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Real Options Value by Monte Carlo Simulation and Fuzzy Numbers

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ABSTRACT

This work presents the development of a methodology based on Monte Carlo Simulation, Fuzzy Numbers and in the Real Options Theory to determine the real options value under technical and market uncertainties. The objective of the proposed methodology is to substantially reduce the computational time involved, facilitating the decision taking process. The methodology involves: fuzzy numbers, to represent certain types of uncertainties that do not have a known stochastic process or probability distribution that can correctly model them; stochastic processes, to represent other kinds of uncertainties; and Monte Carlo simulation, to generate a good approximation of the real option value. This methodology was evaluated in problems of expansion option in the area of oil exploration and production, attaining the same results provided by conventional techniques but with a significant reduction in the necessary computational time.

JEL Classification: *G11, G12, G31, C63*

Keywords: *Real options; Fuzzy numbers; Monte Carlo simulation*

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I. INTRODUCTION

The economic decisions involved in a specific investment, such as acquiring new equipments, increasing the work force or developing new products, as well as the economic evaluation of projects, are affected by economic uncertainties, technical uncertainties and by administrative flexibilities embedded in the projects. Economic uncertainties are generally related to external factors of the project, being represented by stochastic oscillations of the price of the product and by costs. On the other hand, technical uncertainties are associated to internal factors, such as the uncertainty about the production volume and the performance of projects as a result of the use of different technologies. Administrative flexibilities embedded in the projects provide degrees of freedom for the manager to decide whether to invest, to expand, to stop temporarily or to abandon one specific project. If any of these possibilities are ignored in the economic analysis, a sub-evaluation of the project can occur, leading to irreversible errors in the decision making. The real options theory allows the evaluation of this type of projects by considering, in addition to a number of uncertainties, the available administrative flexibilities, having as its objective function the maximization of the investment opportunity value.

The calculation of a real option value (Dixit andPindyck, 1994; Copeland 2001) when some uncertainty factors are present can be accomplished by means of Monte Carlo simulation methods, since they are adequate for problems of high dimensionality or with stochastic parameters. However, these methods demand very high computational time effort, due to the nature of the iterative process algorithm, whose execution time increases with the number of iterations according to the accuracy required. Given the dynamism of the administrative decisions, there is not much time for the decision making process. Therefore, it is important to accelerate the Monte Carlo simulation process without loss in the required precision.

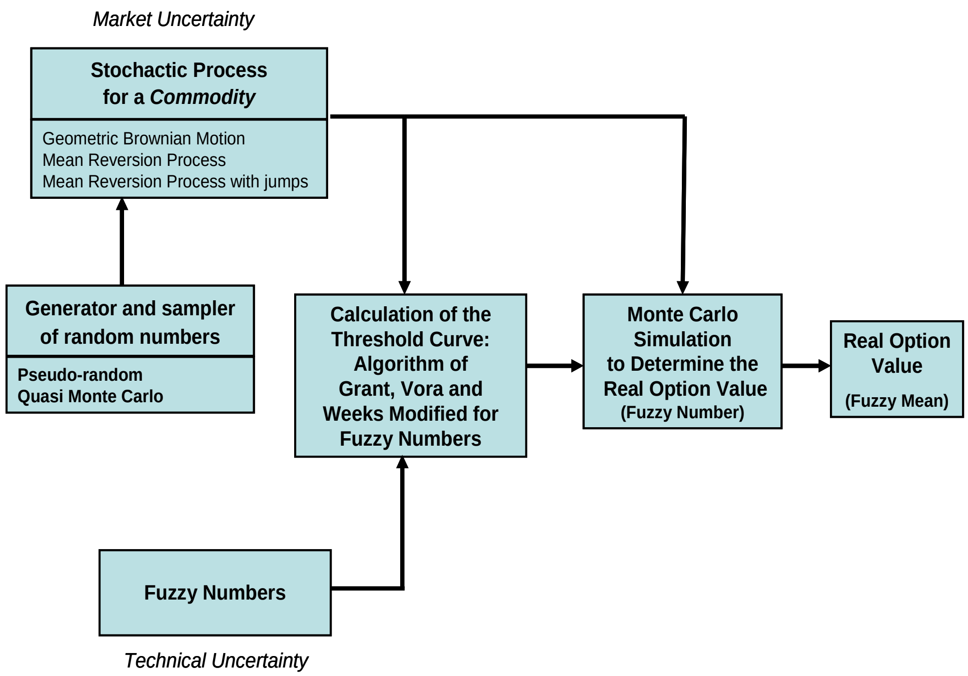
This paper presents a new methodology for the determination of a real option value under technical and market uncertainties with the objective to reduce the computational time. The methodology is based on: fuzzy numbers to represent technical uncertainties; stochastic processes to represent market uncertainties; and Monte Carlo simulation to obtain a good approximation of the real option value. This methodology was applied in the evaluation of an option to invest in an additional well in an oil reserve, considering the size of the reserve as technical uncertainty and the future oil price as the market uncertainty.

