INTERNATIONAL JOURNAL OF BUSINESS, 10(2), 2005 ISSN:1083-4346

On Portfolio Analysis, Market Equilibrium and Corporation Finance with Incomplete Information

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ABSTRACT

This paper presents a new derivation of the Modigliani and Miller (1958, 1963, 1966) propositions using the simple model of capital market equilibrium with incomplete information presented in Merton (1987). The model is used to relate the maximization of stockholder expected utility to the selection of assets and to the financing and investment decisions within firms in a context of incomplete information. Expressions of the cost of capital are presented with and without corporate taxes in the presence of shadow costs of incomplete information.

JEL: *G11, G12, G31, G32*

Keywords: *Incomplete information; Cost of capital; Arbitrage*

I. INTRODUCTION

The effects of uncertainty in financial and economic decision making have been studied without an explicit treatment of the implications of information uncertainty. The main conceptual frameworks are provided first by Modigliani and Miller, MM, (1958, 1963,1966). These authors develop a homogenous risk-class concept in order to eliminate the need for a general equilibrium model. Markowitz (1952), Sharpe (1964), Lintner (1965) and Mossin (1966) provide a basis for making investment decisions using the market model or the capital asset pricing model. Hirshleifer (1964, 1965, 1966) provides a time-state preference approach to prove the Modigliani and Miller, MM no-tax proposition I. Unfortunately, this last approach does not lead to practical decision rules for capital budgeting within the firm.

This paper derives the three MM Propositions using Merton's (1987) model of capital market equilibrium under incomplete information. The use of Merton's (1987) model is motivated by the increasing importance of information and its role in the process of valuation of firms and their assets. An important question in financial economics is how frictions affect equilibrium in capital markets since in a world of costly information, some investors will have incomplete information. As in Hamada (1969), this approach avoids the arbitrage proof in MM, the risk class assumption. The new derivation follows the analysis in Hamada (1969). A model is used relating the maximization of stockholder expected utility to the portfolio selection to, the financing and investment decisions in the corporation. This analysis provides a link between two branches of finance.

Differences in information are important in financial and real markets. They are used in several contexts to explain some puzzling phenomena like the “home equity bias”, the “weekend effect”, “the smile effect”, etc. Kadlec and McConnell (1994) conclude that Merton's shadow cost of incomplete information, reflects also the elasticity of demand and that it may proxy for the adverse price movement aspect of liquidity.(footnote 19, page 629). Foerster and Karolyi (1999) construct an empirical proxy for the shadow cost of incomplete information for each firm, using the methodology in Kadlec and McConnell (1994). Their results are supportive of the Merton (1987) hypothesis and consistent with Kadlec and McConnell (1994). There is a main difference between Merton's (1987) model and the model in Peress (2000). In both models agents spend time and resources to gather information about the security's payoff, but in Merton's model investors are not all aware of the existence of the security but, if they are, they have information of the same quality.

Merton (1987) advances the investor recognition hypothesis in a mean-variance model. This assumption explains the portfolio formation of informationally constrained investors (ICI). The investor recognition hypothesis (IRH) in Merton's context states that investors buy and hold only those securities about which they have enough information.

Merton (1987) adapts the rational framework of the static CAPM to account for incomplete information. Increasing empirical support for IRH-consistent behavior appeared in Falkenstein (1996), Huberman (1999), Shapiro (2000) among others. Even if these authors, among others, study only the case of a portfolio of stocks, the analysis can be applied to derivative assets since options can be duplicated using the underlying asset and riskless bonds.

For these reasons, it is important to account for information costs in the pricing of assets. Bellalah (2001 a, b) develops some applications of the simple model of capital equilibrium with incomplete information CAPMI of Merton (1987) to the pricing of real options and to the valuation of the firms and their assets. Bellalah and Zhen (2002) develop also a model for market closure and international portfolio management within incomplete information.

The structure of this paper is as follows. In section 2, we recall the assumptions and Merton's (1987) model of capital market equilibrium with incomplete information, CAPMI. In section 3, we generalize the MM's Propositions I and II for the no corporate income tax case within information uncertainty. In section 4, the effects of corporate taxes on the financing decision are analyzed. In section 5, the cost of capital is studied in the no tax case. In section 6, the cost of capital is studied in the corporate tax case within information uncertainty.

