demand

elasticities are equal to 1. Figure 2 shows the resulting time path of the spot rate and the expected exchange rate, in the absence of risk considerations. Figure 2 uses the parameters employed in the upper-left diagram of Figure 1, which we also use later as a benchmark case. The time paths show that the changes in the expected rate follow with a lag the actual spot rate. This is a typical feature of chartists' behavior, reflecting extrapolative expectations. As a consequence, during certain periods, the forward rate correctly predicts the direction of the exchange rate changes, but it is unable to predict turning points. In any case, expectational errors occur regularly.

Figure 2
Spot exchange rate and exchange rate expectation with chartists only
(assumptions: $s_0=1$; $s_1=1.05$; $\beta_1=0.2$; $\beta_2=0.7$)

The described time path assumes that there is a single shock occuring only in period 1. In reality, it can be assumed that frequent shocks of different sizes occur. As a result, the time paths of spot and forward rates are unlikely to follow as smooth a pattern as shown in Figures 1 and 2. If, in addition, random shocks are accompanied by changes in fundamentals, exchange rates cannot be expected to fluctuate around the same long-term rate. Hence, an important role of chartists in the foreign exchange market can be used as one explanation or a very uneven path of high frequency exchange rates.[5]

IV. THE INTERACTION OF CHARTISTS AND FUNDAMENTALISTS

We now add, as a second group of speculators in the foreign exchange market, fundamentalists, i.e., speculators with regressive expectations. This implies that $\beta=1-\alpha$ in equation (5). Using the resulting expression of equation (5) in equation (1) yields

$$s_t = \alpha\theta_1(s_t - s_{t-1}) + \alpha\theta_2(s_{t-1} - s_{t-2}) + (1-\alpha)\gamma(s_t^f - s_t) + m_t + u_t \qquad (15)$$

and after rearranging

$$s_t + \frac{\alpha(\theta_1-\theta_2)}{1-\alpha\theta_1+(1-\alpha)\gamma}s_{t-1} + \frac{\alpha\theta_2}{1-\alpha\theta_1+(1-\alpha)\gamma}s_{t-2} = \frac{m_t+(1-\alpha)\gamma s_t^f + u_t}{1-\alpha\theta_1+(1-\alpha)\gamma}. \qquad (16)$$

Here, s_t^f denotes the exchange rate, which fundamentalists expect on the basis of their knowledge about the values of fundamentals. To simplify equation (16), we use the following definitions:

$$a_3 = \frac{\alpha(\theta_1-\theta_2)}{1-\alpha\theta_1+(1-\alpha)\gamma}, \qquad (17a)$$

$$a_4 = \frac{\alpha\theta_2}{1-\alpha\theta_1+(1-\alpha)\gamma}, \qquad (17b)$$

and

$$c = \frac{m_t+(1-\alpha)\gamma s_t^f + u_t}{1-\alpha\theta_1+(1-\alpha)\gamma}, \qquad (17c)$$

where m_t and s_t^f are assumed to be constant and $u_t=0$ for all $t>0$. The dynamics of the spot exchange rate can then be described by the second order difference equation

$$s_t + a_3 s_{t-1} + a_4 s_{t-2} = c\,. \tag{18}$$

Again, three types of solutions can be distinguished. In what follows, we focus again on the solution that involves oscillations. Figure 3 shows the dynamics on the basis of the same parameter values as in the upper-left diagram in Figure 1. As before, a single random shock is assumed to occur in period one. The bold line, which depicts the exchange rate dynamics with chartists and fundamentalists, shows that, in comparison with a market with chartists only (shown by the solid line), exchange rate fluctuations are less pronounced. Thus, the higher the share of fundamentalists among speculators, the lower is the volatility of exchange rates. However, this requires that fundamentalists know the long-run exchange rate. If this is not the case, their losses drive them out of the market.[6]

More formally, the dampening effect of speculators with regressive expectations can be seen by comparing the coefficients of equation (7) with the coefficients of the second-order difference equation above. This shows that

$$a_3 = \frac{\alpha(\theta_1 - \theta_2)}{1 - \alpha\theta_1 + (1-\alpha)\gamma} < a_1 = \frac{\theta_1 - \theta_2}{1 - \theta_1} \tag{19}$$

and

$$a_4 = \frac{\alpha\theta_2}{1 - \alpha\theta_1 - (1-\alpha)\gamma} < a_2 = \frac{\theta_2}{1 - \theta_1}\,. \tag{20}$$