II. METHODOLOGY FOR EVALUATING REAL OPTIONS VALUE BY APPROXIMATION WITH FUZZY NUMBERS

In the proposed methodology, market uncertainties are represented by known stochastic processes (such as Geometric Brownian Motion, Mean Reversion Process and Mean Reversion Process with Jumps), while technical uncertainties are modeled by triangular fuzzy numbers, instead of the triangular probability distribution used in the traditional solution of real options evaluation by stochastic simulation. The main modules of the proposed methodology are depicted in Figure 1.

Figure1

Main modules of the proposed methodology that determines real options value under uncertainties



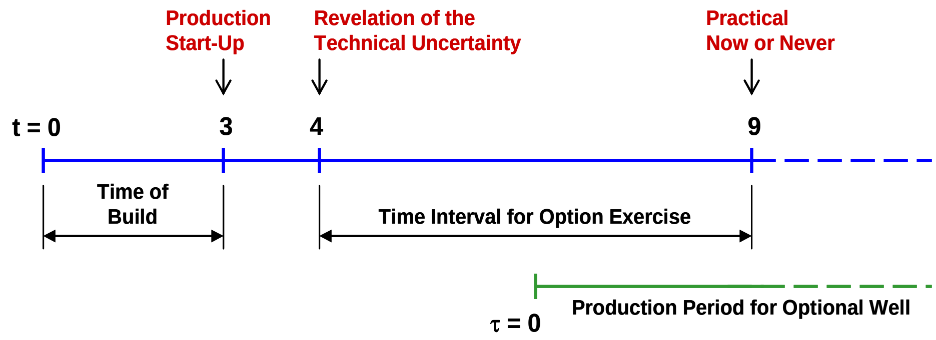
III. THE PROPOSED METHODOLOGY TO SOLVE THE EXPANSION OPTION PROBLEM

The expansion option problem investigates an alternative for the development of an oil production strategy where there is the possibility to expand the production by means of adding an additional well in the future, depending on market conditions and technical information generated by the initial production of the field. The oil price is the market uncertainty and it is assumed to follow one of two possible stochastic processes: Geometric Brownian Motion or Mean Reversion Process.

The technical uncertainty, in this case, corresponds to the volume of the oil reserve that could be drained in the area of the optional well. This technical uncertainty is denoted by B.

In this work it is assumed that three years are necessary to construct the additional well. After the end of construction, the field starts to produce oil; however, the technical sceneries of B are revealed after one year of production (i.e. at the end of the 4^{th} year, since the beginning of the construction). The option to invest in an additional well is limited to a five year period (Figure 2), that is, the option to invest in an additional well initiates with the revelation of the technical sceneries and from this moment (year t = 4) the option can be exercised at any time during the next 5 years. After this period, the option expires (in year t = 9). After the exercise of the option, the additional well is able to produce oil for up to 30 years.

Figure 2
Period for exercising the option for an additional well



To determine the option value a hybrid methodology is applied that joins the stochastic simulation with fuzzy numbers. In this methodology technical uncertainties are represented by fuzzy numbers, instead of the common triangular probability distributions used in traditional methods of option value evaluation.

The triangular fuzzy number comprises the same parameters used in the triangular probability distribution. The fuzzy number allows dealing with the technical uncertainty as a whole, avoiding the need to sample it, as it would be the case for the triangular probability distribution; this method greatly speeds up the process of the Monte Carlo simulation.