II. ASSUMPTIONS AND THE EQUILIBRIUM

We use the same assumptions as those in Merton (1987). In addition, the following three assumptions are used: (1) Expected bankruptcy or default risk associated with debt financing, the interest rate risk and the purchasing power fluctuation are assumed to be negligible when compared to variability risk on equity. (2) Dividend policy does not affect the market value of a firm's equity or cost of capital. (3) Futures investment opportunities are reflected in the current market price as assumption (7) in Hamada (1969).

Merton's model may be stated as follows :

$$E(R_i) = R_F + \beta_i[E(R_m) - R_F] + \lambda_i - \beta_i\lambda_m \tag{1}$$

Where: $E(R_i)$: the equilibrium expected return on security i, $E(R_m)$: the equilibrium expected return on the market portfolio, R_F : the riskless rate of interest, $\beta_i = \frac{cov(R_i, R_m)}{var(R_m)}$: the beta of security i, λ_i : the equilibrium aggregate "shadow cost" for the security i. It is of the same dimension as the expected rate of return on this security S, and λ_m : the weighted average shadow cost of incomplete information over all securities.

For the case of notation, Merton's (1987) model is written as:

$$E(R_i) = (R_F + \lambda_i) + \beta_i[E(R_m) - R_F - \lambda_m]$$

with $\beta_i = \frac{cov(R_i, R_m)}{var(R_m)}$ or as:

$$E(R_i) = (R_F + \lambda_i) + \frac{cov(R_i, R_m)}{var(R_m)}[E(R_m) - R_F - \lambda_m]$$

This can be written as

$$E(R_i) = (R_F + \lambda_i) + \gamma \operatorname{cov}(R_i, R_m) \quad (1a)$$

where $\gamma = \frac{[E(R_m) - R_F - \lambda_m]}{\operatorname{var}(R_m)}$ = is the same for all assets. It is the price of a dollar of risk and can be seen as a measure of market risk aversion. This equation shows the market relationship between the required rate of return and the risk of any security i.

III. THE FINANCING DECISION WHEN THERE ARE NO CORPORATE TAXES

This section studies the effects of capital structure changes on the value of equity and perceived risk in the context of MM's Proposition I.

A. The Effect of Leverage on Equity

Following Hamada (1969), we use the following notations:

S^*_A: the present equilibrium market value of the equity of a debt-free firm A,

S_A : the present market value of the equity of a debt-free firm A, with the hypothesis of Merton (1987) that $S_A = \frac{R_F S^*_A}{R_F + \lambda_A}$

$E(S_{AT})$: the expected market value of firm A one period later,

E(div): the expected dividends paid over this period, and

$E(X_A)$: the expected earnings net of depreciation but before interest and taxes.

Using assumptions (b) and (c), the dollar return can be written as

$$E(X_A) = E(div) + E(S_{AT}) - S_A \quad (2)$$

The definition of an expected rate of return allows to write the rate of return required by shareholders of corporation A as

$$E(R_A) = \frac{E(div) + E(S_{AT}) - S_A}{S_A} = \frac{E(X_A)}{S_A} \quad (3)$$

Let us denote by B the equity of firm A after the issuance of debt. Firm A issues debt and uses the proceeds to buy as much of its equity as it can. When investors acquire securities of firm A, they suffer also information costs for debt. Therefore, even if we assume that debt is issued at a risk-free rate, there is a shadow cost for this asset. Using (2) and (3), the rate of return required by the remaining shareholders becomes:

$$E(R_B) = \frac{E(X_A) - R_F D_B}{S_B} \quad (4)$$

Note that interest payments $R_F D_B$ is certain since the debt is also risk free. However, a fraction λ_B is applied to debt to reflect the shadow cost of incomplete information about the firm's debt. In fact, bond holders do not accept to lend money and buy bonds if they do not know about the firm. In the absence of infinite uncertainty λ_B=0 and the debt comes the risk free rate.

Using (1a), the equilibrium required rates of return from (3) and (4) are

$$(R_F + \lambda_A) + \gamma \operatorname{cov}(R_A, R_m) = \frac{E(X_A)}{S_A} \quad (3a)$$

and

$$(R_F + \lambda_B) + \gamma \operatorname{cov}(R_B, R_m) = \frac{E(X_A) - R_F D_B}{S_B} \quad (4a)$$

In our analysis, it is possible to assume that $\lambda_A = \lambda_B$. This means that the shadow cost of incomplete information about the firm and its assets is the same before and after debt issuance. We can also assume different shadow costs. Hence, if for example $\lambda_B > \lambda_A$, this means that we have less information about the firm after debt issuance and investors suffer a higher shadow cost to get informed about the firm after the issuance of debt. Since equity B is riskier than A, then $\operatorname{cov}(R_B, R_m) > \operatorname{cov}(R_A, R_m)$.

