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Real Options: Valuation of the Option to Invest Including Corporate Tax and Information Costs

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ABSTRACT

The valuation of the option to invest has been established by Dixit and Pindyck (1994). Using their methodology (Diffusion process, Ito's lemma, partial differential equation, boundary conditions), it is possible to determine the value of a project, including costs of information and corporate tax. The underlying assumptions of the formula are that input and output define Brownian geometrical motions, with the same Wiener increment, and that the manufactured activity is dependent on the selling price of the products. All of this enables to obtain a valuation formula of the real option to invest, which is compatible with Dixit and Pindyck's results.

JEL Classification: *G20, G31*

Keywords: *Real options; Information costs*

I. INTRODUCTION

An important part of the recent financial literature is dedicated to the contribution of the models, based on real options, to assess a project of investment and determine the optimal timing to invest, considering the perspectives of a project's future cash flows. The goal of this paper is to present a model which suggests integrating the costs of information and taxation into capital budgeting based on the use of the real options. This approach joins in a double lineage. Indeed, it consists at first in supposing, in the continuation of Merton (1987) that the investor has to engage costs to analyze the appropriate information for the project. Furthermore, this approach is based on the methodology of Dixit and Pindyck (1994), who consider that the price of a product defines a Brownian geometrical motion. The value of the project, which consists in manufacturing the aforementioned product, then arises from the resolution of an equation in the partial derivatives stemming from the application of the Ito's lemma and from usual boundary conditions (*value matching* and *smooth pasting*). Besides, it integrates the value of the option to invest or to delay the date of the investment, which can be specifically determined.

In this context, this paper is divided into three parts. The first part presents the formalization of the value of a project of investment in the presence of costs of information and corporate tax. The second part is centered, in this context of imperfect information and taxation, on the determination of the value of the option to invest at the most convenient date. The decision to realize the project of investment obliges the company to exercise this option. Then it bears a sunk cost, which corresponds to the value of the option that is de facto definitively given up. The third part proposes a series of numerical simulations, which underlines the characteristics of this approach of capital budgeting.

II. VALUE OF A PROJECT INCLUDING CORPORATE TAX AND INFORMATION COSTS

The model, which is presented hereafter, is based on the principle according to which the output price P_t of a product, as well as the input price C_t, defines a Brownian geometrical motion. So:

$$dP_t = \alpha_p P_t dt + \sigma P_t dB_t \tag{1}$$

and

$$dC_t = \alpha_c C_t dt + \sigma C_t dB_t \tag{2}$$

where α_p and α_c represent respectively the trend of the evolution of the selling price of the product and its unit production cost. Besides, σ corresponds to the instantaneous volatility for the production, which is related to the project and dB_t is the increment of a Wiener process. This allows integrating the uncertainty, which is inherent to the market of the product.

Equation (1) allows deducting that:

$$d\ln P_t = \ln P_t - \ln P_0 = \ln\frac{P_t}{P_0} = \left(\alpha_p - \frac{\sigma^2}{2}\right)t + \sigma B_t \tag{3}$$

Hence,

$$\frac{P_t}{P_0} = e^{(\alpha_p - \frac{\sigma^2}{2})t + \sigma B_t} \tag{4}$$

And finally,

$$P_t = P_0 \cdot e^{(\alpha_p - \frac{\sigma^2}{2})t} e^{\sigma B_t} \tag{5}$$

In the same way,

$$C_t = C_0 \cdot e^{(\alpha_c - \frac{\sigma^2}{2})t} e^{\sigma B_t} \tag{6}$$

We now suppose that R_t is the after tax cash flow generated by the project at the date t, Q_t is the quantity produced at the same date t, and τ is the corporate tax rate. From then on, $R_t = (P_t - C_t)Q_t \cdot (1-\tau)$.

Assuming that the manufactured quantity depends on the selling price of product, it is henceforth possible to note $Q_t = P_t^b$. So,

$$\begin{aligned} R_t &= (P_t - C_t)Q_t \cdot (1-\tau) \\ &= (1-\tau)P_0^b e^{b(\alpha_p - \frac{\sigma^2}{2})t} e^{b\sigma B_t} \left(P_0 e^{(\alpha_p - \frac{\sigma^2}{2})t} e^{\sigma B_t} - C_0 e^{(\alpha_c - \frac{\sigma^2}{2})t} e^{\sigma B_t} \right) \end{aligned} \tag{7}$$

Hence:

$$R_t = (1-\tau)P_0^b e^{b(\alpha_p - \frac{\sigma^2}{2})t} e^{(b+1)\sigma B_t} \left(P_0 e^{(\alpha_p - \frac{\sigma^2}{2})t} - C_0 e^{(\alpha_c - \frac{\sigma^2}{2})t} \right) \tag{8}$$