To determine the bound of optimal exercise (or threshold curve), the algorithm of Grant, Vora and Weeks (1997) was adapted to work with fuzzy numbers (Lazo et al. 2004). After the construction of the threshold curve, the proposed methodology to makes simulations for the oil price from the initial price, the option value, will be the mean fuzzy (Gao 1999; Carlsson and Fullér 2001) of all the values that reach or surpass the threshold curve in the simulation, brought to the present value.

After the threshold curve has been calculated,, the behavior of the oil price is simulated using the chosen underlying stochastic process for a given initial price. The option value will be the mean fuzzy of all the values that reach or surpass the threshold curve in the simulation, brought to the present value.

IV. RESULTS WITH THE FUZZY NUMBER METHODOLOGY

This section present the experiments carried out with both methodologies: the traditional methodology of Monte Carlo simulation (ROV, from now on) and the proposed hybrid methodology with fuzzy numbers (FROV, from now on). The same parameters have been considered for both methodologies, substituting only the triangular probability distribution used in ROV for a triangular fuzzy number in FROV. The first of two experiments considers that the oil price follows a Geometric Brownian Motion while the second experiment considers the use of a Mean Reversion Process.

The experiments have been executed in a personal computer with microprocessor AMD Athlon of 1.5GHz with 256Mb of memory RAM.

Table 1 presents the values of all parameters used in the first experiment. This experiment was executed 41 times. Table 2 presents the results obtained with both methodologies (the hybrid methodology stochastic with fuzzy numbers and the methodology of stochastic simulation).

It can be observed from Table 2 that the mean error between the two methodologies is very small (less 1%), which is totally acceptable for this type of application. The number of path simulations used for the oil price can be increased to improve the final precision, as well as the number of samples of the technical uncertainty in the methodology of stochastic simulation. However, these increments imply in a significant increase in the final computational time.

It is important to emphasize the significant difference in the necessary computational time between both methodologies. The same experiment took 5:58 hours when using the stochastic simulation and only 41 minutes when using the hybrid methodology proposed, resulting in an excellent improvement in computational efficiency (8.7 times faster).

The second experiment represents the technical uncertainty by an asymmetric triangular distribution and by an asymmetric fuzzy number. Table 3 presents the values of the parameters used in this experiment.

Table 1
Parameters used in experiment 1

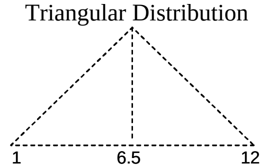
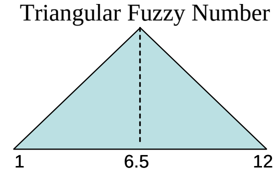
	ROV ***(Monte Carlo Simulation)***	**FROV** ***(Hybrid Model with Fuzzy Numbers)***
Uncertainty (B): Reserve to be drained by well	Triangular Distribution 	Triangular Fuzzy Number 
Oil price Simulations (*Paths*)	1 000	1 000
Number of Samples for Technical Uncertainty	500	Triangular Fuzzy Number
Stochastic Process for Oil Price	Geometric Brownian Motion	Geometric Brownian Motion
Type of Sampler	Quasi Monte Carlo	Quasi Monte Carlo
Discreteness of the Time	0.08333333	0.08333333
Solution by:	Arbitrage	Arbitrage
Investment Cost: I	10 MM US$	10 MM US$

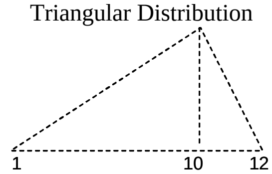
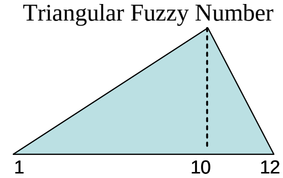
Table 2
Results obtained with Geometric Brownian Motion