Since a_3 and a_4 are smaller than the corresponding parameters a_1 and a_2 from the version of the model with chartists only, the fluctuations with fundamentalists are unambiguously smaller. Moreover, the expression for a_3 implies that the critical value, which the sum of $\theta_1+\theta_2$ must not exceed in order to ensure stability, is higher in the case in which some speculators have regressive expectations.[7]

Figure 3

Exchange rate dynamics with chartists and fundamentalists (assumptions: $s_0=1$; $s_1=1.05$; $\theta_1=0.2$; $\theta_2=0.7$; $\alpha=0.9$; $\gamma=0.15$)

So far, we have assumed that fundamentalists know the long-run exchange rate. However, one may realistically assume that they are uncertain about the exchange rate level that can be explained by fundamentals. In this case, they may try to infer from the observation of new exchange rate data what the true fundamentally determined exchange rate level is. In order to incorporate this idea into the model, we use the following modified foreign exchange demand function of fundamentalists:

$$d_t^f = \gamma_1(s_t^f - s_t) + \gamma_2(s_{t-1}^f - s_t) \quad (21)$$

This demand function allows for varying values of s^f over time. How expectations about the value of s^f change over time depends on initial beliefs and observed exchange rate levels. This can be modeled in different ways. One possibility is to apply a Bayesian scheme. Another possibility is to use a moving average of the spot exchange rate. For simplicity, we apply a relatively

simple function which states that the expected long-run level of the exchange rate (s_t^f) is a function of the belief in the previous period and the difference between an initial perception of the long-run exchange rate level and the spot rate in the previous period

$$s_t^f = s_{t-1}^f + \delta(s^* - s_{t-1}); \quad 0 < \delta < 1. \tag{22}$$

The parameter δ denotes the extent to which fundamentalists deviate from initial beliefs when the exchange rate shows a different development. For simplicity, we assume that s^* is exogenous. However, this is not necessarily the case. For example, if fundamentalists deviate from their initial beliefs and subsequently only take into account their revised beliefs, s^* becomes endogeneous and has to be interpreted as the perceived long-run exchange rate level. Using this assumption in the model yields exchange rate dynamics triggered by a random effect in the foreign exchange market as illustrated in Figure 4. Again, as a benchmark case, we choose the specific parameter values of the upper-left diagram of Figure 1. The effects of fundamentalists on the time path of the exchange rate are shown by the bold line in Figure 4. The other line shows the benchmark case from Figure 1. The illustration highlights that, despite the presence of fundamentalists, the exchange rate deviates for quite some time from the long-run level. This is caused by the fundamentalists, who infer from the deviation of the exchange rate from the initial steady state level that the long-run equilibrium value may have changed. The more slowly fundamentalists revert to their initially perceived long-run exchange rate level, the longer the system needs to get back to the inital equilibrium value.

A similar deviation of the mean exchange rate from the long-run equilibrium can be caused if the belief of fundamentalists is biased by a peso problem. In this case, fundamentalists expect a sizable change in the value of at least one fundamental variable at some point in the future. Assuming that they expect a change that leads to a devaluation of the domestic currency, the expected long-run exchange rate exceeds the level justified by current fundamentals. Then, the exchange rate dynamics follow a similar time path as illustrated by the bold line in Figure 4. However, as long as fundamentalists are convinced of the future change in the fundamentals, the mean exchange rate will be different from the initial long-run level. If, at a specific point in time, fundamentalists revise their expectations in the sense that they revert to initial beliefs about the long-run exchange rate, the mean of the oscillations will immediately jump to the initial steady state value. If, by contrast, the

fundamentals change as anticipated, the oscillations will converge to the new steady state value. The more often fundamentalists revise their beliefs about the future time path of fundamentals, the higher will be the volatility of exchange rates. This implies that another reason for short-term exchange rate fluctuations is the change in the beliefs of speculators about the future time path of fundamentals.

Figure 4

Chartists, fundamentalists, and the effects of learning (assumptions: $s_0=1$; $s_1=1.05$; $\square_1=0.2$; $\alpha=0.9$; $\gamma_1=\gamma_2=0.15$)

There is another interesting aspect that can be incorporated into the model with chartists and fundamentalists discussed here. In case of relatively large uncertainties about the fundamentally determined exchange rate, it can be assumed that speculators form some expectation about the true long-run equilibrium value of the exchange rate but that they do not engage in speculative purchases or sales of foreign assets whenever the spot exchange rate deviates from the expected long-run value. Rather, it is more likely that these speculators wait until the spot rate deviates by a certain margin from the

expected long-run level. Denoting the threshold beyond which fundamentalists enter the market by x, their relative demand for foreign assets can be described by the following function:

$$d_t^f = \begin{cases} 0 & if \quad |s_t^f - s_t| < x. \\ \gamma(s_t^f - s_t) & if \quad |s_t^f - s_t| \geq x. \end{cases} \tag{23}$$