Or still

$$R_t = Ke^{(b+1)\sigma B_t} \cdot F(t) \quad \text{and} \quad K = (1-\tau)P_0^b \tag{9}$$

and

$$F(t) = e^{b(\alpha_p - \frac{\sigma^2}{2})t} \left(P_0 \cdot e^{(\alpha_p - \frac{\sigma^2}{2})t} - C_0 \cdot e^{(\alpha_c - \frac{\sigma^2}{2})t} \right). \tag{10}$$

From then on, by applying Taylor's formula to R_t which is a function of both variables B_t and t and by truncating the expression in the order 2, we have

$$dR_t = \frac{\partial R_t}{\partial t} dt + \frac{\partial R_t}{\partial B_t} dB_t + \frac{1}{2}\frac{\partial^2 R_t}{\partial B_t^2}(dB_t)^2 \tag{11}$$

As far as dB_t is supposed to be the increment of a Wiener process $dB_t = \varepsilon\sqrt{dt}$, where ε is a normally distributed random variable with a zero mean and a standard deviation of 1. Moreover, $(dB_t)^2 = \varepsilon^2 dt$ and $Var(\varepsilon) = E(\varepsilon^2) - [E(\varepsilon)]^2$, where E is the mean and Var is the variance.

So, as $[E(\varepsilon)] = 0$, it can be deducted that $E(\varepsilon^2) = 1$. Consequently, $E(\varepsilon^2 dt) = dt$. Besides, $Var(\varepsilon^2 dt) = (dt)^2 Var(\varepsilon^2)$, which converges towards 0 when dt aims towards 0. Then, it can be gathered that $\varepsilon^2 dt$ is equal to dt when dt is very small. So,

$$dR_t = \frac{\partial R_t}{\partial t} dt + \frac{\partial R_t}{\partial B_t} dB_t + \frac{1}{2}\frac{\partial^2 R_t}{\partial B_t^2} dt \tag{12}$$

Consequently,

$$\begin{aligned} dR_t &= Ke^{(b+1)\sigma B_t} \cdot F'(t)dt + K(b+1)\cdot\sigma\cdot e^{(b+1)\sigma B_t} \cdot F(t)dB_t \\ &\quad + \frac{1}{2}(b+1)^2\sigma^2 K e^{(b+1)\sigma B_t} \cdot F(t)dt \\ &= R_t(b+1)\sigma\cdot dB_t + R_t\left[\frac{1}{2}(b+1)^2\sigma^2 + \frac{F'(t)}{F(t)}\right]dt \\ &= R_t(b+1)\sigma\cdot dB_t + R_t \cdot f(t)\cdot dt \end{aligned} \tag{13}$$

where $f(t) = \frac{1}{2}(b+1)^2\sigma^2 + \frac{F'(t)}{F(t)}$ and is different from 0.

To determine the V value of the project, an arbitrage portfolio can be constituted. It consists in buying the project and in selling n units of cash flow, n being determined so that the portfolio is risk free. The holding of the asset corresponding to the project enables to receive the Rdt income on the brief interval of time (t, t+dt). Besides, because of the short position on a unit of cash flow a dividend has to be paid to the holder of the long position. By considering a δ dividend yield, the dividend paid on the (t, t+dt) interval of time by the holder of a short position on n units of cash flow stands is equal to $n\delta Rdt$. So, the holder of the arbitrage portfolio receives, on the (t, t+dt) interval of time, a $(R - n\delta R)dt$ net dividend, of which Rdt because he has a long position on the project and $-n\delta Rdt$ because of his short position. Moreover, he realizes a capital gain, which is equal to $dV(R) - ndR_t$.

The formalization of the amount of the capital gain can be obtained by applying directly the lemma of Ito in V, which is a function of both variables R_t and t. So,

$$dV(R,t) = \frac{\partial V}{\partial t}dt + \frac{\partial V}{\partial R}dR + \frac{1}{2}R^2(b+1)^2\sigma^2\frac{\partial^2 V}{\partial R^2}dt \tag{14}$$

with $dR = R\cdot f(t)\cdot dt + R(b+1)\sigma\cdot dB_t$.

By considering that the project can be delayed in perpetuity: $\frac{\partial V}{\partial t}=0$. So,

$$\begin{aligned} dV(R,t) &= \frac{\partial V}{\partial R_t}dt + \frac{1}{2}R^2(b+1)^2\sigma^2\frac{\partial^2 V}{\partial R^2}dt \\ &= V'(R)\left[R(b+1)\sigma\cdot dB_t + R\cdot f(t)\cdot dt\right] + \frac{1}{2}R^2(b+1)^2\sigma^2 V''(R)dt \end{aligned} \tag{15}$$

