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Advancement to the Real Option Models in Valuing R&D

Yu-Wen Lan
Department of Finance and Banking, Lunghwa University of Science and Technology, No. 300, Sec. 1, Wanshou Rd., Guishan, Taoyuan County 33306, Taiwan
93441007@cc.ncu.edu.tw

ABSTRACT

Bellalah (2003b) followed the context of Merton (1987) and firstly incorporated the information cost into the real option model (ROM) in valuing R&D. However, the information cost though affecting R&D's market value has no influence to R&D's payoff. We extend Bellalah's models as to allow exogenous factors to influence R&D's payoff. We also made discussion on the individual effects of each factor and the real level of information cost which were not addressed by Bellalah.

JEL Classifications: *D81, G13*

Keywords: *R&D; Real Options*

I. INTRODUCTION

Merton (1987) asserted the importance of information cost and documented that an investor shall demand higher stock return if higher information cost is expensed. Following the context of Merton, Bellalah (1999a, 2003b) incorporated the information cost factor in valuing both options and R&D. However, in Bellalah's setting only the factors influencing R&D's market value were considered. The truth is that R&D value will depreciate while time elapses; its value could also be vanished overnight because of any unexpected evolution. These facts imply some other exogenous factors which influence the R&D's payoff deserve to be comprehended. This study attempts to modify Bellalah's ROM as to incorporate factors like exponential decay (θ) and Poisson event (ξ) into consideration.

There are three types of information cost defined including the average cost prevailed in market (λ_M), the cost affiliated with R&D options (λ_F) and the cost affiliated with R&D yield's price (λ_P). The disposal in Bellalah (1999a, 2003b) may have caused two issues: first, the individual effect of information costs was unknown and, secondly, the reason of why λ_M , λ_F and λ_P were set to be 4% for example was unknown. For the level of information cost, Bellalah stressed the hardness in defining it and proposed an alternative as to find proxies from derivates markets; though this idea was not taken eventually. We are going to observe the individual effect of information costs; we are also going to actualize Bellalah's proposal to see what the real level of information cost could be and. To sum up, the existing ROM in valuing R&D could either be too optimistic or too pessimistic. This inaccuracy could be caused by either inappropriate model setting or inappropriate parameter level setting; we are trying to reduce the mentioned inappropriateness through both statistical and mathematical means.

II. RE-MODELING

The factors of exponential decay θ and Poisson event ξ are going to be considered. μ means the required rate of return which is the sum of expected capital gain α and dividend δ. While exponential decay and Poisson event are jointly considered, the project value can be:

$$V(P)=\int_0^{\infty}\theta e^{-\theta T}P(1-e^{-(\delta+\xi\phi)T})/(\delta+\xi\phi)dT=\theta P/(\delta+\xi\phi)\left[\int_0^{\infty}e^{-\theta T}dT-\int_0^{\infty}e^{-(\delta+\theta+\xi\phi)T}dT\right]$$
$$=P/(\delta+\theta+\xi\phi)\,. \tag{1}$$

The R&D project value can be deemed the function of product price 'P'. P follows the mean reverting process $dPdt+\alpha PdZ\text{-}\phi P\xi dt,$ ϕ represents the loss rate of the sudden death, ξ represents the probability of sudden death. The revenue of an investment can be expressed as $u=\alpha+\delta$, α is drift term, δ represents dividend yield, μ is the risk adjusted expected rate of return which equals to the risk free rate r under the premise of arbitrage free. The expected present value of cash flow will become:

$E[\int_0^T P_t e^{-\mu t} dt] = \int_0^T P e^{(\alpha-\xi\phi)t} e^{-\mu t} dt = \frac{P}{(\delta+\xi\phi)}[1-e^{-(\delta+\xi\phi)T}]$, which is the integral base of the value of entire investment V. If the stochastic process of exponential decay is further considered, its p.d.f. function will be incorporated as the (1) showed. Through (1), a spiky event like θ and ξ can be smoothened as an additional discount factor in the denominator.

