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A General Formula for the WACC: A Reply

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ABSTRACT

Farber, Gillet and Szafarz (2006) propose a general formula for the WACC in which the expected return on the tax shield appears explicitly. The classical Modigliani-Miller and Harris-Pringle WACC formulas for specific debt policies are then derived from the general formula after having determined the corresponding tax shield expected returns. Replying to Fernandez' (2007) comment, this note explores, in addition, the validity of the general formula in the Miles-Ezzel setup with annual adjustment of the level of debt to maintain a constant market-value debt ratio.

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I. INTRODUCTION

In his comment, Fernandez (2007) claims that the general formulation of the WACC proposed in Farber, Gillet, Szafarz (FGS, 2006) is only correct under the assumption that the required returns are constant over time. Moreover, he argues that "it is not possible to derive a debt policy such that the appropriate discount rate for the tax shield is r_A in all periods" (where r_A is the cost of capital for the unlevered firm).

These statements are incorrect. First, the general formula of the WACC in FGS (2006) is derived from the equality between the average return on both side of the market-value balance sheet. It should be satisfied at any point in time and, more importantly, it does not imply that the WACC is a constant. Of course, if all variable entering the formula are constant, the resulting WACC is a constant. But, as shown in this paper, a constant WACC can also be obtained when some of the variables vary over time.

Second, as shown by several authors (see, for instance, Taggart, 1991 or Arzac and Glosten 2005), the expected return on the tax shield is equal to the cost of capital of the unlevered firm for a leverage policy with continuous rebalancing of the level of debt to maintain a fixed leverage ratio. This financial policy was initially analyzed by Harris and Pringle (1985). Fernandez' comment refers to the Miles and Ezzell (1980) setting where the firm rebalances its debt once a year instead of continuously, a case not considered in FGS (2006).

The rest of this paper proceeds as follow. Section 1 shows that the formulation of the WACC in FGS (2006) is indeed general and does not require assuming constant returns over time. Section 2 derives the classical WACC formulas of Modigliani Miller (1963) and Harris Pringle (1985), the latter corresponding to the Miles and Ezzel (1980) case with continuous rebalancing. Section 3 examines whether the general formula applies in a Miles-Ezzel setup with annual rebalancing and arbitrary cash-flows. Section 4 considers the use of the general formula under perpetual growth. General conclusions are presented in section 5.

II. THE GENERAL WACC FORMULA

The general formula for the WACC (Equation (18) in FGS, 2006) is:

$$\mathrm{WACC} = r_A\left(1-\frac{VTS}{V}\right) - r_D T_C \frac{D}{V} + r_{TS}\frac{VTS}{V} \tag{1}$$

where r_A, r_D and r_{TS} are the expected returns respectively on the unlevered firm, the debt and the tax shield, V is the value of the levered firm, VTS is the value of the tax shield, D is the value of the debt and T_C is the corporate tax rate.

This formula is derived from the balance sheet identity and the definition of the weighted average cost of capital (for a similar presentation, see Berk and De Marzo, 2007). It should therefore be verified at any point in time, whatever returns are annually or continuously compounded.

To exhibit this, start with the accounting identity (time indices have been added for clarification):

$$V_t = VU_t + VTS_t = E_t + D_t \tag{2}$$

where VU_t is the value of the unlevered firm.

Assuming, for simplicity, that the corporate tax rate is constant over time, the weighted average cost of capital is defined as:

$$WACC_t \equiv r_{E,t}\frac{E_t}{V_t} + r_{D,t}(1 - T_C)\frac{D_t}{V_t} \tag{3}$$

From (2), the expected returns on both side of the balance sheet should be equal:

$$r_{A,t}\frac{VU_t}{V_t} + r_{TS,t}\frac{VTS_t}{V_t} = r_{E,t}\frac{E_t}{V_t} + r_{D,t}\frac{D_t}{V_t} \tag{4}$$

As:

$$VU_t = V_t - VTS_t \tag{5}$$

and, from (3)

$$r_{E,t}\frac{E_t}{V_t} = WACC - r_{D,t}(1 - T_C)\frac{D_t}{V_t} \tag{6}$$

the general formula for the WACC is:

$$WACC_t = r_{A,t}(1 - \frac{VTS_t}{V_t}) - r_{D,t}T_C\frac{D_t}{V_t} + r_{TS,t}\frac{VTS_t}{V_t} \tag{7}$$

This equation is the general formula in FGS (2006).

III. THE MODIGLIANI-MILLER AND HARRIS-PRINGLE FORMULAS

Although Eq. (7) is fully general, its practical applicability requires additional conditions. Indeed, when the WACC is constant over time, the value of a levered firm can be computed by discounting with the WACC the unlevered free cash-flows. Therefore, we pay a special attention to the special cases that make the WACC constant. The resulting particular formulas can also be found in textbooks (Brealey et al., 2006; Ross et al., 2005). As in all papers devoted to this topic, we assume here that $r_{A,t} = r_A$ and $r_{D,t} = r_D$ are constant.

First, Modigliani and Miller (1963) assume that the level of debt D is constant. Then, as the expected after-tax cash-flow of the unlevered firm is fixed, VU is also constant. By assumption, $r_{TS} = r_D$ and the value of the tax shield is $VTS_t = T_C D$. Therefore, the value of the firm V is a constant and the general WACC formula (7) simplifies to a constant WACC:

$$\mathrm{WACC_t} = r_A - r_A T_C L \text{ for all t} \tag{8}$$

with L = D/V.