To simplify the writings, $dV(R,t)$ will be noted $dV(R)$ below. Consequently,

$$\begin{aligned} dV(R) - ndR_t &= V'(R)\left[R(b+1)\sigma\cdot dB_t + R\cdot f(t)\cdot dt\right] + \frac{1}{2}R^2(b+1)^2\sigma^2 V''(R)dt \\ &\quad - n\left[R\cdot f(t)\cdot dt + R(b+1)\sigma\cdot dB_t\right] \\ &= \left\{Rf(t)\left[V'(R)-n\right] + \frac{1}{2}R^2(b+1)^2\sigma^2 V''(R)\right\}dt + R(b+1)\sigma\left[V'(R)-n\right]dB_t \end{aligned} \tag{15}$$

Besides, by choosing n=V'(R), the global payment of the arbitrage portfolio's owner is equal to

$$(R - n\delta R)dt + dV(R) - ndR_t = \left[R - \delta RV'(R) + \frac{1}{2}R^2(b+1)^2\sigma^2 V''(R)dt\right]. \tag{16}$$

On principle, return of an arbitrage portfolio is equal to the r risk free rate. However, considering the λ_V costs of information, which have to be paid to study the project, and the λ_R costs of information, which are related to the cash flows analysis, the project return must be equal to $(r+\lambda_V)$ and the return of the future cash flows must be equal to $(r+\lambda_R)$. It will be supposed, afterwards, that $\lambda_V > \lambda_R$. These parameters represent sunk costs, which must be engaged before realizing a project of investment. That is why it is advisable to integrate them into any discount calculation.

On the dt brief interval of time, the project return is $(r+\lambda_V)V(R)dt$ and the return of the n - or V'(R) - sold cash flows return is $(r+\lambda_R)Rdt$. Then:

$$\left[R - \delta RV'(R) + \frac{1}{2}R^2(b+1)^2\sigma^2 V''(R)\right]dt = (r+\lambda_V)V(R)dt - RV'(R)(r+\lambda_R)dt \tag{17}$$

The grouping of the terms of Equation (17) and the simplification by dt drives to the following differential equation

$$\underbrace{\frac{1}{2}R^2(b+1)^2\sigma^2 V''(R) + (r+\lambda_R-\delta)RV'(R) - (r+\lambda_V)V(R)}_{\text{Homogeneous part}} + \underbrace{R}_{\substack{\text{second}\\ \text{member}}} = 0 \tag{18}$$

By using the principle according to which the function $R \mapsto AR^{\overline{\beta}}$ satisfies the equation

$$\frac{1}{2}R^2(b+1)^2\sigma^2V''(R) + (r+\lambda_R - \delta)RV'(R) - (r+\lambda_V)V(R) = 0, \qquad (19)$$

it is possible to write down the associated quadratic equation

$$\frac{1}{2}R^2(b+1)^2\sigma^2A\overline{\beta}(\overline{\beta}-1)R^{\overline{\beta}-2} + (r+\lambda_R - \delta)RA\overline{\beta}R^{\overline{\beta}-1} - (r+\lambda_V)AR^{\overline{\beta}} = 0. \qquad (20)$$

Or, by simplifying by $AR^{\overline{\beta}}$:

$$\frac{1}{2}(b+1)^2\sigma^2A\overline{\beta}(\overline{\beta}-1) + (r+\lambda_R - \delta)\overline{\beta} - (r+\lambda_V) = 0. \qquad (21)$$

The general solution of Equation (21) has the following shape:

$$V(R) = B_1R^{\overline{\beta}_1} + B_2R^{\overline{\beta}_2} + \frac{R}{\delta + \lambda_V - \lambda_R} \qquad (22)$$

where $\overline{\beta}_1$ and $\overline{\beta}_2$ are the both roots of the quadratic Equation (21). The solving of Equation (21) enables to find that

$$\overline{\beta}_1 = \frac{1}{2} - \frac{r+\lambda_R-\delta}{(b+1)^2\sigma^2} - \sqrt{\left[\frac{r+\lambda_R-\delta}{(b+1)^2\sigma^2} - \frac{1}{2}\right]^2 + \frac{2(r+\lambda_V)}{(b+1)^2\sigma^2}} > 1 \qquad (23)$$

$$\overline{\beta}_2 = \frac{1}{2} - \frac{r+\lambda_R-\delta}{(b+1)^2\sigma^2} - \sqrt{\left[\frac{r+\lambda_R-\delta}{(b+1)^2\sigma^2} - \frac{1}{2}\right]^2 + \frac{2(r+\lambda_V)}{(b+1)^2\sigma^2}} < 0. \qquad (24)$$

So, if the project did not benefit from a perpetual option to invest, its value would be equal to

$$V(R) = \frac{R}{\delta + \lambda_V - \lambda_R}. \qquad (25)$$

III. DECISION OF INVESTMENT AND VALUE OF THE OPTION TO INVEST INCLUDING CORPORATE TAX AND INFORMATION COSTS

This second part of is centered on the determination of the value of the option to invest which allows delaying the investment up to the most convenient date. This option must be exercised when the critical values reached by the project and the cash flows are respectively V* and R*. In that case, the invested amount is noted I.