According to ROM, an R&D project value V can be seen as a combination of investment I and option value F therefore V(P)=I+F(P). We may utilize a portfolio $\Phi = F(P) - nP$ as to long one unit of option and to short n units output with price P and let its payoff be:

$$r[F - nP]dt = dF - ndP - n\delta P dt \tag{2}$$

From (2) we can derive a corresponding Bellman equation:

$$(1/2)\sigma^2 P^2 F_{PP} + (r-\delta)PF_P - rF = 0 \tag{3}$$

In (3), we set $n = F'(P)$ to eliminate the disturbance term dz. (3) is a Partial Differential Equation (PDE) and we can solve F by either analytical, if it has a close form solution, or numerical way. When the exponential decay, Poisson event and information cost are jointly considered, the Bellman equation becomes:

$$(1/2)F_{PP}\sigma^2 P^2 + (r-\delta+\lambda_P)F_P P - (r+\xi+\lambda_F)F + \xi F((1-\phi)P) = 0 \tag{4}$$

F solved from (4) is the value of a simple option and we denote it F_1 in latter expressions.

We further consider a complex situation as to let the option compound with succeeding replacement options. P* means a threshold which is optimal to exercise the R&D project. When $P < P^*$, the value of the compound option over next interval is:

$$F = Pdt + (1-\theta dt)e^{-(r+\lambda_F)dt}E[F(P+dP)] + \theta dt e^{-(r+\lambda_{F'})dt}E[F'(P+dP)] \tag{5}$$

This means an installed investment could either survive with probability $(1-\theta dt)$ or die with probability θdt in next short interval. When $P < P^*$, (5) can be expanded as:

$$\begin{aligned} F &= Pdt + (1-\theta dt)(1-(r+\lambda_F)dt)\left[F + F_P(\alpha+\lambda_P)Pdt + (1/2)F_{PP}\sigma^2 P^2 dt - \xi F dt + \xi F((1-\phi)P)dt\right] \\ &+ \theta dt(1-(r+\lambda_{F'})dt)\left[F' + F'_P(\alpha+\lambda_P)dt + (1/2)F'_{PP}\sigma^2 P^2 dt - \xi F' dt + \xi F'((1-\phi)P)dt\right] \end{aligned}$$

when $P > P^*$, (5) can be expanded as:

$$\begin{aligned} F &= Pdt + (1-\theta dt)e^{-(r+\lambda_F)dt}E[F(P+dP)] + \theta dte^{-(r+\lambda_F)dt}E[F(P+dP)-I] \\ &= Pdt + (1-\theta dt)(1-(r+\lambda_F)dt)\Big[F + F_P(\alpha+\lambda_P)Pdt + (1/2)F_{PP}\sigma^2P^2dt - \xi Fdt + \xi F((1-\phi)P)dt\Big] \\ &\quad + \theta dt(1-(r+\lambda_F)dt)\Big[F + F_P(\alpha+\lambda_P)dt + (1/2)F_{PP}\sigma^2P^2dt - \xi Fdt + \xi F((1-\phi)P)dt - I\Big] \end{aligned}$$

The respective Bellman equation becomes:

$$(1/2)F_{PP}\sigma^2P^2 + (r-\delta+\lambda_P)F_PP - (\theta+r+\lambda_F+\xi)F + \theta F' + P = 0 \tag{6}$$

$$(1/2)F_{PP}\sigma^2P^2 + (r-\delta+\lambda_P)F_PP - (r+\lambda_F+\xi)F - \theta I + P = 0 \tag{7}$$

Be noted that (6) and (7) will meet tangentially on P*. F solved from (6) and (7) is the value of a compound option and we denote it F_2 in latter expressions.