Second, Harris and Pringle (1985) assume that the level of debt is rebalanced continuously so as to maintain a constant debt ratio D_t/V_t = L. Moreover, as the level of debt is always proportional to the value of the unlevered firm, the expected return on the tax shield is the same as the cost of capital of the unlevered firm (r_{TS} = r_A). As a consequence, this implies a constant WACC:

$$\mathrm{WACC_t} = r_A - r_D T_C L \text{ for all t} \tag{9}$$

where r_A and r_D are continuously compounded returns (see: Taggart, 1991, and Arzac and Glosten, 2005). It is worth noting that formula (9) does not require the calculation of VTS. Moreover, Eq. (9) shows that, with constant r_A, r_D and T_C , a constant WACC may only be result from a constant debt ratio L. Any other financial policy makes the WACC time-varying.

IV. MILES-EZZEL WITH ANNUAL REBALANCING AND ARBITRARY CASH-FLOWS

We now apply the general formula (7) to the Miles and Ezzel (1980) setup where the debt is rebalanced once a year and cash-flows are arbitrary, a case not considered in FGS (2006).

Formula (7) can be written as:

$$\mathrm{WACC_t} = r_A - r_D T_C L + (r_{TS,t} - r_A)\frac{VTS_t}{V_t} \tag{10}$$

With arbitrary cash-flows, both $r_{TS,t}$ and VTS_t /V_t may vary over time. Under what condition does the WACC remain constant?

From Eq. (10), the condition is: the product ($r_{TS,t}$ - r_A)(VTS_t/V_t) is constant. This condition is met by the Miles-Ezzell leverage policy with annual rebalancing. Indeed, these authors obtain the following value for the tax shield at time t:

$$VTS_t = \frac{T_C r_D L V_t}{1 + r_D} + \frac{VTS_{t+1}}{1 + r_A} \tag{11}$$

The next year tax shield is discounted at the cost of debt whereas the expected future value of the tax shield is discounted at the cost of capital of the unlevered firm.

By definition, the expected return on the tax shield is:

$$r_{TS,t} = \frac{T_C r_D L V_t + (VTS_{t+1} - VTS_t)}{VTS_t} \tag{12}$$

Using (11):

$$VTS_{t+1} - VTS_t = r_A VTS_t - \frac{T_C r_D)(1 + r_A)V_t}{1 + r_D} \tag{13}$$

Therefore:

$$r_{TS,t} = r_A + (r_D - r_A)\frac{T_C r_D L}{1 + r_D} \cdot \frac{V_t}{VTS_t} \tag{14}$$

Formula (14) shows that $r_{TS,t}$ is inversely related to the fraction of the value accounted by the tax shield. For instance, with a finite horizon T and $VTS_T = 0$, the value of the tax shield at time T-1 is obtained from (11) as :

$$VTS_{t-1} = \frac{T_C r_D L V_{t-1}}{1 + r_D} \tag{15}$$

and:

$$r_{TS,t-1} = r_D \quad (16$$

Since the dynamics of the unlevered cash-flows is arbitrary, V_t may well vary with time, so that both $r_{TS,t}$ and VTS_t / V_t change as well. However, the WACC remains constant because:

$$(r_{TS,t} - r_A)\frac{VTS_t}{V_t} = (r_D - r_A)\frac{T_C r_D L}{1 + r_D} \text{ for all t} \tag{17}$$

Replacing in the general formula (7), we obtain the Miles-Ezzell formula for the WACC with annual rebalancing:

$$WACC_t = r_A - r_D T_C L \frac{1 + r_A}{1 + r_D} \text{ for all t} \tag{18}$$

V. MILES EZZEL WITH ANNUAL REBALANCING AND CONSTANT GROWTH

Consider now a firm whose (unlevered) free cash-flows grow at a constant rate *g*. In order to derive the expected return on the tax shield for this firm, let us start from the formula for VTS_t obtained by Arzac and Glosten (2005) (their Eq. (13)):

$$VTS_t = \frac{T_C r_D D_t (1 + r_A)}{(r_A - g)(1 + r_D)} \tag{19}$$

Therefore, the constant ratio of the value of the levered firm accounted to the value the tax shield is:

$$\frac{V_t}{VTS_t} = \frac{(r_A - g)(1 + r_D)}{T_C r_D L(1 + r_A)} \tag{20}$$

Replacing in (14), the constant expected return on the tax shield is given by:

$$r_{TS} = r_A + (r_D - r_A)\frac{(r_A - g)}{(1 + r_A)} \tag{21}$$

With perpetual constant growth, the expected return on the tax shield is constant over time and positively related to the growth rate. In the special case where g = 0, it simplifies to:

$$r_{TS} = r_A \frac{1 + r_D}{(1 + r_A)} \tag{22}$$

where the ratio $(1+r_D)/(1+r_A)$ captures the impact of annual rebalancing (as opposed to continuous rebalancing) on the expected return of the tax shield.

Finally, appropriate replacements in the general formula lead again to Eq. (18), the Miles-Ezzell formula with annual rebalancing.

VI. CONCLUSIONS

The formula proposed in FGS (2006) is general as claimed originally. It is a time-independent identity which does not depend on the model used to value the tax shield. The WACC formulas of Modigliani-Miller (constant debt) and Harris-Pringle (constant debt ratio with continuous rebalancing) can be seen as special cases of this general formula. Moreover, in the Miles-Ezzell setup (constant debt ratio with annual rebalancing), a constant WACC is obtained because variations in the expected return on the tax shield compensate variations in the fraction of the firm accounted by the value of the tax shield.

Whether the Miles and Ezzel model is realistic or not regarding firms' debt structure remains an open question lying outside the scope of the current paper. Further work could also address the discrete vs. continuous time scale assumptions and their consequence regarding the WACC calculations.

Finally, we thank Pablo Fernandez for pointing out the importance and complexity of these issues and offering us the possibility to clarify our point and exhibiting additional applications of the WACC general formula.

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