The option value can be obtained by constituting again an arbitrage portfolio. This one consists in acquiring an option to invest in the project and in selling n units of cash flows generated by the aforementioned project, n being determined so that the portfolio is risk free. Besides, because of the short position, δRdt has to be paid for each unit of cash flow, which has been sold, where δ is the dividend yield. Furthermore, the capital gain on the portfolio is

$$dF(R) - ndR_t = \left\{Rf(t)\left[F'(R) - n\right] + \frac{1}{2}R^2(b+1)^2\sigma^2F''(R)\right\}dt + R(b+1)\sigma\left[F'(R) - n\right]dB_t \quad (26)$$

By choosing n=F'(R), the amount which is received by the owner of the arbitrage portfolio of is equal to

$$\begin{aligned} -n\delta Rdt + dF(R) - ndR_t &= -F'(R)\delta Rdt + dF(R) - ndR_t \\ &= \left[\frac{1}{2}R^2(b+1)^2\sigma^2F''(R) - \delta RF'(R)\right]dt \end{aligned} \quad (27)$$

On principle, the return of the arbitrage portfolio is the r risk free rate. However, considering the λ_F costs of information, which are related to the option to invest at the convenient date and the λ_R costs of information, which are related to the cash flow analysis, the return of the option must be equal to $(r + \lambda_F)$ and the return of the cash flows is $(r + \lambda_R)$. In this context,

$$\left[\frac{1}{2}R^2(b+1)^2\sigma^2F''(R) - \delta RF'(R)\right]dt = (r+\lambda_F)F(R)dt - RF'(R)(r+\lambda_R)dt. \quad (28)$$

This allows obtaining the second order homogeneous differential equation:

$$\frac{1}{2}R^2(b+1)^2\sigma^2F''(R) + (r+\lambda_R-\delta)RF'(R) - (r+\lambda_F)F(R) = 0 \quad (29)$$

The general solution to Equation (29) is $F(R) = A_1R^{\beta_1} + A_2R^{\beta_2}$, where A_1 and A_2 are real unknown constants which have to determined and where β_1 and β_2 are the solutions of the quadratic equation:

$$\frac{1}{2}(b+1)^2\sigma^2\beta(\beta-1) + (r+\lambda_R-\delta)\beta - (r+\lambda_F) = 0 \quad (30)$$

Its resolution drives to the following solutions:

$$\beta_1 = \frac{1}{2} - \frac{r+\lambda_R-\delta}{(b+1)^2\sigma^2} + \sqrt{\left[\frac{r+\lambda_R-\delta}{(b+1)^2\sigma^2} - \frac{1}{2}\right]^2 + \frac{2(r+\lambda_F)}{(b+1)^2\sigma^2}} > 1 \tag{31}$$

$$\overline{\beta}_2 = \frac{1}{2} - \frac{r+\lambda_R-\delta}{(b+1)^2\sigma^2} - \sqrt{\left[\frac{r+\lambda_R-\delta}{(b+1)^2\sigma^2} - \frac{1}{2}\right]^2 + \frac{2(r+\lambda_V)}{(b+1)^2\sigma^2}} < 0. \tag{32}$$

As far as F(0)=0, A_2=0. Consequently,

$$F(R) = A_1 R^{\beta_1}. \tag{33}$$

Assuming that the I investment is the strike price of the option, it is possible to determine the R* critical value of the cash flows by using the usual boundary conditions, issued from the methodology of Dixit and Pindyck (1994). The value matching condition allows writing:

$$F(R^*) = V(R^*) - I \tag{34}$$

So, by using Equations (25) and (33), Equation (34) becomes

$$A_1 R^{*\beta_1} = \frac{R^*}{\delta+\lambda_V-\lambda_R} - I \tag{35}$$

The *smooth pasting* condition enables to write:

$$F'(R^*) = V'(R^*) \tag{36}$$

So, by deriving both members of Equation (35) with regard to R, Equation (36) becomes:

$$\beta_1 A_1 R^{*\beta_1 - 1} = \frac{1}{\delta+\lambda_V-\lambda_R} \tag{37}$$

From then on, by dividing Equation (35) by Equation (37), we obtain:

$$\frac{R^*}{\beta_1} = \left[\frac{R^*}{\delta+\lambda_V-\lambda_R} - I\right](\delta+\lambda_V-\lambda_R)$$

$$\frac{R^*}{\beta_1} = R^* - (\delta+\lambda_V-\lambda_R)I$$

$$R^* = \frac{\beta_1}{\beta_1-1}(\delta+\lambda_V-\lambda_R)I \tag{38}$$

By substituting Equation (38) into Equation (37), it is possible to get A_1. So,

$$\beta_1 A_1 \left(\frac{\beta_1}{\beta_1 - 1}\right)^{\beta_1 - 1} (\delta + \lambda_V - \lambda_R)^{\beta_1 - 1} I^{\beta_1 - 1} = \frac{1}{\delta + \lambda_V - \lambda_R} \qquad (39)$$

