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Real Options with Information Costs: A Synthesis

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ABSTRACT

This paper presents some new results regarding the pricing of real options in the presence of market frictions. Since the standard literature ignores the role of market frictions and the effect of incomplete information, we rely on Merton's (1987) model of capital market equilibrium with incomplete information (CAPMI) to introduce information costs in the pricing of real assets. Using this model instead of the standard CAPM of Sharpe (1964) allows computing the value of the firm and its assets in the presence of information uncertainty. In the original derivation of the Black-Scholes (1973) model, the CAPM was used. In the same context, the CAPMI model can be used. Using the methodology in Bellalah (1999, 2001, 2002) and in Paxson (2003) for the pricing of real options, we extend the standard models to account for the effects of shadow costs of incomplete information. The models can be used for the valuation of several real options, R&D projects as well as projects with several stages like joint ventures.

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I. INTRODUCTION

The standard literature on capital budgeting techniques uses the net present value as a reference criterion in investment decisions. The analysis is mainly based on the use of the cost of capital in the discounting of future cash flows. A project is accepted if its extended Net Present Value, NPV, is positive, otherwise it is rejected. The extended NPV corresponds to the standard NPV plus the flexibility in investment decisions. The standard technique for calculating the NPV has not changed much since Fisher (1907) by discounting the expected cash flow at an appropriate discount rate. The research in this area is based on the specification and estimation of the discount rate.

Over the last two decades, a body of academic research takes the methodology used in financial option pricing and applies it to real options in what is well known as real options theory. This approach recognizes the importance of flexibility in business activities. Today, options are worth more than ever because of the new realities of the actual economy: information intensity, instantaneous communications, high volatility, etc[2].The literature on real options and discounted cash flow techniques ignores the role of information uncertainty. However, these costs play a central role in financial markets and capital budgeting decisions. Financial models based on complete information might be inadequate to capture the complexity of rationality in action.

Some factors and constraints, like entry into a business are not costless and may influence the short run behaviour of asset prices. The treatment of information and its associated costs play a central role in capital markets. If an investor does not know about a trading opportunity, he will not act to implement an appropriate strategy to benefit from it. However, the investor must determine if potential gains are sufficient to warrant the costs of implementing the strategy. These costs include time and expenses required to create data base to support the strategy, to build models and to get informed about the technology. This argument applies in varying degrees to the adoption in practice of new structural models of evaluation.

This reasoning holds not only for individual investors but also for professional managers who spend resources and time in the same spirit. It is also valid for the elaboration and implementation of option pricing models.

Hence, recognition of information costs might be important in asset valuation and has the potential to explain empirical biases exhibited by prices computed from complete information models. As shown in Merton (1987), the "true" discounting rate for future risky cash flows must be coherent with his simple model of capital market equilibrium with incomplete information. This model can be used in the valuation of real assets. Nowadays, a rich set of criteria is used to recognize the company’s real options. Consultants look beyond traditional financial analysis techniques to get reasonable guidelines in investment practices. Actual decision making in firms resorts to real options. The value of the firm can have two components: the value of the existing projects and the value of the options hold by the firm to do other things. The use of standard option valuation techniques in the valuation of real assets is based on some important assumptions[3]. Managers are interested not only in real options, but also in the latest outgrowth in DCF analysis.

The structure of the paper is as follows:

Section II presents a simple framework for the valuation of the firm and its assets in the presence of information costs. Using Merton’s (1987) model of capital market equilibrium with incomplete information, we show how to extend the standard valuation context to account for the effects of incomplete information in the computation of the firm value. We use the main results in the real option literature to make the standard analogy between financial and real options. This allows the presentation of the main applications of the real option pricing theory. Section III develops a simple context for the pricing of real options with information costs. We develop simple analytic formulas for the pricing of commodity options in the presence of information costs. The models are simulated and compared to the models in Black and Sholes (1973) and Black (1976). Section IV extends the results in some real option pricing models to account for information costs. This allows us to study the investment timing and the pricing of real assets using standard options.

Section V extends the formulas in Geske (1979) for the valuation of the firm and its assets in the presence of information costs. Section VI extends the results in Lint and Pennings (1998) for the pricing of the option on market introduction with information costs. Section VII develops some simple models for the pricing of real options in a discrete time setting by accounting for the role of shadow costs of incomplete information. We first extend the Cox, Ross and Rubinstein (1979) model to account for information costs in the valuation of managerial flexibility and the option to abandon. Then, we use the generalization in Trigeorgis (1990) for the pricing of several complex investment opportunities with embedded real options to account for the effects of information costs. Most of the models presented in this paper can be applied to the valuation of biotechnology projects and investments with several stages.

II. FROM FINANCIAL OPTIONS TO REAL OPTIONS AND THE EFFECTS OF INFORMATION COSTS: SOME STANDARD APPLICATIONS

The standard analysis in corporate investments needs the projection of the project's cash flows and then to perform an NPV analysis. The discount rate is set with regard to the risk of the project. The riskier the project, the higher the manager sets the discount rate.

This standard approach ignores the presence of information costs. However, information plays a central role in the valuation of financial assets and must be accounted for in the valuation process.

Managers recognize that the NPV analysis is incomplete and short-sighted. This analysis ensures in theory perpetual profitability for a company. The NPV fails because it assumes the decision to invest in a project is all or nothing. Hence, it ignores the presence of many incremental points in a project where the option exists to go forward or abort[4]. Realistic view of the capital budgeting process portrays projects as a sequence of options[5].

Merton (1987) presents a simple context to account for information costs. Before applying the main implications of Merton's model, we remind first this model and the definition of the shadow costs of incomplete information.

A. Merton's model

Merton's model is a two period model of capital market equilibrium in an economy where each investor has information about only a subset of the available securities.

The main assumption in the Merton's model is that an investor includes an asset S in his portfolio only if he has some information about the first and second moment of the distribution of its returns. In this model, information costs have two components: the costs of gathering and processing data for the analysis and the valuation of the firm and its assets, and the costs of information transmission from an economic agent to another.