Using this demand function in equation (5), which describes the dynamics of the exchange rate, implies that, within a certain zone around the long-run exchange rate level, the spot rate is determined by random factors and chartists only. The demand function (23) can be justified also by taking into account that market participants have heterogenous expectations about the long-run exchange rate level. However, most expected long-run levels may be within a certain band. Hence, only when the spot rate leaves this band, pronounced speculation of fundamentalists are likely to occur.

These considerations can be further modified by assuming, as in Frankel and Froot [8], that chartists believe in a random walk process governing most of the exchange rate dynamics. In this case, the spot rate could indeed be viewed as such a process within a certain margin around the long-run level. The stabilizing effect of fundamentalists would occur only whenever the spot rate leaves this band.[8] A prerequisite of this view of the spot rate dynamics is that the long-run exchange rate level lies indeed within this band and that the belief of speculators about the path of fundamental variables does not constantly change.

Figure 5
Chartists and fundamentalists with an uncertainty zone

V. CHARTISTS, FUNDAMENTALISTS, AND SPECULATORS WITH RATIONAL EXPECTATIONS

We now add to the model speculators with rational expectations (referred to in this model as "rational speculators"). Their demand in the foreign exchange market is given by equation (4), which we repeat here for convenience:

$$d_t^r = \mu E_t(s_{t+1} - s_t). \tag{4}$$

Using this function together with the demand functions (2) and (3) for chartists and fundamentalists, respectively, in the function for total foreign exchange demand (5) yields

$$d_t = \alpha\theta_1(s_t - s_{t-1}) + \alpha\theta_2(s_{t-1} - s_{t-2}) + \beta\gamma(s_t^f - s_t) + (1-\alpha-\beta)\mu E_t(s_{t+1} - s_t) \tag{24}$$

We assume that fundamentalists use the true long-run exchange rate in their portfolio decisions. This demand function implies, as the equilibrium exchange rate,

$$s_t = -\eta\alpha(\theta_1 - \theta_2)s_{t-1} - \eta\alpha\theta_2 s_{t-2} + \eta\beta\gamma s_t^f + \eta(1-\alpha-\beta)\mu E_t(s_{t+1} - s_t) + \eta m_t = \mu_t, \tag{25}$$

where

$$\eta = \frac{1}{1-\alpha\theta_1+\beta\gamma} . \tag{26}$$

Equation (25) implies that the exchange rate is determined by past exchange rate data as well as by expected exchange rate levels. The expected exchange rate affects the demand of fundamentalists and of rational speculators.

Since rational speculators know the dynamics of the system and since the latter is determined by past data as well as by their assumption about the equilibrium exchange rate used by fundamentalists, the future dynamics of the expectation about the future change in the spot rate can be derived from actual data. Thus, equation (25) can be used to derive the exchange rate change that rational speculators expect for the next period:

$$E_t(s_{t+1}-s_t) = \frac{1}{(1-\alpha-\beta)\mu\eta}s_t+\frac{\alpha(\theta_1-\theta_2)}{(1-\alpha-\beta)\mu}s_{t-1}+\frac{\alpha\theta_2}{(1-\alpha-\beta)\mu}s_{t-2}-\frac{\beta\gamma s_t^f+m_t+u_t/\eta}{(1-\alpha-\beta)\mu} . \tag{27}$$

There are three possibilities under which rational speculators may go wrong. First, they may incorrectly anticipate the beliefs of fundamentalists. Second, the actual development of fundamentals (here m_t) may be different from the expected value. Third, a random shock may occur ($u_t \neq 0$). If none of these factors occurs, the participation of rational speculators in the foreign exchange market leads to a further dampening of exchange fluctuations. This is illustrated in Figure 6 by again simulating the dynamic effects of a one-time random shock to the exchange rate in period 1. As before, we compare the dynamics to the spot exchange rate path shown in the benchmark case of Figure 1 (upper-left diagram) with chartists only. In addition, Figure 6 shows the spot exchange rate dynamics in the case in which chartists and fundamentalists are active in the foreign exchange market.