Hence,

$$A_1 = \frac{(\beta_1 - 1)^{\beta_1 - 1} I^{-(\beta_1 - 1)}}{\left[(\delta + \lambda_V - \lambda_R)^{\beta_1} \beta_1\right]^{\beta_1}} \qquad (40)$$

Finally, by substituting of Equation (38) into Equation (25), we have

$$V^* = \frac{\beta_1}{\beta_1 - 1} \cdot I \qquad (41)$$

In other words, according to the operational conclusions of Dixit and Pindyck (1994), a project can be undertaken provided that the value of its cash flows is equal to a multiple of the amount of the envisaged investment. The value of β_1 in Equation (23) is different from that of the formula established by Dixit and Pindyck (1994). However, the by replacing b, λ_R and λ_V by 0, Equation (41) comes down to Dixit and Pindyck's formula In that case, the critical value of the investment stemming from Equation (41) and that stemming from the formula obtained by Dixit and Pindyck (1994) are identical.

IV. SIMULATIONS

The results of the second part of this paper allow formulating a decision of investment rule under uncomplete information. In the lineage of Dixit and Pindyck (1994), it emerges that an investment must be realized when the amount R cash flows is superior to the critical level R*. Correlatively, if R is lower than R* then the V(R) value of the project is lower than the sum of the amount of the investment I and of the value F(R) of the option of waiting for the optimal timing to invest.

The figure 1 below shows that the value of the option of wait is an increasing function of R. This figure is built by considering a r risk free rate equal to 3.5 %, a $\delta = 1.5\%$ dividend yield, a $\lambda_R = 1\%$ cost of information about the cash flows, a $\lambda_F = 2.5\%$ cost of information about the option, a $\lambda_V = 5\%$ cost of information about the project, b = -2, and an investment I = 1. Besides, three assumptions of volatility have been taken into account: $\sigma = 20\%$, $\sigma = 25\%$ and $\sigma = 30\%$.

Figure 1
Analysis of sensibility of the value of the option of waiting in the R
Cash flows and in the volatility

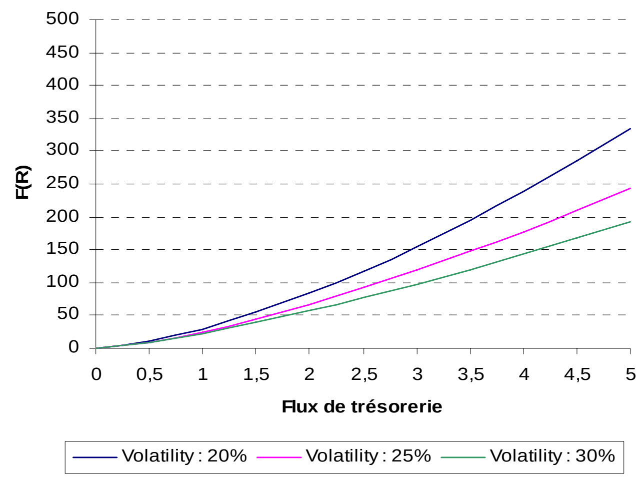


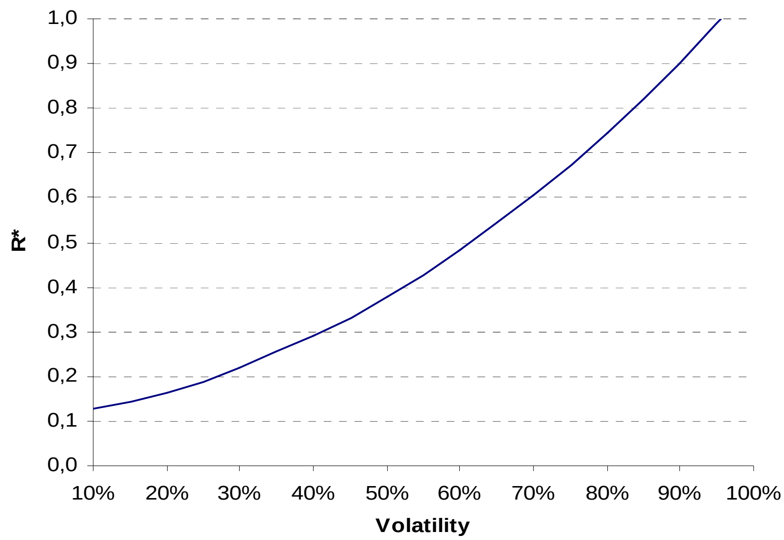
Table 1 below recapitulates the various values of *F(R)* which have been represented graphically:

Table 1
Value of F(R) according to R and the volatility

σ	*R*										
	0	*0,5*	*1,0*	*1,5*	*2,0*	*2,5*	*3,0*	*3,5*	*4,0*	*4,5*	*5,0*
20%	0	11	30	55	84	118	155	195	239	285	334
25%	0	10	25	45	67	92	119	147	178	210	243
30%	0	9	23	39	57	77	98	120	143	168	193

Figure 2 below shows that the critical R* value of the cash flows is also an increasing function of the level of volatility.