Merton's model may be stated as follows:

$$\overline{R}_s - \gamma = \beta_s[\overline{R}_m - \gamma] + \lambda_s - \beta_s \lambda_m \tag{1}$$

where: $\overline{R}_s$: the equilibrium expected return on security S ,

$\overline{R}_m$: the equilibrium expected return on the market portfolio,

R : one plus the riskless rate of interest, r,

$\beta = \dfrac{\mathrm{cov}(\overline{R}s/\overline{R}m)}{\mathrm{Var}(\overline{R}m)}$: the beta of security S,

λ_s: the equilibrium aggregate "shadow cost" for the security S ,

λ_m: the weighted average shadow cost of incomplete information

The CAPM of Merton (1987), referred to as the CAPMI is an extension of the standard CAPM to a context of incomplete information. Note that when $\lambda_m = \lambda_s = 0$, this model reduces to the standard CAPM of Sharpe (1964).

B. Application to a biotechnology firm

For a biotechnology firm, the development of a drug needs several stages: discovery, pre-clinical, Phase I clinical trials, Phase II clinical trials, Phase III clinical trials, submission for review and post approval. We can apply in this setting Merton's (1987) model of capital market equilibrium with incomplete information for the computation of the cost of capital, the expected net present value (ENPV) in the decision tree method. Following the analysis in Kellogg and Charnes (1999), we will generalize their decision-tree method and the application of the binomial model to account for shadow costs of incomplete information.

A model is constructed to compute the expected net present value (ENPV) without accounting for growth options. The (ENPV) can be computed in the presence of information costs. In the decision tree method, the ENPV is computed as:

$$\mathrm{ENPV} = \sum_{i=1}^{7} \rho_i \sum_{t=1}^{T} \frac{\mathrm{DCF}_{it}}{(1+r_d)^t} + \rho_7 \sum_{j=1}^{5} \frac{q_j \mathrm{CCF}_{j,t}}{(1+r_c)^t} \tag{2}$$

where: i = 1, …,7 : an index of the 7 stages in the project,

ρ_i : the probability that stage i is the end stage for product i ,

T : the time at which all future cash flows become zero,

DCF_{it} : the expected development stage cash flow at time t given that stage i is the end stage,

r_d : the discount rate for development cash flows,
j = 1 to 5: an index of quality for the product,
q_j : the probability that the product is of quality j ,
r_c : the discount rate for commercialization cash flows.

The discounting rates rd and r_c can be estimated using Merton's CAPMI as in Bellalah (2000, 2001). This method is easy to implement and accounts for the effects of information costs in project valuation.

C. From Financial options to real options: some examples

Real option valuation maps out the possibilities available to a company, including those not readily apparent in the decision tree. By varying the discount rate through the tree, it accounts for the relative level of risk for different cash flows. Real option valuation can also identify the optimal course of the company at each stage in the process.

a. The standard analogy between financial and real options

There is a well established analogy between financial options and corporate investments that lead to future opportunities. It is evident for a manager why investing today in research and development or in a new marketing program can lead to a possibility of new markets in the future. Dixit (1992, 1995) and Dixit and Pindyck (1994) suggest that option theory provides helpful explanations since the goal of the investments is to reveal information about technological possibilities, production costs or market potential.

Consider for example a generic investment opportunity or a capital budgeting project to see the analogy with financial options. The difficult task lies in mapping a project onto an option. A corporate investment opportunity looks like a call because the firm has the right but not the obligation to acquire a given underlying, (the operating assets of a project or a new business). If the manager finds a call option in the market similar to the investment opportunity, then the value of that option can give him information about the value of the investment opportunity. Using this analogy between financial options and real options allows to know more about the project. This approach is more interesting than the standard discounting cash flow techniques DCT.

The option implicit in the project (the real option) and the NPV without the option are easily compared when the project can no longer be delayed. The reader can refer to Kogut (1991), and Mac Donald and Siegel (1984, 1986) among others.

A real option confers flexibilities to its holder and can be economically important. Paddock, Siegel and Smith (1988) and Berger et al (1996) show that the value of a firm is the combined value of the assets already in use and the present value of the future investment opportunities. There are several situations that lead to real options in different sectors in the economy.

b. Standard and complex real options and their applications: some examples

If you consider the example of high-tech start-up companies, these firms are valued mainly for their real options rather than their existing projects. The market recognizes today the value of these options. While standard options are easily identified, it is more difficult to identify compound and learning options.

Compound options generate other options among exercise. These options involve sequenced or staged investments. When a manager makes an initial investment, he has the right to make a second investment, which in turn gives the right to make a third investment, and so on.

Learning options allow the manager to pay to learn about an uncertain technology or system. Staged investments give managers the right to abandon or scale up projects, to expand into new geographic areas and investing in research and development[6].

A first example of compound options can be found in a staged investment, which may be assimilated to a sequence of stages where each stage is contingent on the completion of its predecessor. This is the case for a company seeking to expand in foreign markets. The firm might start in a single territory. It can then learn and modify the specific features of its product. The first experience enables the firm to expand into similar overseas markets. However, the manager must weigh the value of the option to expand cautiously against the potential costs of coming second in some or all of these markets. This situation corresponds also to joint ventures and the valuation of joint ventures and biotechnology products where each stage is contingent on the subsequent stages.

A second example is given in the market for corporate control and acquisitions. A sequence of acquisitions represents a staged series of investments and can be assimilated to compound options. Real options can be used in this context to value all possible contingencies. In this case, the literature regarding exotic options can be applied to value the different real options.

A third example corresponds to mining companies. Mining companies must often give an answer to the following question: when to develop the properties they own and how much to bid for the right to implement additional properties. These decisions refer to a combination of options: the option to learn about the quantity of ore and the option to defer the development waiting for favourable prices[7].

A fourth example is given for the development of a natural gas field (compound rainbow options). Combinations of learning options and rainbow options can arise for some firms[8].

A fifth example is given by R&D in pharmaceuticals (Rainbow options) Projects in R&D combine learning and compound options. R&D projects contain both technological and product uncertainties[9].