Figure 6

Chartists, fundamentalists, and speculators with rational expectations
(assumptions: $s_0=1$; $s_1=1.05$; $\alpha=0.8$; $\beta=0.1$; $\mu=0.5$)

Figure 6 shows that even in the presense of rational speculators, the spot exchange rate may exhibit cyclical fluctuations. The larger the share of these speculators in the foreign exchange market, the faster the system converges.[9]

As in the version of the model with chartists and fundamentalists only, the exchange rate again may be driven away from the long-run value if there is uncertainty among speculators about the true long-run equilibrium spot rate. Such uncertainty can exist among both fundamentalists and rational speculators. If speculators revise their opinion about the true value of the long-run level only over time, the short-term fluctuations converge only after some time (for example, in the case of a peso problem or the use of wrong fundamentals) or over time (for example, in the case of a learning process) to the true long-run level. Moreover, the revision of expectations later on can trigger additional fluctuations as the exchange rate is likely to jump at the time of the revision of expectations. If these effects are pronounced, the volatility of spot rates in a market with rational speculators or fundamentalists could even be greater than with chartists only. For the same reason, this also holds for the dynamics of the forward exchange rate.

VI. SUMMARY AND CONCLUSIONS

The model shows how the behavior of chartists, fundamentalists, and rational speculators in the foreign exchange market can result in fluctuations of the spot rate triggered by a random shock to the market. Although the exact exchange rate path depends on the extent of the shock and specific parameters in the demand function of the different types of speculators as well as their market share, the heterogeneity of expectations can explain several characteristics of short-term exchange rate developments, which have often been emphasized in empirical studies on exchange rate dynamics. Among these is the observed positive serial correlation of exchange rates in the very short run and the reversal of these "runs" after some time.

The model not only suggests that, contrary to Dornbusch's [4] overshooting model, short term exchange rate changes consist of a number of smaller changes but also shows that Friedman's [10] hypothesis, according to which profitable speculation is always stabilizing, does not always apply. In our framework, destabilizing chartists can make profits, especially in the short run. Furthermore, the model discussed here can be used to explain the often observed lack of efficiency of foreign exchange markets in the short run. Together with uncertainty of fundamentalists about the true long-run equilibrium rate and the possibility of random shocks, the model illustrates why exchange rate projections in the short run are so difficult. Finally, the model also implies characteristics which were emphasized as quasi-rationality for speculative markets.

The disadvantage of the model lies in its ad-hoc character of the specification of the demand functions and the difficulty to directly test it. Instead, the hypotheses underlying the model have to be tested. However, the analyses of the short-term behavior of speculators support, in particular, the described behavior of chartists.

The model can be extended in several directions. For example, the interaction of financial markets and goods markets as well as policy parameters could be introduced more explicitely. In addition, the shares of the speculator groups, which are exogenous in our model, could be modeled as functions of the profitability of the different types of speculators.

NOTES

1. The poor prediction quality of traditional exchange rate approaches was already shown by Meese and Rogoff [22].
2. For simplicity, we abstract from valuation effects of exchange rate changes on relative supply and demand. A more detailed analysis would have to take into account that a depreciation reduces the relative value of domestic assets.
3. Frankel and Froot [8] distinguish between chartists and fundamentalist but model their behavior differently. In particular, in their model chartists assume that a random walk process underlies the exchange rate. In addition, portfolio managers constantly apply a Bayes approach in order to find out whether they should base their own decisions on the predictions of chartists or fundamentalists whose behavior is based on the expectation of some long-term exchange rate concept.
4. With initial values of the exchange rate equal to s_0 and s_1, the parameters are $C_1=s_0-m$ and $C_2=[s_1-m-(s_0-m)R\cos\varphi]/(R\sin\varphi)$.
5. The dynamics shown in figures 1 and 2 do not necessarily imply a zero profit for each chartist. If expectations are heterogeneous, the profit situations of different chartists can be very different. This is consistent with the results of the study of Schulmeister [23].
6. The diagram in Fig. 3 assumes that the share of fundamentalists is 10%.
7. Note that, given the values of θ_1 and θ_2, the dampening effect depends not only on $1 - \alpha$ (the share of fundamentalists among speculators) but also on γ (the elasticity of fundamentalists' demand).
8. This view is consistent with what has been called "quasi-rationality" in the foreign exchange market; see, for example, De Grauwe [3].
9. A more formal consequence is that, with rational speculators, the conditions for the fluctuation to converge are less rigid than otherwise.

REFERENCES

[1] Chinn, M. and Frankel, J. (1991). "Are Exchange Rate Expectations Biased?-Tests for a Cross-Section of 25 Currencies." *NBER Working Paper.* No. 3807.