Figure 2
Analysis of sensibility of the critical *R**value of the cash flows in the volatility



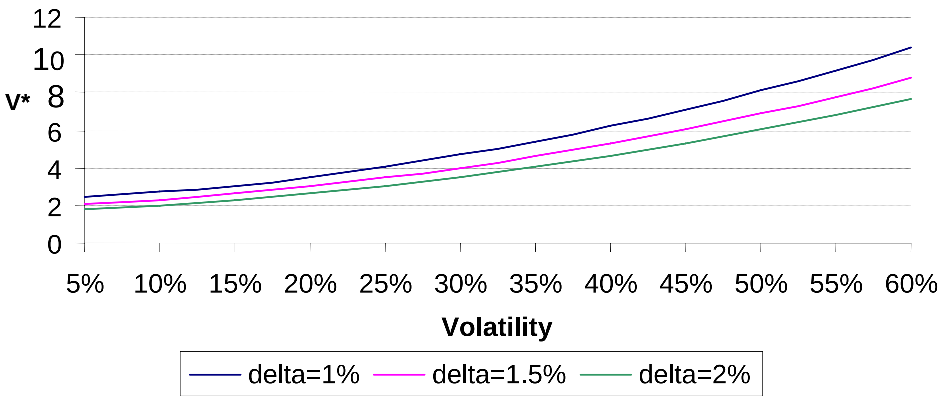
This figure is built by considering a r risk free rate equal to 3.5%, a $\delta = 1.5\%$ dividend yield, a $\lambda_R = 1\%$ cost of information about the cash flows, a $\lambda_F = 2.5\%$ cost of information about the option, a $\lambda_V = 5\%$ cost of information about the project, b = -2, and an investment I = 1. Table 2 below recapitulates the various values of R* represented graphically:

Table 2
Value of *R** according to the volatility

σ									
10%	*20%*	*30%*	*40%*	*50%*	*60%*	*70%*	*80%*	*90%*	*100%*
0,1	0,2	0,2	0,3	0,4	0,5	0,6	0,7	0,9	1,1

Figure 3 below shows the impact of the evolution of the σ volatility and of δ dividend yield on the critical value V* of the project. This figure is based on a r risk free rate equal to 3.5%, a $\lambda_R = 1\%$ cost of information about the cash flows, a $\lambda_F = 2.5\%$ cost of information about the option, a $\lambda_V = 5\%$ cost of information about the project, b = -2, and an investment $I = 1$. Besides, three assumptions of dividend yield are taken into account: $\delta = 1\%$, $\delta = 1.5\%$, $\delta = 2\%$.

Figure 3
Analysis of sensibility of the critical value of the project
in the σ volatility and the δ dividend yield



It emerges that the V* critical value of the project is an increasing function of the volatility. Consequently, the progress of the volatility is translated by a reduction of investments. Besides, any increase of the dividend yield increases the V* critical value of the project from which the company can invest.

Table 3 below recapitulates the various values of V* represented graphically.

Table 3
Value of V* according to the σ volatility and the δ dividend yield

δ	σ											
	5%	*10%*	*15%*	*20%*	*25%*	*30%*	*35%*	*40%*	*45%*	*50%*	*55%*	*60%*
1,0%	2,5	2,7	3,1	3,5	4,1	4,7	5,4	6,2	7,1	8,1	9,2	10,4
1,5%	2,1	2,3	2,6	3,0	3,5	4,0	4,6	5,3	6,0	6,9	7,8	8,8
2,0%	1,8	2,0	2,3	2,6	3,0	3,5	4,0	4,6	5,3	6,0	6,8	7,6

Figure 4 below shows that the F(R) value of the option of waiting and the V(R) value of the project are increasing functions of R and decreasing functions of the dividend yield which takes the following values: $\delta = 1\%$, $\delta = 1.5\%$ and $\delta = 2\%$ The figure 4 is based on a r risk free rate equal to 3.5 %, a $\lambda_R = 1\%$ cost of information about the cash flows, a $\lambda_F = 2.5\%$ cost of information about the option, a $\lambda_V = 5\%$ cost of information about the project, b = -2, and an investment I = 20. It emerges from this graph that when the dividend yield increases, the R* critical amount of the cash flows decreases. R* corresponds to the abscissa of the tangential point of the representative curves of the F(R) value of the option of waiting and the V(R)-I net present value of the project. So, when $\delta = 1\%$, then R*=6.2 ; when $\delta = 1.5\%$, then R*=5.8 and when $\delta = 2\%$, then R*=5.6.