Using (3a) and (4a) to isolate $E(X_A)$, we have:

$$\begin{aligned} & S_A[(R_F + \lambda_A) + \gamma \operatorname{cov}(R_A, R_m)] \\ & = S_B\left[\gamma \operatorname{cov}(R_B, R_m) + R_F \frac{D_B}{S_B} + (R_F + \lambda_B)\right] \end{aligned} \quad (5)$$

Since

$$\operatorname{cov}(R_A, R_m) = E\left(\frac{X_A}{S_A}\right) - E\left(\frac{X_A}{S_A}\right)[R_m - E(R_m) + \lambda_m] = \left[\frac{1}{S_A}\operatorname{cov}(X_A, R_m)\right] \quad (6)$$

In the same way, we have

$$\operatorname{cov}(R_B, R_m) = \frac{1}{S_A} \times \operatorname{cov}(X_A, R_m) \quad (7)$$

When (6) and (7) are substituted into (5), we have[2]

$$S_A\left[\frac{\gamma}{S_A}\operatorname{cov}(X_A, R_m) + \lambda_A + R_F\right] = S_B\left[\frac{\gamma}{S_B}\operatorname{cov}(X_A, R_m) + R_F\frac{D_B}{S_B} + (R_F + \lambda_B)\right]$$

This reduces to

$$S_A(R_F+\lambda_A)=\lambda_B S_B+(S_B+D_B)R_F$$

$$\frac{S_A(R_F+\lambda_A)-\lambda_B S_B}{R_F}=S_B+D_B \tag{8}$$

which is equivalent to

$$S_A^*=S_B^*+D_B \tag{8'}$$

Recall that by definition $V=S_B+D_B$. And

$$V=\frac{S_A(R_F+\lambda_A)}{R_F}=\frac{E(X_A)(R_F+\lambda_A)}{E(R_A)R_F}-\frac{\lambda_B(E(X_A)-D_B R_F)}{E(R_B)R_F}$$

$$\Leftrightarrow$$

$$V=S_A+\frac{\lambda_A S_A-\lambda_B S_B}{R_F} \tag{9}$$

The total value of the firm is a function of the expected earnings of the assets, their uncertainty, $cov(R_A, R_m)$, the market factors γ, the shadow costs of incomplete information, λ_A and λ_B and the riskless rate R_F. This is the theory of value in the presence of shadow costs of incomplete information without the use of the homogeneous risk-class assumption.

Hamada (1969) presents a switching mechanism to replace the MM arbitrage operation in the presence of complete information. Substituting (4) for $E(R_i)$ and (7a) for $cov(R_i, R_m)$ in (1a), the value of γ is given by

$$\gamma = \frac{E(X_A)-R_F D_B-n_B P_B(R_F+\lambda_B)}{\frac{1}{S_T}\sum_{k=1}^{T}cov(X_A,X_k)} \tag{10}$$

where the number of shares n_B times the price per share P_B gives the value of equity S_B. Equation (10) shows that the ratio of the expected return above the risk-free rate to the risk of any asset is a constant γ corresponding to the market price per unit of risk in equilibrium.

If P_B is above its equilibrium price, then the right hand side of (10) would fall below γ. Investors will sell B and buy another security giving γ. This switching drives down the price of B and restores the equality (10) in equilibrium. If P_B is below its equilibrium price, then the right hand side of (10) would rise above γ . Since the excess rate of return is higher, investors will bid for B, driving up P_B. This switching operation is substituted for the MM arbitrage operation.

B. Particular Case

If we have $\lambda_A = \lambda_B = \lambda$ equation (8) gives:

$$\frac{S_A R_F - \lambda(S_A - S_B)}{R_F} = S_B + D_B \tag{8a}$$

$$\Leftrightarrow$$

$$S_A = S_B + D_B + \frac{\lambda}{R_F}(S_B / S_A - 1) \tag{8a}$$

C. Leverage and the Expected Rate of Return

Equation (8) can be used in the study of the effect of leverage on the expected rate of return (MM's Proposition II). The expected return for A and B can be obtained by substituting (6) and (7) into (1a):

$$E(R_A) = (R_F + \lambda_A) + \frac{\gamma}{S_A} \mathrm{cov}(X_A, R_m) \tag{11}$$

$$E(R_B) = (R_F + \lambda_B) + \frac{\gamma}{S_B} \mathrm{cov}(X_A, R_m) \tag{11a}$$

where equity B corresponds to the same physical firm as A except that it has debt in its capital structure.