III. THE VALUATION OF STANDARD AND REAL OPTIONS WITH INFORMATION COSTS: THE GENERAL CONTEXT

Several models in financial economics are proposed to deal with the ability to delay an irreversible investment expenditure[10]. Before presenting some models for the valuation of real options in a continuous time setting, we present the general context for the valuation of financial options with information costs. Our definition of information costs refers to the models in Merton (1987) and Bellalah (1999). Merton (1987) introduced a modified capital asset pricing model where each investor can participate only in markets contained in an exogenous, investor specific subset of all asset markets.

He examines how this modification affects the standard CAPM and shows that limited market participation can explain empirical anomalies. Since the acquisition of information and its dissimination are central activities in finance and in the investment process, Merton's (1987) simple model of capital market equilibrium with incomplete information might provide some insights into the behaviour of security prices. The model allows studying the equilibrium structure of asset prices and its connection with empirical anomalies in the same context.

The appendix provides the derivation of a Differential equation for a derivative security on a spot asset in the presence of a continuous dividend yield and information costs. It provides the general differential equation for the pricing of derivative assets with information costs.

It gives also the valuation of simple European and American Commodity options with information costs.

The following tables provide simulations results regarding our model with incomplete information and the Black and Scholes model. Option values are compared for different levels of the underlying asset (from 70 to 120) and different information costs regarding the option and its underlying asset.

Table 1
Call options values using the following parameters:
K =100, r=0.08, t=0.25, σ =0.4

Black & Scholes		Incomplete Information			(λs, λc)		
		.01 , .001	.01 , 0	.03 , .001	.03 , 0	.2 , .01	.2 , 0
S: 70	.0000	.0000	.0000	.0000	.0000	.0000	.0000
S:80	.9800	.9829	.9775	.9780	.9727	.9374	.9322
S: 90	0.8972	4.1189	4.1103	4.0984	4.0898	3.9278	3.9196
S:100	5.0177	8.9085	8.8941	8.8641	8.8497	8.4953	8.4815
S:110	12.6520	15.6119	15.5912	15.540	15.5134	14.887	14.867
S:120	22.0877	23.660	23.7409	23.6489	23.6225	22.664	22.639

Table 2
Call options values using the following parameters:
K =100, r =0.08, t=0.5, σ =0.2

Black & Scholes		Incomplete Information			(λs, λc)		
		.01, .001	.01 , 0	.03 , .001	.03 , 0	.2 , .01	.2 , 0
S: 70	.0000	.0000	.0000	.0000	.0000	.0000	.0000
S:80	.9800	.0000	.0000	.0000	.0000	.0000	.0000
S: 90	0.8972	2.7336	2.7174	2.7064	2.6904	2.4859	2.4712
S:100	5.0177	7.7024	7.6705	7.6257	7.5942	7.0044	6.9754
S:110	12.6520	15.2463	15.1936	15.0887	15.0425	13.856	13.816
S:120	22.0877	24.2790	24.2223	24.0374	23.9814	22.078	22.0227

Table 3
Call options values using the following parameters:
K=100, r=0.12, t=0.25, σ =0.2

Black & Scholes		Incomplete Information			(λs, λc)		
		.01, .001	.01 , 0	.03, .001	.03 , 0	.2 , .01	.2 , 0
S: 70	.0000	.0000	.0000	.0000	.0000	.0000	.0000
S:80	.9800	.0000	.0000	.0000	.0000	.0000	.0000
S: 90	0.8972	2.7336	2.7174	2.7064	2.6904	2.4859	2.4712
S:100	5.0177	7.7024	7.6705	7.6257	7.5942	7.0044	6.9754
S:110	12.6520	15.2463	15.1936	15.0887	15.0425	13.856	13.816
S:120	23.0877	23.0110	22.2223	21.0374	21.9230	21.038	21.011

The following tables provide simulation results for the extended Black model in the presence of information costs. Simulations are given for different levels of the underlying futures price and several information costs.

Table 4
Europeen futures call values using the following parameters:
K=100, r=0.08, T=0.25, σ = 0.2

Complete Information Black's model		Incomplete Information model	
Futures price	Option price	λc = 1%	λc = 5%
F = 70	0.0000	0.0000	0.0000
F = 80	0.0000	0.0000	0.0000
F = 90	0.4363	0.4353	0.4310
F = 100	3.8990	3.8990	3.8602
F = 110	10.7627	10.7260	10.6290
F = 120	19.7754	19.7260	19.5290

Table 5
Europeen futures call values using the following parameters:
K=100, r =0.12, T = 0.25, σ = 0.2

Complete Information Black's model		Incomplete Information model	
Futures price	Option price	λc = 1%	λc = 5%
F = 70	0.0000	0.0000	0.0000
F = 80	0.0000	0.0000	0.0000
F = 90	0.4320	0.4353	0.4267
F = 100	3.8698	3.8990	3.8217
F = 110	10.6556	10.6260	10.5237
F = 120	19.5786	19.5260	19.3354

Table 6
Europeen futures call values using the following parameters:
K=100, r=0.08, T=0.5, σ =0.2

Complete Information Black's model		Incomplete Information model	
Futures price	Option price	λc = 1%	λc = 5%
F = 70	0.0000	0.0000	0.0000
F = 80	0.0000	0.0000	0.0000
F = 90	1.5913	1.5833	1.5520
F = 100	5.4161	5.3890	5.2823
F = 110	11.7525	10.6939	11.4235
F = 120	19.9557	19.8562	19.4630

IV. THE INVESTMENT TIMING AND THE PRICING OF REAL ASSETS WITHIN INFORMATION UNCERTAINTY

The investment opportunity is analogous to a call option on a common stock since it gives the right to make investment expenditure at the strike price and to receive the project. The firm's option to invest refers to the possibility to pay a sunk cost I and to receive a project which is worth, V[11]. Irreversibility is an important component of the investment process[12]. The dynamics of the project's value can be described by the following equation:

$$dV/V= \alpha dt + \sigma\, dz \tag{3}$$

where α and σ refer to the instantaneous rate of return and the standard deviation of the project, and dz is a geometric Brownian motion.

This equation shows that the current project value is known, whereas its future values are log-normally distributed. Following Bellalah (2001 a), we denote by X the price of an asset perfectly correlated with V.