Figure 4

Analysis of sensibility of the value of the F(R) option of waiting and of the V (R)-I net present value of the project in the R cash flows and the δ dividend yield

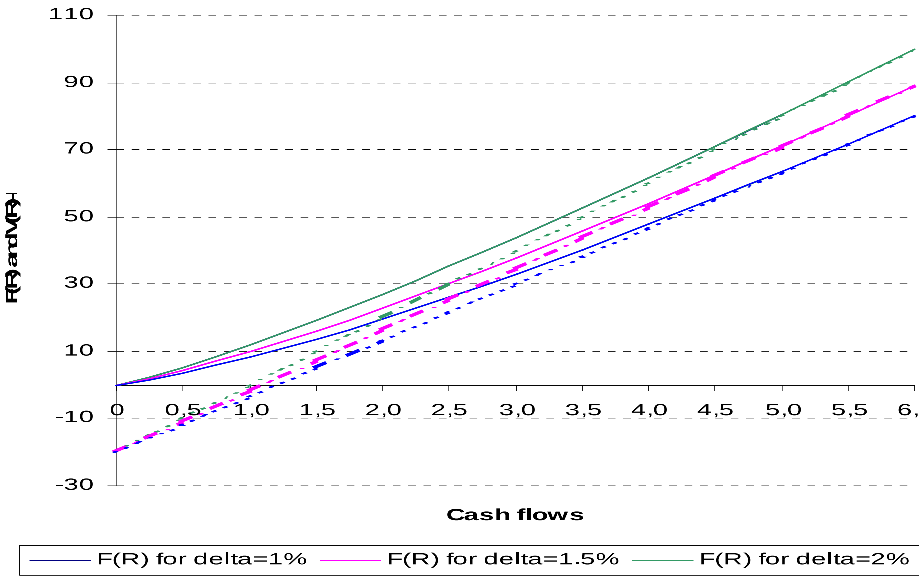


Table 4 below recapitulates the various values of F(R) represented graphically.

Table 4

F(R) value according to the R cash flows
and of the δ dividend yield

δ	R												
	0	*0,5*	*1,0*	*1,5*	*2,0*	*2,5*	*3,0*	*3,5*	*4,0*	*4,5*	*5,0*	*5,5*	*6,0*
1,0%	0	5	12	19	27	35	44	53	62	71	80	90	100
1,5%	0	4	10	16	23	30	38	46	54	62	71	80	89
2,0%	0	3	8	14	20	26	33	40	48	55	63	72	80

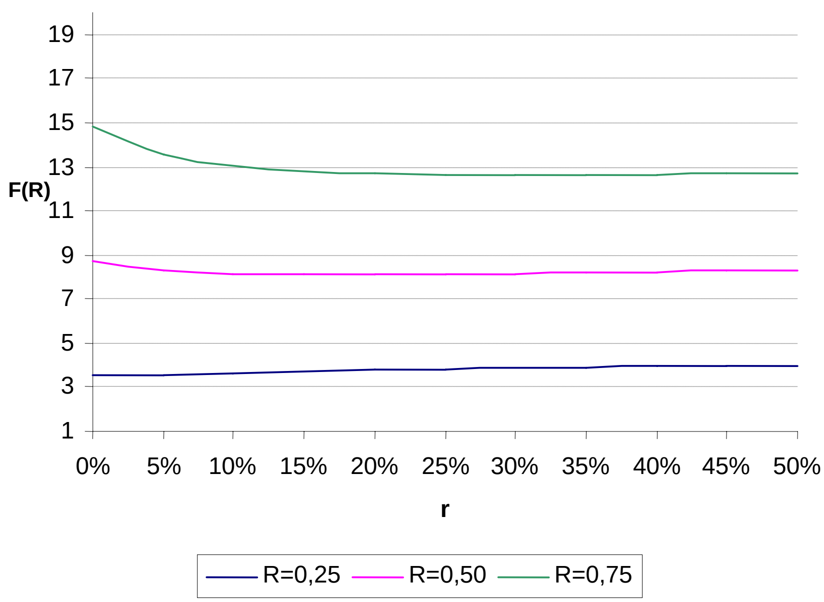
Besides, Table 5 displays the values of V(R)-I which are represented by dotted lines on Figure 4.