III. SIMULATIONS

To illustrate the F_1 and F_2, we shall exploit an industrial case as the background to keep the simulations ‘virtual’. The ‘Local Area Network’ (LAN) industry in Taiwan was selected due to its high R&D orientation. The LAN industry in Taiwan is eye-catching referring to its annual global share 76.5%, 53%, 90.9% and 84% on NIC, Hub / Switch, SOHO router and WLAN (wireless LAN). We focused on the listed LAN companies and collect their financial and stock parameters from both Taiwan Economic Journal (TEJ) and the website of Taiwan Stock Exchange Corporation (TSEC). Sample period is from January 1st, 1999 to March 31st, 2006.

We set the parameters σ, r, δ to equal the practical level and let ξ, λ_F and λ_P innovate in following simulations.

Figure 1
Value plane of F_1

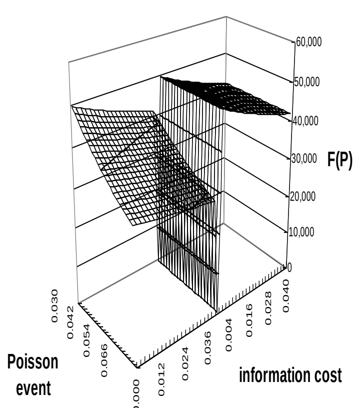


Figure 2
Value plane of F_2

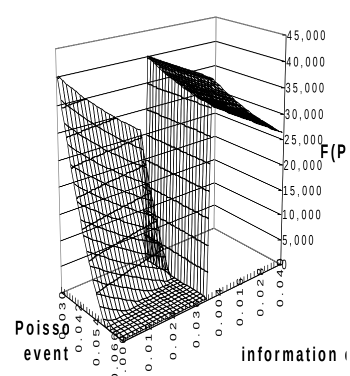


Figure 3
Value plane of F_1
(λ_P moves from 0 to 4%)

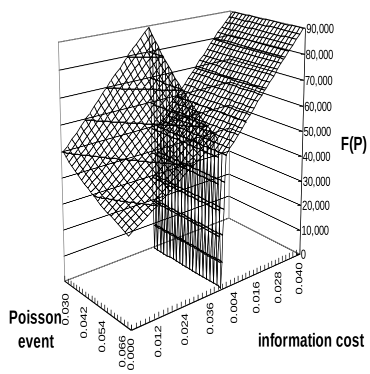
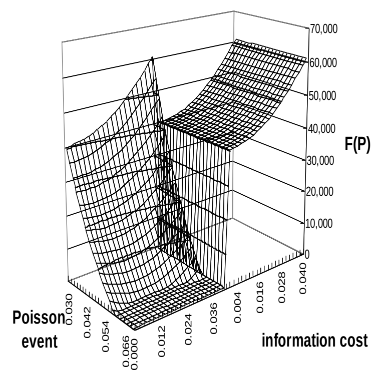


Figure 4
Value plane of F_2
(λ_P moves from 0 to 4%)



Figures 1 and 2 demonstrate the F_1 and F_2 value plane under influence of λ_F and ξ. Figure 3 and 4 demonstrate an additional influence caused by λ_P. In Figure 1, the back (right) plane exhibits F_1 which moves with information cost λ_F while keeping ξ fixed; the front (left) plane exhibits F_1 which moves with information cost λ_F and Poisson event ξ simultaneously. As shown, the plane will mainly incline toward ξ axis if ξ is considered. This expounds that ξ is a more influential factor; the scenario of Figure 2 is similar also. In Figure 3 and 4, we let the λ_P innovate with λ_F, which makes the plane toward information cost axis becoming a positive slope. The result implies that the appreciation of λ_P will raise the option value and partly cancel the influence of λ_F. The value depreciation caused by ξ can somehow be alleviated by the raise of λ_P but not much; ξ is still the major strength to domain the plane. Situations are similar if let the θ join except the influence of θ is minor than ξ. The simulations elucidate two things: first, the incorporation of exogenous factors which influence to R&D's payoff should be important since the new factors outweighs the information cost and, secondly, spending λ_P will improve the stochastic control on price thus a positive relationship with option value was observed.