	Time	Fuzzy ROV		ROV		Time
	min	Well 1	Option Value	Well 1	Option Value	min
1	0:00:59	17.37	7.37	17.33	7.33	0:05:20
2	0:00:58	18.81	8.81	19.12	9.12	0:10:12
3	0:00:57	19.51	9.51	18.96	8.96	0:10:25
4	0:01:08	18.98	8.98	18.92	8.92	0:07:46
5	0:00:57	19.54	9.54	17.04	7.04	0:14:15
6	0:00:55	17.18	7.18	18.35	8.35	0:05:20
7	0:00:51	19.71	9.71	18.83	8.83	0:13:02
8	0:00:57	19.73	9.73	19.39	9.39	0:07:22
9	0:01:01	18.45	8.45	18.74	8.74	0:09:54
10	0:00:53	17.59	7.59	19.81	9.81	0:08:33
11	0:00:54	18.83	8.83	17.79	7.79	0:12:17
12	0:01:04	18.21	8.21	18.58	8.58	0:05:35
13	0:00:56	18.44	8.44	19.12	9.12	0:08:58
14	0:01:28	18.13	8.13	19.12	9.12	0:10:29
15	0:00:56	19.05	9.05	18.79	8.79	0:10:07
16	0:01:05	18.91	8.91	16.9	6.9	0:12:08
17	0:00:49	16.05	6.05	18.68	8.68	0:10:10
18	0:01:03	18.25	8.25	18.54	8.54	0:06:20
19	0:01:09	19.4	9.4	19.47	9.47	0:08:16
20	0:01:03	19.17	9.17	18.75	8.75	0:06:18
21	0:01:03	18.67	8.67	19.46	9.46	0:06:39
22	0:01:05	18.98	8.98	19.17	9.17	0:07:34
23	0:00:51	17.99	7.99	19.26	9.26	0:10:26
24	0:01:02	20.08	10.08	20.31	10.31	0:06:21
25	0:00:50	18.76	8.76	19.1	9.1	0:07:44
26	0:00:59	19.14	9.14	18.92	8.92	0:07:23
27	0:00:58	16.85	6.85	16.77	6.77	0:17:04
28	0:01:05	19.73	9.73	18.6	8.6	0:04:56
29	0:01:11	19.05	9.05	18.68	8.68	0:09:26
30	0:01:10	18.12	8.12	19.03	9.03	0:08:30
31	0:00:53	19.1	9.1	18.65	8.65	0:07:49
32	0:01:00	18.53	8.53	18.88	8.88	0:06:43
33	0:01:11	22.4	12.4	18.86	8.86	0:09:28
34	0:00:53	18.72	8.72	20.11	10.11	0:06:47
35	0:00:57	18.95	8.95	19.84	9.84	0:07:03
36	0:01:04	19.64	9.64	17.85	7.85	0:09:29
37	0:01:01	19.13	9.13	19.95	9.95	0:12:59
38	0:01:00	18.73	8.73	18.5	8.5	0:06:12
39	0:01:03	18.56	8.56	18.44	8.44	0:10:06
40	0:00:58	18.76	8.76	21.19	10.19	0:06:12
41	0:00:54	18.08	8.08	17.32	7.32	0:06:53

		Fuzzy ROV		ROV		
	Time	Well 1	Option Value	Well 1	Option Value	Time
MEAN OF THE EXPERIMENTS	0:01:00	18.76	**8.76**	18.81	**8.78**	0:08:45
VARIANCE OF THE EXPERIMENTS			**1.01**		**0.73**	
TOTAL TIME	0:41:11					5:58:31

MEAN ERROR	**0.23%**	
AVERAGE EFFICIENCY	**8.71**	**Times Faster with Fuzzy Numbers**

Table 3
Parameters used in Experiment 2

	ROV ***(Monte Carlo Simulation)***	**FROV** ***(Hybrid Model with Fuzzy Numbers)***
Uncertainty (B): Reserve to be drained by well	Triangular Distribution 	Triangular Fuzzy Number 
Oil price Simulations (*Paths*)	5 000	5 000
Number of Samples for Technical Uncertainty	1 000	Triangular Fuzzy Number
Stochastic Process for Oil Price	Mean Reversion Process	Mean Reversion Process
Type of Sampler	Quasi Monte Carlo	Quasi Monte Carlo
Discreteness of the Time	0.08333333	0.08333333
Solution by:	Arbitrage	Arbitrage
Investment Cost: I	10 MM US$	10 MM US$

This second experiment was executed 21 times. Table 4 presents the results obtained with both methodologies (the hybrid methodology with fuzzy numbers and the methodology of stochastic simulation).