[2] Cultler, D.M., Poterba, J.M., and Summers, L.H. (1990). "Speculative Dynamics and the Role of Feedback Traders." *American Economic Review, Papers and Proceedings.* May, 63-68.

[3] De Grauwe, P. (1989). *International Money*. Oxford.

[4] Dornbusch, R. (1976). "Expectations and Exchange Rate Dynamics."

Journal of Political Economy. Vol. 78, 255-276.

[5] Edwards, S. (1983). "Floating Exchange Rates, Expectations and New Information." *Journal of Monetary Economics*. Vol. 11, 321-336.

[6] Frankel, J.A. and Chinn, M. (1991). *Exchange Rate Expectations and the Risk Premium: Tests for a Cross Section of 17 Currencies.* Institute for International Economics. Washington.

[7] Frankel, J.A. and Froot, K. (1987). "Using Survey Data to Test Standard Proportions Regarding Exchange Rate Expectations." *American Economic Review.* Vol. 77(1), 133-153.

[8] Frankel, J.A and Froot, K. (1988). "Chartists, Fundamentalists and the Demand for Dollars". *Greek Economic Review.* Vol. 10, 49-102.

[9] Frankel, J.A. and Froot, K. (1990). "Exchange Rate Forecasting Techniques, Survey Data, and Implications for the Foreign Exchange Market." *International Monetary Fund WP/90/42*. Washington.

[10] Friedman, M. (1953). "The Case for Flexible Exchange Rates." In: Friedman, M. (Ed.). *Essays in Positive Economics.* Chicago: University of Chicago Press, 157-203.

[11] Froot, K.A. and Ito, T. (1989). "On the Consistency of Short-run and Long-run Exchange Rate Expectations." *Journal of International Money and Finance*. Vol. 8, 487-510.

[12] Garber, P.M. (1990a). "Famous First Bubbles." *Economic Perspectives.* Vol. 4, 35-54.

[13] Garber, P.M. (1990b). "The Dollar as a Bubble." In: Petri, P., and Gerlitz, S. (Eds.). *The Economics of the Dollar Cycles*. 129-147.

[14] Goodhart, C. (1988). "The Foreign Exchange Market: A Random Walk with a Dragging Anchor." *Econometrica.* Vol. 55, 437-460.

[15] Hartley, P. (1983). "Rational Expectations and the Foreign Exchange Market." In: Frenkel, J.A. (Ed.). *Exchange Rates and International Macroeconomics.* Chicago. University of Chicago Press. 153-188.

[16] Ito, T. (1990). "Foreign Exchange Rate Expectations: Micro Survey Data." *American Economic Review.* Vol. 80, 434-449.

[17] Ito, T. and Roley, V.V. (1988). "Intraday Yen/Dollar Exchange Rate Movements: New or Noise?" *NBER Working Paper. No. 2703*.

[18] Krasker, W. (1980). "The Peso Problem in Testing the Efficiency of Forward Exchange Markets." *Journal of Monetary Economics.* Vol. 6, 269-276.

[19] Lewis, K.K. (1989a). "Can Learning Affect Exchange-Rate Behavior? The Case of the Dollar in the Early 1980s." *Journal of Monetary*

Economics. Vol 23. 79-100.

[20] Lewis, K.K. (1989b). "Changing Beliefs and Systematic Rational Forecast Errors with Evidence from Foreign Exchange." *American Economic Review*. Vol. 79, 621-636.

[21] Liu, P.C. and Maddala, G.S. (1992). "Rationality of Survey Data and Tests for Market Efficiency in the Foreign Exchange Market." *Journal of International Money and Finance.* Vol. 11, 366-381.

[22] Meese, R.A. and Rogoff, K. (1983). "The Out-of-Sample Failure of Empirical Exchange Rate Models: Sampling Errors or Misspecification?" In: Frenkel, J.A. (ed*.) Exchange Rates and International Macroeconomics*. Chicago, 67-105.

[23] Schulmeister, S. (1987). "An Essay on Exchange Rate Dynamics." Working Paper. Austrian Institute of Economic Research. Wien.

[24] Takagi, S. (1991). "Exchange Rate Expectations: A Survey of Survey Studies." *International Monetary Fund. Staff Papers*. Vol. 38, 156-183.

[25] Taylor, M.P. (1989). "Expectations, Risk and Uncertainty in the Foreign Exchange Market: Some Results Based on Survey Data." *The Manchester School.* Vol. 57, 142-153.

[26] Taylor, M.P. and H. Allen. (1992). "The Use of Technical Analysis in the Foreign Exchange Market*." Journal of International Money and Finance.* Vol. 11, 304-314.