IV. AN EXPLORATION TO THE LEVEL OF INFORMATION COST

Bellalah (2003b) stressed that the magnitude of information cost is hard to define and proposed an alternative as to collect proxies from derivatives markets. We are going to actualize Bellalah's idea to find these proxies. The plausibility of proxies will be tested by the regression analysis:

$$\beta_{it} = \alpha_0 + \alpha_1 Finan_{it} + \alpha_2 DE_{it} + \alpha_3 LQ_{it} + \alpha_4 ROE_{it} + \sum_{k=1} \alpha_{5k} (V_j / S)_{i,t-k} + u_{it} \quad (8)$$

i denotes the sample companies, j=1,2, V_j=I+F_j, t denotes time. β means the beta coefficient belonging to $CAPM_i$ which represents the risk level. Since the higher R&D investment will incur a higher company's risk (Black and Scholes, 1976; Ho et al., 2004), we take β as a dependent variable to be regressed and a positive coefficient of V_j/S is expected. The financial leverage (Finan), debt-equity ratio (DE), liquidity (LQ) and profitability (ROE) are comprehended as control variables. We let V be divided by contemporaneous sales to eliminate the idiosyncratic scale effect. (8) implies that β is a function of multi-period R&D value. Be noted the multicollinearity could happen on V_j/S therefore a polynomial distributed lags (PDL) technique is exploited here.

Before collecting the proxies of information costs, we need to clarify two issues including what the adequate proxy should be and how the proxy can be collected. We followed Amihud and Mendelson (1989) who asserted that the bid-ask spread an adequate proxy of information cost. In order to collect proxies adequately, this paper utilized a bulletin called 'statistics of close' (code TF7) extracted from the intraday data of Taiwan Economic Journal (TEJ). In this bulleting, the final and correspondent bid (column #9) and ask (column #10) prices within the last hour before close were recorded. According to Amihud and Mendelson (1989)'s definition, the information cost can thus be estimated by $|P_{bid} - P_{ask}|$.

When the data of either options or futures are exploited, the simultaneously existing contracts (with its different term) shall confuse us while getting the information cost. To solve this poser, we followed the Chicago Board Options Exchange (CBOE) disciplines in estimating the volatility index (VIX). CBOE demands the contract series of 'near-the-money', 'nearby' and 'second-nearby' being applied for VIX estimation. For the contracts with days less than six to the expiration, CBOE demands the contract series of second-nearby and third-nearby being applied to avoid the possible fluctuation on price, see also Whaley (2000). λ_M, λ_F and λ_P can be estimated by Taiwan weighted stock index (TAIEX), stock options and common stocks. Anyway, we utilized the data from Taiwan stock index options (TXO) for λ_M and Taiwan electronics options (TEO) for λ_F since there's no TAIEX transactions and no individual stock options offered by sample companies. The proxies collected from markets are deemed the real level of information cost.

Table 1 shows the situation while $\lambda_M = \lambda_F = \lambda_P = 0$. In Table 2 we start to consider the non-zero situation and let the cost be either Bellalah's (2003b) or real level. The AdjR^2 slightly changed between Table 1 and 2 while letting the cost be the Bellalah's level. The change becomes remarkable if let the cost be the 'real'. Be noticed that the averaged λ_M, λ_F and λ_P are 2.14%, 23.24% and 0.23%; which is much different with Bellalah's setting. Be noted, the universal 4% level set by Bellalah could be a random level just for illustration. However, comparing with the respective real information level 2%, 20% and 0.2%, the large difference could reflect that the Bellalah's setting is too unreal. According to Table 2, the explanation power of option value to the R&D value is much improved (the adjR^2 is averagely 48% improved) if the actual level of information cost is adopted, an evidence of Bellalah's unreality. Of course, this inference can be the yield of market bias thus more empirical research other than the Taiwan case shall be needed; based upon the same reason, the discussion here is heuristic.