Table 5
V (R)-I value according to the R cash flows
and the δ dividend yield

δ	R												
	0	*0,5*	*1,0*	*1,5*	*2,0*	*2,5*	*3,0*	*3,5*	*4,0*	*4,5*	*5,0*	*5,5*	*6,0*
1,0%	-20	-10	0	10	20	30	40	50	60	70	80	90	100
1,5%	-20	-11	-2	7	16	25	35	44	53	62	71	80	89
2,0%	-20	-12	-3	5	13	22	30	38	47	55	63	72	80

Figure 5 below illustrates the impact of an increase in the r risk free rate on the F(R) value of the option to invest. The figure is based on a 1.5% dividend yield, a 40% volatility, a $\lambda_R = 1\%$ cost of information about the cash flows, a $\lambda_F = 2.5\%$ cost of information about the option, a $\lambda_V = 5\%$ cost of information about the project, b = -2, and an investment I = 1.

Figure 5
Analysis of sensibility of the value of the F(R) option of waiting
In the r risk free rate and in the R cash flows



Three assumptions of the future cash flows are taken into account: R=0,25, R=0,50 and R=0,75. It emerges that the F(R) value increases with the r risk free rate. So, a higher interest rate increases the cost of opportunity of the immediate investment and is translated, de facto, by a decrease in investments. The table 6 below recapitulates the F(R) various values of represented graphically:

Table 6

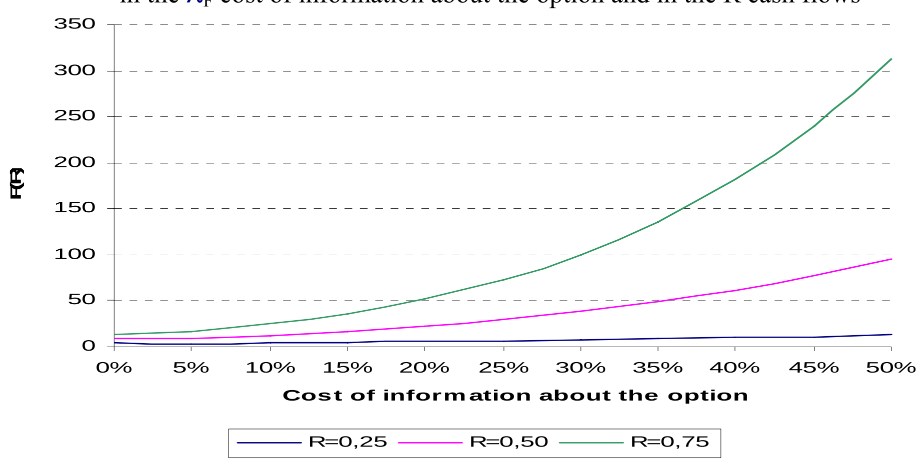
F (R) value according to the *R* cash flows and to the *r* risk free rate

R	*r*										
	0%	*5%*	*10%*	*15%*	*20%*	*25%*	*30%*	*35%*	*40%*	*45%*	*50%*
0,25	3,5	3,6	3,6	3,7	3,8	3,8	3,9	3,9	3,9	4,0	4,0
0,50	8,7	8,3	8,1	8,1	8,1	8,1	8,2	8,2	8,2	8,3	8,3
0,75	14,8	13,5	13,0	12,8	12,7	12,6	12,6	12,6	12,7	12,7	12,7

Figure 6 below shows that the value of the option of waiting is an increasing function of the λ_F costs of information on the option. This graph is built by considering a r = 4% risk free rate, a 2% dividend yield, a $\lambda_R = 1\%$ cost of information about the cash flows, a $\lambda_V = 5\%$ cost of information about the project, b = -2, and an investment I = 1. Three level assumptions of the cash flows are taken into account: R=0,25, R=0,50 and R=0,75. It emerges from this graph that the increase in the λ_F cost of information about the option is translated by an increase in the value of the option of waiting. In other words, the cost of opportunity of the immediate investment increases with the λ_F cost of information about the option, which is translated by a decrease in investments.

Figure 6

Analysis of sensibility of the value of the F(R) option of waiting in the λ_F cost of information about the option and in the R cash flows



It also emerges from this graph that the value of the option is an increasing function of the R cash flows.

Table 7 below recapitulates the various F(R) values represented graphically.

Table 7
F (R) value according to the R cash flows
and to the λ_F cost of information about the option

R	λ_F										
	0%	*5%*	*10%*	*15%*	*20%*	*25%*	*30%*	*35%*	*40%*	*45%*	*50%*
0,25	4	4	4	5	5	6	7	8	10	11	13
0,50	8	9	13	17	22	29	38	49	62	77	96
0,75	13	17	25	36	52	73	100	136	181	240	313

All in all, the consideration of the costs of information within the framework of the decision of investment does not question the spirit of the practical conclusions of Dixit and Pindyck (1994). So, a project can be undertaken if its cash flows are at least equal to a multiple of the investment to be achieved. This multiple integrates the costs of information into the sense of Merton's CAPM (1987). Besides, in the theoretical assumption where the investor is free of charge of complete information about the envisaged project, the value of the option to invest and the critical value of the cash flows stemming from this model are identical to those obtained by Dixit and Pindyck (1994).

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