The dynamics of X are represented by:

$$dx/ X= \mu dt + \sigma\, dz \tag{4}$$

where μ stands for the expected return from owning a completed project.

We denote by $\delta = \mu-\alpha$. If V were the price of a share, δ would be the dividend rate on the stock. In this context, δ represents an opportunity cost of delaying investment. If δ is zero, then there is no opportunity cost to keeping the option alive.

Let C(V) be the value of the firm's option to invest. Using Merton's (1987) model, Bellalah (2001 a) obtain option prices in the context of incomplete information. Consider the return on the following portfolio P: hold an option which is worth C(V) and go short C(V) units of the project where the subscript V refers to the partial derivative with respect to V :

$$P = C - C_V V \tag{5}$$

The total return for this portfolio over a short interval of time dt is:

$$dC - C_V dV - \delta V\, C_V\, dt \tag{6}$$

Since there are information costs supported on the option and on its underlying assets, the return must be equal to (r +λv) for the project and (r +λc) for the option where λv and λc refer respectively to the information costs on the project and he option. In fact, as it appears in the previous analysis and as it is suggested by the referee, the information cost relative to the project λv ncerns the collection of information, the analysis and the study of the project. The information cost regarding the option is λc explained by the sunk costs spend to analyze, value, collect information and implement models regarding the option. Our analysis reveals that the dividend yield δ is adjusted to become δ + λc - λv. The discounting of the option price is done under the risk neutral probability with information costsa. This can be written as r + λc. The risk neutral drift is changed from r - δ to r - δ + λv.

In this context, we have:

$$dC - C_V dV - \delta V\, C_V\, dt = (r + \lambda c)C\, dt + (r + \lambda_v)\, V\, C_V\, dt \tag{7}$$

or:

$$1/2\, \sigma^2 V^2\, C_{VV} + (\, r + \lambda v - \delta)VC_V - (r + \lambda c)\, C = 0 \tag{8}$$

This equation for the value of C(V) must satisfy the following conditions:

$$C(0) = 0$$
$$C(V^*) = V^* - I$$
$$CV(V) = 1$$

The value V* is the price at which it is optimal to invest. At that time, the firm receives the difference V* - I. The solution to the differential equation under the above conditions gives the value of C(V). The solution under the first condition is:

$$C(V) = a\, V^{\beta} \tag{9}$$

where a is a constant and :

$$\beta = \frac{1}{2} - \frac{(r - \delta + \lambda v)}{\sigma^2} + \sqrt{[\frac{(r - \delta + \lambda v)}{\sigma^2} - \frac{1}{2}]^2 + \frac{2(r + \lambda c)}{\sigma^2}} \tag{10}$$

The value of the constant a and the critical value V* are:

$$V^* = \frac{\beta I}{\beta - I}, \qquad \alpha = \frac{(V^* - I)}{(V^*)^{\beta}} \tag{11}$$

In Myers and Majd (1985), the sunk costs are related to the decision to exit or abandon a project for different reasons including severance pay for workers, and land reclamation for the case of a mine. In the Brennan and Schwartz (1985) model, the decision to invest contains the sunk cost of land reclamation. Following the work of Myers (1984), Kester (1984) and Grenadier and Weiss (1997), the option-pricing theory can be applied to real-investment decisions as well as to strategies.

The innovation investment strategy can be viewed as a link in a chain of future investment options. Grenadier and Weiss (1997) identify four potential strategies[13].

V. THE VALUATION OF COMPOUND OPTIONS WITHIN INFORMATION COSTS

Several projects are often valued using the concept of compound options introduced by Geske (1979). For example, the development process for a new product requires several stages where the manager resorts to the new information revealed up to that point to decide whether to abandon or to continue the project. This is particularly the case for a biotechnology firm for which the development of a drug needs several stages.

The idea is that engaging in the development phase is equivalent to buying a call on the value of a subsequent product. Hence, there is the initial option and the growth option. In the presence of only two stages a formula for a call on a call can be used. We show how to value compound options in the presence of information costs. For the sake of simplicity, we use the general context proposed by Geske (1979).

If the stock is considered as an option on the value of the firm, V, then the value of the call as a compound option can be expressed as a function of the firm's value. This analysis follows from the setting in Geske (1979). As suggested by the referee, we can also apply the model in Bellalah and El Farissi (2002) to show how to price options with information costs by generalizing the approaches in Leland (1994) and Leland and Toft (1996).

Following Geske (1979), consider a levered firm for which the debt corresponds to pure discount bonds maturing in T years with a face value M. Under the standard assumptions of liquidating the firm in T years, paying off the bondholders and giving the residual value (if any) to stockholders, the bondholders have given the stockholders the option to buy back the assets of the firm at the debt's maturity date.

In this context, a call on the firm's stock is a compound option, $C(S,t) = f(g(V,t),t)$ where t stands for the current time.

Using the standard dynamics, the return on the firm's assets follows the stochastic differential equation:

$$dV/V = \sigma_v dt + \sigma_v dz_v \tag{12}$$

Where α_v and δ_v refer to the instantaneous rate of return and the standard deviation of the return of the firm per unit time, and dz_v is a Brownian motion.

Using the definition of the call C (V,t) , its return can be described by the following differential equation:

$$dC/C = \alpha_c dt + \sigma_c dz_c \tag{13}$$

where α_c and δ_c refer to the instantaneous rate of return and the standard deviation of the return on the call per unit time, and dzc is a Brownian motion. Using Ito's lemma as before, the dynamics of the call can be expressed as:

$$dC = \frac{1}{2} C_{vv}\sigma^2{}_v + C_u\, dV + C_t dt \tag{14}$$

It is possible to create a riskless hedge with two securities, in this case, between the firm and a call to get the following partial differential equation:

$$\frac{1}{2}\sigma^2{}_v V^2 C_{vv} + (r + \lambda_v)VC_u - (r+\lambda_c)C + C_t = 0 \tag{15}$$

where λ_v in an information cost relative to the firm's or the project's value. At the option's maturity date, the value of the call option on the firm's stock must satisfy the following condition:

$$C_t = \max [S_t - K, 0] \tag{16}$$

where K stands for the strike price. Investors suffer sunk costs to get informed about the equity and the assets of the firm. The costs regarding the equity and the firm's cash-flows reflect the agency costs and the asymmetric information costs. These costs characterize also joint ventures. In this situation, the call formula is given by:

$$C_0 = V_0\, e^{-(\lambda c - \lambda v)T}$$

$$N_2\left(h + \sigma_v \sqrt{t},\ k + \sigma v\sqrt{T},\ \sqrt{\frac{t}{T}} \right) - Me^{-(r+\lambda c)T}\, N_2\left(h,k,\sqrt{\frac{t}{T}}\right) - Ke^{-(r+\lambda c)t}\, N(h) \tag{17}$$

where M is the face value of debt. The value/bar{V} is determined by the following equation:

$$S_t - K = \overline{V}e^{-(\lambda c - \lambda v)(T-t)} N(k + \sigma v\sqrt{T-t}) - Me^{-(r - \lambda v)(T - t)}\, N(k) - K = 0 \tag{18}$$

with:

$$h = [\ \ln(\frac{V}{V}) + (r + \lambda v - \frac{1}{2}\sigma v^2)t] / \sigma v\sqrt{t}$$

$$k = [\ \ln(\frac{V}{M}) + (r + \lambda v - \frac{1}{2}\sigma v^2)T] / \sigma v\sqrt{T}$$

where N(.,.,.) refers to the cumulative bivariate normal distribution. If the information cost is zero, this compound option pricing formula becomes that in Geske (1979). This formula is also useful for the valuation of real options in the presence of information costs.

Table 7
Simulation and comparison of the Black and Scholes model and our model for the values of European equity options.

V	λc=λv= 0	λc= 0.1 %, λv= 1 %	λc= 0 %, λv= 1%	λc= 0.1 %, λv= 3 %
70	0.000	0.000	0.000	0.000
80	0.9800	0.9829	0.9775	0.9780
90	4.1206	4.1189	4.1103	4.0984
100	8.9163	8.9085	8.8941	8.8641
110	15.6302	15.6119	15.5912	15.5340
120	23.8003	23.7660	23.7409	23.6489

Notes: The following parameters are used: M = 100, r = 0.08, T =0.25, δv= 0.4

Table 8
Simulation of equity values as compound options in the presence of information costs using our model for the following parameters

C0	λc=0% λv=0%	λc=2% λv=2%	λc=1% λv=2%	λc=1% λv=2%
110	6.82	7.13	7.16	7.14
120	15.17	15.65	15.70	15.67
130	26.52	27.16	27.25	27.20

Notes: The following parameters are used : K = 20 , M = 100 , r = 0.08 , T =0.25 , t = 0.125 ,δv= 0.4

Table 8 provides the simulation results for the compound option formula with information costs and the Geske's compound call formula using the following parameters: K = 20, M = 100, r = 0.08, T =0.25, t = 0.125, δv= 0.4. The parameters used for information costs are:

case a : (λc= λv=0 %),
case b : (λc= λv= 2 %),
case c : (λc= 1 %, λv=2 %),
case d : (λc= 1%, λv= 2%).

In case (a), we have exactly the same values as those generated by the formula in Geske (1979). The table shows that the compound option price is an increasing function of the firm's or the project's assets. This result is independent of the values attributed to information costs. The compound option price is an increasing function of the information costs regarding the firm's assets, λv. When λv is fixed, this allows the study of the effects of the other information costs on the option value. In this case, the option price seems to be a decreasing function of the information cost λc. We intend to test this model on real data.

VI. RESEARCH AND DEVELOPMENT AND THE OPTION ON MARKET INTRODUCTION IN THE PRESENCE OF INFORMATION COSTS

Several companies face the difficulty of selecting an optimal portfolio of research projects. As it appears in the analysis of Lint and Pennings (1998), the standard DCF techniques for capital budgeting can distort the process of selecting a portfolio of research projects. When managers have the option to abandon a project, it is possible to think of the cost of R&D as an option on major follow-on investments. Newton and Pearson (1994) provide an option pricing framework for R&D investments. Lint and Pennings (1998) report the application of an option pricing model for setting the budget of R&D projects. Their model captures a discontinuous arrival of new information that affects the project's value. R&D options can be viewed as European when two conditions hold[14].

In the Lint and Pennings's (1998) model, the variance of the underlying value σ^2 is given by the product of a parameter representing the number of annual business shifts η and a parameter γ for the expected absolute change in the underlying value at every business shift:

$$\sigma^2 = \eta \gamma^2 \tag{19}$$

Applying asymptotic theory, the option value can be approximated with the Black and Scholes (1973) formula where σ^2 is replaced by $\eta \gamma^2$, or:

$$C(S,T) = S(t)\, N(d + \sqrt{\eta(T-t)\gamma}) - Ie^{-r(T-t)}N(d)$$

$$d = \left[\ \ln\left(\frac{S(t)}{I}\right) + \left(r - \frac{1}{2}\eta\gamma^2\right)(T-t)\ \right] / \sigma\sqrt{\eta(T-t)}\gamma \tag{20}$$

where S(t), r, T-t stand respectively for the underlying value at present, the costs for market introduction, the risk free rate and the option's time to maturity.

Lint and Pennings (1998) use their model in Philips and show that the option value is largely determined by the opportunity to make a final decision on market introduction with more technological and market information. They show that the option value must compensate the R&D costs necessary to create the option. Their estimation of the option value of the potential benefits to market new products based on R&D goes beyond myopic use of DCF analysis.

In the conclusion of their paper, they suggest to classify a variety of past and current R&D projects into sets of similar risks and returns. This can allow the estimation of the value of future idiosyncratic R&D projects by option analysis as in Newton and Pearson (1994). This line of research imposes an information cost in the spirit of the costs in Merton's (1987) model of capital market equilibrium with incomplete information[15].

It is possible to use the methodology in Lint and Pennings (1998) and in Bellalah (1999) to account for the role of information costs. In this case, the option value is given by:

$$C(S,T) = S(t)e^{-(\lambda c-\lambda s)(T-t)}N(d + \sqrt{\eta(T-t)}\ \gamma) - Ie^{-(r+\lambda c)(T-t)}N(d)$$

$$d= [\ln((\frac{S(t)}{I}) + (r + \lambda s - \frac{1}{2}\eta\gamma^2) (T-t)]/ \sigma\sqrt{\eta(T-t)}\ \gamma \tag{21}$$

where λs and λc denote respectively the information costs relative to S and C .