Table 1
The explanatory power of different R&D value approaches

Dependent Var.: CAPM$_i$'s β							
	C	Finan	DE	LQ	ROE	$V_j/S(\sum_{k=1}^{52}\alpha_{5k})$	AdjR^2
V_1/S	-1.32 (-12.34)***	1.94 (21.33)***	-0.36 (-8.89)***	0.00 (2.62)***	1.32 (4.77)***	(13.18)***	0.61
V_2/S	-0.12 (-10.20)***	1.96 (19.37)***	-0.44 (-10.50)***	0.00 (2.04)***	1.28 (4.33)***	(9.25)***	0.55

$p<0.1^{*}$, $p<0.05^{**}$, $p<0.01^{***}$

Table 2
The explanatory power influenced by information cost

	$\lambda_M=\lambda_F=\lambda_P=5\%$		real λ_M, λ_F and λ_P	
	$\sum_{k=1}^{52}\alpha_{5k}$	AdjR^2	$\sum_{k=1}^{52}\alpha_{5k}$	AdjR^2
V_1/S	(13.547)***	0.618	(7.798)***	0.859
V_2/S	(8.782)***	0.545	(7.571)***	0.859

$p<0.1^{*}$, $p<0.05^{**}$, $p<0.01^{***}$

V. CONCLUSION

The Bellalah's (1999a and 2003b) model can though depict the change of R&D's market value due to the spillover effect of information collection, it cannot figure the change of R&D's payoff due to the competitor's activity. This makes Bellalah's model deviating to the reality. We made extension to Bellalah's models as to incorporate exogenous factors including exponential decay θ and Poisson event ξ for compensation on aforesaid deficiency.

The influence of information cost onto R&D value is roughly half to exponential decay θ and one third to Poisson event ξ, indicating that the new added factors outweigh the information cost as well as support our modeling extension. Bellalah (1999a, 2003b) did not observe the information cost individually but a lump-sum effect instead; we made an individual survey and found that the information cost affiliated with price λ_P moves conversely from the others. This finding implies that the cost in pursuing a more adequate price will boost the R&D value.

Bellalah (2003b) commented that the information cost is hard to define therefore suggested to find proxies for replacement from the derivates markets. However, such an idea was not eventually executed but who merely set a random level of information cost instead. We actualized Bellalah's idea and propose a working frame as to exploit the ways of volatility indices estimation. The average level of the proxies of λ_M, λ_F and λ_P are 2.14%, 23.24% and 0.23%. The real level is much different than the presumable recognition and evidences us the better predictability on β - this helps investors being more prudent because he knows better the risk level what have borne by his portfolio.

This paper excavated a bias problem of the Bellalah's (1999a, 2003b) model as the parameters of R&D's depreciation (θ) and sudden death (ξ) did not incorporated. Such an omission will cause the over optimistic tendency while estimating the R&D value. The much stronger effect of θ and ξ than the information cost (λ) which was solely focused by Bellalah proved the necessity of this incorporation. Moreover, Bellalah used the 4% level of information cost for illustration could be too unreal; the true level of λ_M, λ_F and λ_P are close to 2%, 20% and 0.2%, much deviated from the 4% setting. This difference is remarkable nevertheless a suspicious factor of market bias is considered. I found the cost of λ_P will heave the R&D option value, this phenomenon was not addressed by Bellalah; I made a discussion and an explanation on it as well.

APPENDIX

An analytical solution of F_1 and F_2 can be obtained when $\phi = 1$. The process to solve F_1 is introduced herewith: let $F = A_1 P^{\beta_1}$, then its differentiation form can be acquired as $F' = \beta_1 A_1 P^{\beta_1 - 1}$, and $F'' = \beta_1(\beta_1 - 1) A_1 P^{\beta_1 - 2}$; according to equation (1), its fundamental quadratic form can be written as:

$$\frac{1}{2}\beta_1(\beta_1 - 1)\sigma^2 A_1 P^{\beta_1} + (r - \delta + \lambda_P)\beta_1 A_1 P^{\beta_1} - (r + \xi + \lambda_F) A_1 P^{\beta_1} = 0,$$