In this experiment the mean error of between both the methodologies is also very small. Again, it is worthy to mention the significant improvement in the computational time obtained by FROV comparing to ROV. The hybrid methodology proposed performed the simulations 295, 68 times faster than the traditional approach.

This experiment proved that the quality of the results obtained with the proposed methodology is not influenced by the form of the fuzzy number (symmetric or asymmetric).

Comparing the results of both experiments, it can be observed that the first experiment, in which considers that the oil price follows a Geometric Brownian Motion, the computational efficiency of the methodology proposed is smaller than when the Mean Reversion Process is considered. This is due to the fact that when the Geometric Brownian Motion is used the threshold curve is calculated only once (Dias, 2000 and 2001).

Table 4
Results obtained in Experiment 2

	Time	Fuzzy ROV		ROV		Time
	Hours	Well 1	Option Value	Well 1	Option Value	Hours
1	0:03:10	16.7	6.7	16.68	6.68	17:29:41
2	0:03:39	16.47	6.47	16.46	6.46	17:07:53
3	0:02:59	16.21	6.21	16.25	6.25	17:08:07
4	0:03:38	16.22	6.22	15.94	5.97	17:08:45
5	0:03:34	16.76	6.76	16.71	6.71	15:07:33
6	0:04:09	16.63	6.63	16.57	6.57	16:56:22
7	0:03:42	16.6	6.6	16.61	6.61	17:01:21
8	0:03:59	16.95	6.95	16.98	6.98	16:39:36
9	0:03:42	17.05	7.05	16.86	6.86	16:38:19
10	0:03:53	16.58	6.58	16.63	6.63	16:56:47
11	0:03:43	16.27	6.27	16.28	6.28	16:53:02
12	0:03:37	16.68	6.68	16.63	6.63	22:50:52
13	0:03:03	16.25	6.25	16.39	6.39	17:29:27
14	0:03:09	16.26	6.26	16.21	6.21	17:24:28
15	0:03:42	16.73	6.73	16.45	6.45	17:11:38
16	0:03:34	16.68	6.68	17.23	7.23	19:27:35
17	0:03:59	16.76	6.76	16.66	6.66	17:13:44
18	0:03:21	16.48	6.48	16.47	6.47	16:54:46
19	0:03:37	16.71	6.71	16.68	6.68	17:00:46
20	0:03:43	16.75	6.75	16.39	6.39	23:54:43
21	0:03:34	16.44	6.44	16.42	6.42	17:13:46

		Fuzzy ROV		ROV		
	Time	Well 1	Option Value	Well 1	Option Value	Time
MEAN OF THE EXPERIMENTS	0:03:36	16.58	**6.58**	16.55	**6.55**	17:42:21
VARIANCE OF THE EXPERIMENTS			**0.06**		**0.08**	
TOTAL TIME	1:15:27					371:49:11
MEAN ERROR	**0.47%**					
AVERAGE EFFICIENCY	**295.68**	Times Faster with Fuzzy Numbers				

V. CONCLUSIONS

This paper presented the development of a computational viable methodology to determine real options value under technical and market uncertainties, with the objective to reduce the computational time and thus to facilitate the consequent decision making process. Hence, the union of different techniques was proposed: Fuzzy numbers to represent certain types of uncertainties, stochastic processes to represent the others uncertainties and the Monte Carlo simulation to get approximate the real option value.

The results obtained with the proposed methodology have been compared with the traditional methodology of stochastic simulation. The results proved to be satisfactory, with very small average errors and with a significant reduction in the time required to perform the whole process with fuzzy numbers. The hybrid methodology proposed with fuzzy numbers revealed to be efficient to approximate the real option value and to reduce the computational time for two different underlying stochastic processes considered for the oil price.

The reduction of the computational time with small average error is due to the fact that the fuzzy number represents the whole technical uncertainty. Therefore, it is not necessary to sample the technical uncertainty. Then, the algorithm of Grant, Vora and Weeks (that is used to calculate the threshold curve) is executed only one time.

The methodology proposed and verified in this work is a promising alternative for decision support systems in the area of real options. The ability to obtain good approximation for the real option value, without having to carry out heavy stochastic simulations, allows for more agile systems, capable of executing various experiments in little time frames.

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