VII. THE VALUATION OF REAL OPTIONS AND R&D PROJECTS WITHIN INFORMATION COSTS IN A DISCRETE-TIME SETTING

The majority of the papers concerned with the pricing of real assets in a discrete time setting derive from the models for financial options pioneered by Cox, Ross and Rubinstein (1979).

A. The valuation of a biotechnology firm using a discrete-time framework within information costs

Following the analysis in Kellogg and Charnes (1999), the value of the firm can be found also using the binomial lattice with the addition of a growth option. The growth option is represented by a second binomial lattice for a research phase. The current value of the asset S (or $S_{0,0}$ is computed using the discounted value of the expected commercialization cash flows to time zero as :

$$S_{0,0} = S = \sum_{j=1}^{5} qj \sum_{t=1}^{T} \frac{(1+r_c)^t}{CCF_{jt}} \tag{22}$$

where the discount rate is estimated using Merton's CAPMI. The number of stages can be arbitrarily any number.

It is possible to construct an n period binomial lattice of asset values. The value of the underlying asset S goes up by u or down by d. This multiplicative process is continued for n period until the n th lattice.

Kellogg and Charnes (1999) use the fact that $u = e^{\sigma}$ and $d = e^{-\sigma}$ and impose that h = Sul = $Se^{\sigma l}$ where l corresponds to a given number of years. They used an example in which the periods are supposed to have a length of one year.

The next step is to add in the value of the growth option. The idea is that engaging in the development phase is equivalent to buying a call on the value of a subsequent product. Hence, there is the initial option and the growth option. The value of the growth option at the time of the launch of the first product is added to each of the Ek values of the first NME.

Once the binomial tree of asset values is completed, it is possible to compute the possible payoffs and roll back the values using the risk neutral probabilities. The different payoffs are computed as:

$$P_k = \max [E_k(\theta_t) - DCF_{t,} 0] \tag{23}$$

where θ_t : the probability of continuation to the next year in t and DCFt : the R&D payment in year t.

The Pk values are rolled back by multiplying the adjacent values, such as P_1 and P_2 (denoted by Vt+1, k and Vt+1, k+1) by the risk neutral probabilities p and (1-p), the probability of continuation to the next year and a discount factor to obtain Vt,k.

The risk neutral probabilities are calculated as: $p = \dfrac{e^{(r+\lambda s)\Delta t} - d}{(u-d)}$. As the option values are rolled back, they are adjusted for the probability of success at that phase of development and for the cost of development that year. The option values can be obtained at each node as:

$$V_{t,k} = \max[\, (p\, V_{t+1,k} + (1\text{-}p)\, V_{t+1,k+1})\ \theta_t\ e^{-(r+\lambda v)\sqrt{\Delta t}} - DCFt, 0] \qquad (24)$$

B. The generalization of discrete time models for the pricing of projects and real assets within information uncertainty

Trigeorgis (1991) proposed a Log-transformed binomial model for the pricing of several complex investment opportunities with embedded real options. The model can be extended to account for information costs. The value of the expected cash flows or the underlying asset V satisfies the following dynamics:

$$\frac{dV}{V} = \alpha dt + \sigma dz \qquad (25)$$

Consider the variable X = log V and K = σ^2 dt.

If we divide the project's life T into N discrete intervals of length, then K can be approximated from $\sigma^2 \frac{T}{N}$. Within each interval, X moves up by an amount $\Delta X = H$ with probability π or down by the same amount $\Delta X = -H$ with probability $(1 - \Pi)$. The mean of the process is $E(dX) = \mu K$; and its variance is $Var(dX) = K$ with

$$\mu = \frac{(r + \lambda_S\ )}{\sigma 2} - \frac{1}{2}. \qquad (26)$$

The mean and the variance of the discrete process are $E(\Delta X) = 2\pi H - H$ and $Var(\Delta X) = H^2 - [E(\Delta X)]^2$.

The discrete time process is consistent with the continuous diffusion process when $2\pi H - H = K$ with $\mu = \dfrac{(r+\lambda s)}{\sigma^2} - \dfrac{1}{2}$ so $\pi = \dfrac{1}{2}(1 + \dfrac{\mu K}{H})$ and $H^2 - \mu K^2 = K$ so that H $= \sqrt{K + (\mu K)^2}$.

The model can be implemented in four steps. In the first step, the cash flows CF are specified.

In the second step, the model determines the following key variables: the time-step: the drift μ from $\frac{(r+\lambda s)}{\sigma^2 - \frac{1}{2}}$, the state-step H from $\sqrt{K+(\mu K)^2}$, and the probability π from $\frac{1}{2}(1+\frac{\mu K}{H})$.

Let j be the integer of time steps (each of length K), i the integer index for the state variable X (for the net number of ups less downs). Let R(i) be the total investment opportunity value (the project plus its embedded options). In the third step, for each state i , the project's values are $V(i) = e^{(X0 + i H)}$. The total investment opportunity values are given by the terminal condition R(i) = max[V(i), 0].

The fourth step follows an iterative procedure. Between two periods, the value of the opportunity in the earlier period j at state i, R'(i) is given by:

$$R'(i)= e^{-(r+ \lambda c)((\frac{K}{\sigma^2})} [\pi R(i+1) + (1- \pi)R(i-1)] \tag{27}$$

In this setting, the values of the different real options can be calculated by specifying their payoffs. The payoff of the option to switch or abandon for salvage value S is R'= max(R,S).

The payoff of the option to expand by e by investing an amount I4 is R'= R+max (e V - I4,0). The payoff of the option to contract the project scale by c saving an amount I3' is R'= R + max(I3'- cV,0). The payoff of the option to abandon by defaulting on investment I2 is R'=max(R -I2,0). The payoff of the option to defer (until next period) is R'= max(e $e^{-(r+ \lambda c)T}$ $E(R_{j+1})$, Rj).