The difference between (11) and (11a) gives

$$E(R_B) - E(R_A) = \frac{\gamma}{S_B} \mathrm{cov}(X_A, R_m) - \frac{\gamma}{S_A} \mathrm{cov}(X_A, R_m) + \lambda_B - \lambda_A \tag{11'}$$

$$\Leftrightarrow$$

$$E(R_B) - E(R_A) = \frac{\gamma[S_A - S_B]}{S_B S_A} \mathrm{cov}(X_A, R_m) + \lambda_B - \lambda_A \tag{11'}$$

From (9): $$S_A - S_B = \frac{\lambda_B S_B - \lambda_A S_A + R_F D_B}{R_F}$$

Substituting that into (11'), we obtain:

$$E(R_B)-E(R_A)=(\lambda_B-\lambda_A)+\frac{\gamma}{S_B S_A}\operatorname{cov}(X_A,R_m)\frac{\lambda_B S_B-\lambda_A S_A+R_F D_B}{R_F} \qquad (12)$$

From (11), we have :

$$\gamma \operatorname{cov}(X_A,R_m)=S_A\left[E(R_A)-R_F-\lambda_A\right] \qquad (11b)$$

When this last equation is substituted in (12), this gives a generalization of the MM's Proposition II :

$$E(R_B)=(\lambda_B-\lambda_A)+E(R_A)+\frac{\left[E(R_A)-R_F-\lambda_A\right]}{S_B}\times\frac{\lambda_B S_B-\lambda_A S_A+R_F D_B}{R_F} \qquad (13)$$

$$\Leftrightarrow$$

$$E(R_B)=(\lambda_A-\lambda_B)+E(R_A)\left\{1+\frac{\lambda_B S_B-\lambda_A S_A+R_{F\partial}D_B}{R_F S_B}\right\}-(R_F+\lambda_A)\left[\frac{\lambda_B S_B-\lambda_A S_A+R_F D_B}{R_F S_B}\right] \qquad (13)$$

This expression shows that the rate of return required by investors increases linearly with the debt-equity ratio. It is also a function of the information cost.

D. The Financing Decision with Corporate Taxes

The rate of return, R, must be defined on an after corporate income tax basis. The firm's financing decision can be studied by modifying equations (2), (3a) and (4a) to account for the tax rate τ as follows :

$$E[X_A(1-\tau)]=E(div)+E(S_{AT})-S_A \qquad (2a)$$

$$E(R_A)=\frac{E\left[X_A(1-\tau)\right]}{S_A}=(R_F+\lambda_A)+\gamma\operatorname{cov}(R_A,R_m) \qquad (3b)$$

$$E(R_B)=\frac{E\left[(X_A-R_F D_B)(1-\tau)\right]}{S_B}=(R_F+\lambda_B)+\gamma\operatorname{cov}(R_B,R_m) \qquad (4b)$$

When (3b) and (4b) are used to isolate the tax-adjusted expected asset earnings, (1-τ) E(X_A), and equating the two relations gives:

$$S_A[(R_F+\lambda_A)+\gamma\operatorname{cov}(R_A,R_m)]=S_B\left[\gamma\operatorname{cov}(R_B,R_m)+R_F+\lambda_B+\frac{R_F D_B}{S_B}(1-\tau)\right] \qquad (14)$$

The two covariance terms can be written as

$$\operatorname{cov}(R_A, R_m) = \frac{(1-\tau)}{S_A}\operatorname{cov}(X_A, R_m) \tag{15}$$

and

$$\operatorname{cov}(R_B, R_m) = \frac{(1-\tau)}{S_B}\operatorname{cov}(X_A, R_m) \tag{16}$$