The solution of β_1 will thus be:

$$\frac{1}{2} - (r - \delta + \lambda_P)/\sigma^2 + \sqrt{((r - \delta + \lambda_P)/\sigma^2 - \frac{1}{2})^2 + 2(r + \xi + \lambda_F)/\sigma^2}\ .$$

Three boundary conditions including 'value matching', 'smooth pasting' and 'absorption' mentioned in our text will be applied in order to solve the variables in 'F' (option value). Through the mentioned conditions we shall get $P^* = \dfrac{\beta_1^* I(\theta + \delta + \xi\phi)}{\beta_1^* - 1}$, and

$A_1^* = \dfrac{(P^* - I(\theta+\delta+\xi\phi))/(\theta+\delta+\xi\phi)}{(\beta_1^* I(\theta+\delta+\xi\phi)/(\beta_1^*-1))^{\beta_1^*}}$. A_1^* can be re-written in a simpler form:

$A_1^* = I^{(1-\beta_1^*)}(\theta+\delta+\xi\phi)^{-\beta_1^*}(\beta_1^*-1)^{\beta_1^*-1}\beta_1^{*-\beta_1^*}$; F_1 is solved.

The deriving procedure of F_2 will be more complicate since it belongs to a compound option. According to equation (2) and (3) and their corresponding fundamental quadratic, β_1, β_2 and $\beta_1^{'}$ will be the first batch of variables we are going to solve. We do not address details on β_2 since it is just the negative root dealing in same way as β_1, the solution of $\beta_1^{'}$ is slightly different from β_1 and β_2 since its quadratic function is different, the solution of $\beta_1^{'}$ is:

$$\beta_1^{'*} = \frac{1}{2} - (r-\delta+\lambda_P)/\sigma^2 + \sqrt{((r-\delta+\lambda_P)/\sigma^2 - \frac{1}{2})^2 + 2(\theta+r+\xi+\lambda_F)/\sigma^2} \ .$$

The way of partial differential equation (PDE) techniques used by Hull (2000) will be utilized here. According to Hull, we shall express the option value as:

$F = A_1P^{\beta_1} = B_1P^{\beta_1^{'}} + P/(\theta+\delta+\xi\phi) + A_1P^{\beta_1} - 1$. Its first order differential become $F^{'} = \beta_1A_1P^{\beta_1-1} = \beta_1^{'}B_1P^{\beta_1^{'}-1} + 1/(\theta+\delta+\xi\phi) + \beta_1A_1P^{\beta_1-1}$. We are going to use the three conditions again plus with the fourth condition called 'tangency' as the outcome of (6) and (7) should be congruent on the point of P*. We shall have:

$P^* = \dfrac{\beta_1^{'*}I(\theta+\delta+\xi\phi)}{\beta_1^{'*}-1}$, $B_2^* = \dfrac{\beta_1^*B_1^*P^{\beta_1^{'*}} + (P^*\beta_1^*/\theta+\delta+\xi\phi) + (\theta I\beta_1^*/r) - (P^*(\beta_1^*-1)/\delta+\xi\phi)}{P^{*\beta_2^*}(\beta_1^*-\beta_2^*)}$,

$A_1^* = \dfrac{\beta_2^*B_2^*P^{*\beta_2^*} + (P^*/\delta+\xi\phi)}{\beta_1^*P^{*\beta_1^*}}$. F_2 is thus solved.

An example may help readers to practice and to verify as well. Let the revenue's annual variance of an R&D project be $\sigma = 0.20$, risk free rate $r = 0.10$, information cast $\lambda_P = \lambda_F = 0.02$, dividend yield $\delta = 0.08$, exponential decay rate $\theta = 0.05$, probability of a Poisson event $\xi = 0.05$, and R&D investment $I = 100{,}000$; through the derived equations above, we can obtain $\beta_1 = 2.458$, $\beta_2 = -3.458$, $\beta_1^{'} = 2.854$, the F_1 threshold price P^*=30,345, the F_2 threshold price P^*=27,708, and, the most important, the option value F_1=68,585, and F_2=72,934.

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