When a real option is encountered in the backward procedure, then the total opportunity value is revised to reflect the asymmetry introduced by that flexibility or real option. This general procedure can be applied for the valuation of several projects and firms in the presence of information costs.

VII. SUMMARY

This paper provides the main results in the literature regarding the valuation of the firm and its assets using the real option theory when we account for the effects of information uncertainty.

We propose some simple models for the analysis of the investment decision under uncertainty, irreversibility and sunk costs.

First, we use Merton (1987) model of capital market equilibrium with incomplete information to determine the appropriate rate for the discounting of future risky cash flows under incomplete information. The context of incomplete information allows the extension of the standard theory of firm valuation.

Second, we present the main potential applications of option pricing theory to the valuation of simple and complex real options. Third, we develop some simple models for the pricing of European and American commodity options in the presence of information costs. We propose also simple analytic formulas for the pricing of compound options in the presence of information costs. These formulas are useful in the study of the main results in the literature regarding the investment timing and the pricing of real assets using standard and complex options in the presence of incomplete information. The analysis is extended to the valuation of research and development and the option on market introduction. It is also applied to the valuation of flexibility as a compound option in the same context. Fourth, a general context is proposed for the valuation of real options and the pricing of real assets in a discrete-time setting. Using the Trigeorgis (1991) general Log-transformed binomial model for the pricing of complex investment opportunities, we provide a context for the valuation of real options under incomplete information. Our approach can be extended to price most well-known real options in the presence of information costs. While the estimation of the magnitude of these costs is done in Bellalah and Jacquillat (1995) for financial options, it is possible to look for a convenient approach to estimate these costs for real options. We let this point for a future research.

APPENDIX

The valuation equations for a derivative security in the presence of information costs

1. The valuation equations derivation of a Differential equation for a derivative security on a spot asset in the presence ofa continuous dividend yield and information costs

Following the analysis in Bellalah (2001), we denote by C the price of a derivative security on a stock with a continuous dividend yield δ.

The dynamics of the underlying asset are given by $dS = \mu S dt + \sigma S\, dz$, where the drift term \mu and the volatility \sigma are constants and dz is a Wiener process.

Using Ito's lemma for the function C(S,t) gives

$$dC= \left(\frac{\partial C}{\partial S}\ \mu S + \frac{1}{2}\ \frac{\partial^2 C}{\partial S^2}\sigma^2 S^2\right)dt + \frac{\partial C}{\partial S}\sigma S dz \qquad (28)$$

It is possible to construct a portfolio \Pi by holding a position in the derivative security and a certain number of units of the underlying asset

$$\Pi = - C + \frac{\partial C}{\partial S} S \qquad (29)$$

Over a short time interval, the change in the portfolio value can be written as

$$\Delta\Pi= \left(- \frac{\partial C}{\partial S}\ \mu S - \frac{1}{2}\ \frac{\partial^2 C}{\partial S^2}\sigma^2 S^2\right) \Delta t \qquad (30)$$

Over the same time interval, dividends are given by $\delta S \frac{\partial C}{\partial S} \Delta t$ We denote by δW the change in the wealth of the portfolio holder. In this case, we have

$$\Delta W = \left(-\frac{\partial C}{\partial t} - \frac{1}{2}\frac{\partial^2 C}{\partial S^2}\sigma^2 S^2 + \delta S\frac{\partial C}{dS}\right)\Delta t \tag{31}$$

Since this change is independent of the Wiener process, the portfolio is instantaneously risk-less and must earn the risk-free rate plus information costs or

$$\left(-\frac{\partial C}{\partial t} - \frac{1}{2}\frac{\partial^2 C}{\partial S^2}\sigma^2 S^2 + \delta S\frac{\partial C}{dS}\right)\Delta t = -(r+\lambda c)C\Delta t + (r+\lambda s)S\frac{\partial C}{dS}\,)\Delta t \tag{32}$$

Where λ_i refers to these costs. This gives

$$\frac{\partial C}{\partial t} + (r+\lambda s-\delta)\,S\frac{\partial C}{\partial S}\;\Delta t + \frac{1}{2}\frac{\partial^2 C}{\partial S^2}\sigma^2 S^2 = (r+\lambda c)C \tag{33}$$

This equation must be satisfied by the derivative security in the presence of information costs and a continuous dividend yield.

2. The general differential equation for the pricing of derivative assets with information costs

We consider the general case for the valuation of any contingent claim with information cost. We denote by

- θ_i : value of ith state variable,
- m_i : expected growth in ith state variable,
- γ_i : market price of risk of ith state variable,
- S_i : volatility of ith state variable,
- r : instantaneous risk-free rate,
- λ_i : shadow cost of incomplete information of ith state variable, where i takes the values from 1 to n .

As it is well known, the price of any contingent claim must satisfy the following partial differential equation where $\rho_{i,j}$ stands for the correlation coefficient between the variables θ_i and

$$\sum_i \theta_i\frac{\partial C}{\partial\theta_i}(m_i-\gamma_i S_i) + \frac{1}{2}\sum_{i,k}\rho_{i,k},s_i s_k\theta_i\theta_k\;\frac{\partial^2 f}{\partial\theta_i\partial\theta_k} = rC \tag{34}$$

We can show as in Bellalah (2001) how to obtain a similar equation in the presence of incomplete information. In this context, the equation becomes

$$\sum_i \theta_i\frac{\partial C}{\partial\theta_i}(m_i-\gamma_i S_i) + \frac{1}{2}\sum_{i,k}\rho_{i,k},s_i s_k\theta_i\theta_k\;\frac{\partial^2 f}{\partial\theta_i\partial\theta_k} = (r+\lambda)\,C \tag{35}$$

In the presence of a single state variable, θ, equation (2) becomes

$$\frac{\partial C}{\partial t} + \theta \frac{\partial C}{\partial \theta 2}(m - \gamma s) + \frac{1}{2} s^2 \theta^2 \frac{\partial^2 C}{\partial^2 \theta} = (r + \lambda)C \tag{36}$$

For a non-dividend paying security, the expected return and volatility must satisfy $m - r - \lambda = \gamma s$ and $m - \gamma s = r + \lambda$.

In this case, equation (36) becomes the extended Black-Scholes equation in the presence of information costs.

For a dividend-paying security at a rate δ, we have $\delta + m - r - \lambda = \gamma s$ or $m - \gamma s = r + \lambda - \delta$.

In this case, equation (36) becomes

$$\frac{\partial C}{\partial t} + (r + \lambda_S - \delta)S \frac{\partial C}{\partial S} + \frac{1}{2} s^2 S^2 \frac{\partial^2 C}{\partial^2 S} = (r + \lambda)C \tag{37}$$

This equation is a generalization of the Black and Scholes (1973) equation in the presence of information costs.

3. The valuation of simple European and American Commodity options with information costs

Following Black (1976), we assume that all the parameters of the Merton's (1987) CAPMI are constant through time.

Under these assumptions, the value of the commodity option, C(S,t), can be written as a function of the underlying price and time.

As suggested by the referee, we can use some limiting arguments as in Bellalah and Prigent (2001) to show that the application of Merton's (1987) model is possible in a continuous-time setting.We can show as in Bellalah (1999) the following equation for the pricing of commodity options :

$$\frac{1}{2}\sigma^2 S^2 C_{ss} + (b + \lambda_S) SC_S - (r + \lambda_c)C + C_t = 0 \tag{38}$$

When the information costs λS and λc are set equal to zero, this equation collapses to that in Barone-Adesi and Whaley (1987). The term b indicates the cost of carrying the commodity. The value of a European commodity call is:

$$C(S,T) = Se^{((b-r-(\lambda c - \lambda s))T)} N(d_1) - Ke^{-(r+\lambda c)T} N(d_2) \tag{39}$$

with:

$$d_1 = \left[\ln\left(\left(\frac{S}{K}\right) + (b + \frac{1}{2}\sigma^2 + \lambda s)T \right] / \sigma\sqrt{T}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

where N(.) is the cumulative normal density function.

When λ_S and λ_c are equal to zero and b = r, this formula is the same as that in Black and Scholes. A direct application of the approach in Barone-Adesi and Whaley (1987), allows writing down immediately the formulas for American commodity options with information costs.

ENDNOTES

1. This paper benefits from the comments of professors Gordon Sick, Lenos rigeorgis and the participants in the real option conference in Cyprus, July 2002. I would like to thank Professors Richard Roll, Giovanni Barone-Adesi and Robert Webb and an anonymous referee for their helpful comments.
2. For a survey of important results in the literature, the reader can refer to Brealey and Myers (1985), Copeland and Weston (1988), Smith and Nau (1994) and Bellalah (1998) among others.
3. For a survey of the important results in the standard literature regarding option pricing, the reader can refer to Black and Scholes (1973), Merton (1973, 1992), Cox, Ross and Rubinstein (1979), Cox and Rubinstein (1985), Hull (2000), Briys-Bellalah et al. (1998) among others.
4. For a survey of the literature on real options, the reader can refer to Trigeorgis (1990, 1993 a, b, c, 1995, 1996), Pindyck (1991), Padock, Siegel and Smith (1988), Myers (1984), McDonald and Siegel (1984, 1986), Myers and Majd (1990) among others.
5. For a review of the main results in this literature, the reader can refer to Luehrman (1997, 1998), etc.}
6. The main element in the determination of profitability in certain cyclical activities is the ability of timing a business cycle to build for example a new factory. The manager does not have to commit himself outright to a new factory. He has the option of staging the investment over a given period by paying a certain amount up front for design, another amount in a period for pre-construction work and an other outlay to complete construction at the end of the year. This gives him the flexibility to walk away if profit projections fall below a given level or to abandon at the end of the initial construction phase and save a given additional outlay. The factory is designed to convert an input into an output and its profitability would be a function of the spread between these prices. The manager can invest in new factories only when the input output spread is higher than its long-term average. The NPV assumes that the factory is built and operated, ignoring the flexibility offered to managers
7. In general, learning options appear when a company has the possibility to speed up the arrival of information by making an investment. Real option theory can be used to determine the optimal time to exercise the option. When the company does not know the quantity of ore in its mine, it has a learning option: to pay money to find out. Here also, the main models for the pricing of exotic options can be applied.
8. Consider a company deciding on how much production capacity to install in an undeveloped natural gas field. The company can create a decision tree for a real option valuation model (ROV) to weigh up the various decisions in view of the uncertainty regarding the price and quantity. Using the information regarding the

volatility of gas prices and quantity, the ROV model can estimate the total value of the different courses open to the company. The reader can refer to the work of Brennan (1991), Brennan and Schwartz (1985), Pickles and Smith (1993), etc.

9. Consider a pharmaceutical company ranking different R&D projects in order of priority. The real option approach handles both uncertainties. R&D projects can be classified as compound rainbow options, each contingent on the preceding options and on multiple sources of uncertainty (rainbow options and multi factor options). In this context, the models for the pricing of exotic options can be applied. For the general approach regarding the pricing of these options, the reader can refer to Bellalah (2002).
10. These models undermine the theoretical foundation of standard neoclassical investment models and invalidate the net present value criteria in investment choice under uncertainty. For a survey of this literature, the reader can refer to Pindyck (1991) and the references in that paper.
11. Unlike standard options, this call is perpetual and has no expiration date. This result is used in McDonald and Siegel (1986) and Pindyck (1991). In this context, the investment opportunity is equivalent to a perpetual call. The decision regarding the timing of the investment is equivalent to the choice of the exercise time of this option
12. Pindyck's (1991) presents a survey of some applications of this theory to a variety of investment problems.
13. For an extension of their model to account for the effects of incomplete information, see Bellalah (2001, 2002)}.
14. Lint and Pennings (1998) assume that the costs associated with the irreversible investment, required for market introduction, and the time for completing R&D are given with reasonable accuracy. By ignoring dividends, they propose a simple model which is an extension for R&D option pricing in practice. The approach in Lint and Pennings (1998) is based on a discontinuous arrival of information affecting the project.
15. These costs are discussed in Bellalah and Jacquillat (1995) and Bellalah (1999) for the pricing of financial options

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