When (15) and (16) are substituted in (14), this gives

$$S_A(R_F + \lambda_A) = S_B(R_F + \lambda_B) + (1-\tau)D_B R_F \tag{17}$$

Or: Since the total market value of a firm $V = S_B + D_B$, then using (17) gives

$$V = \frac{S_A(R_F + \lambda_A) - \lambda_B S_B}{R_F} + \tau D_B \tag{18}$$

Using the first half of (3b) for S_A, (18) can be written as

$$V = \frac{(1-\tau)E(X_A)}{R_F}\left[\frac{(R_F + \lambda_A)}{E(R_A)}\right] - \frac{\lambda_B S_B}{R_F} + \tau D_B \tag{19}$$

Using the first half of (4b) for S_B, (19) can be written as

$$V = \frac{(1-\tau)E(X_A)}{R_F}\left[\frac{(R_F + \lambda_A)}{E(R_A)} - \frac{\lambda_B}{E(R_B)}\right] + \frac{\lambda_B D_B (1-\tau)}{E(R_B)} + \tau Db \tag{19'}$$

This is the MM's result in a market equilibrium setting.

E. Investment Analysis and the Cost of Capital Assuming No Corporate Taxes

It is assumed that managers maximize their expected utility of terminal wealth. The change in equity value as a result of project selection must be at least larger than any equity required to finance the project. Let us denote dI as the purchase cost of the incremental investment, and dE.F as the new equity required to finance this investment.

In this context, a project dI is acceptable when the following capital budgeting criterion is satisfied:[3]

$$\frac{dS}{dI} \geq \frac{dE.F}{dI} \tag{20}$$

1. Derivation of the cost of capital

Recall from section 3 that

$$S_A = \frac{E(X_A)}{R_F + \lambda_A} - \frac{S_A \gamma \operatorname{cov}(R_A, R_m)}{R_F + \lambda_A}$$

$$S_B = \frac{E(X_A) - R_F D_B}{R_F + \lambda_B} - \frac{S_B \gamma \operatorname{cov}(R_B, R_m)}{R_F + \lambda_B}, \text{ and}$$

$$V = \frac{Sa(R_F + \lambda_A) - \lambda_B S_B}{R_F}$$

In this context, it is possible to show that

$$V = \frac{E(X_A)}{R_F + \lambda_B} + \frac{\lambda_B D_B}{R_F + \lambda_B} - \frac{\left[E(X_T) - (R_F + \lambda_m) S_T\right] \sum_k \operatorname{cov}(X_A, X_k)}{(R_F + \lambda_B)\sigma^2(X_T)} \tag{21}$$

where X_T corresponds to the sum of all dollar earnings from risky assets combined.[4] Besides (21) is equivalent to

$$V = \frac{E(X_A)}{R_F + \lambda_B} - \frac{\frac{\gamma}{S_T} \sum_k \operatorname{cov}(X_A, X_k)}{R_F + \lambda_B} + \frac{\lambda_B D_B}{R_F + \lambda_B} \tag{21a}$$

If we define the market value of all equity except A as $S_T' = \sum_{k \neq A}^{t} S_k$, then $S_T = S'_T + S_A$. When this is substitued in (21),this gives

$$S_B = \frac{(R_F + \lambda_A)}{R_F} \left\{ \frac{E(X_A)\sigma^2(X_T) - \left[E(X_T) - (R_F + \lambda_m)(S_T' - D)\right] \sum_k \operatorname{cov}(X_A, X_k)}{(R_F + \lambda_A)\sigma^2(X_T) - (R_F + \lambda_m) \sum_k \operatorname{cov}(X_A, X_k)} \right\} - D - \frac{\lambda bSb}{Rf}$$

When the capital budgeting criterion (20) to (21 b) is used, noting that $\frac{dD}{dI} + \frac{dE.F.}{dI} = 1$ and solving for the dollar return gives marginal internal rate of return

$$\frac{dE(X_A)}{dI} \geq (R_F + \lambda_B) + \frac{\gamma}{S_T} \left[\frac{d\sum_k \operatorname{cov}(X_A, X_k)}{dI} - Z \frac{d\sigma^2(X_T)}{dI} \right] + Z \left[\frac{dE(X_T)}{dI} - (R_F + \lambda_m)(1 + \frac{dS'_T}{dI} - \frac{dD}{dI}) \right] \tag{22}$$

where Z is given by $\frac{\sum_k \text{cov}(X_A, X_k)}{\sigma^2(X_T)} = \frac{V}{S_T}\left[\frac{E(R_A) - R_F - \lambda_A}{E(R_M) - R_F - \lambda_m}\right]$.

Now, consider the effect of the incremental investment on the expected value and variance of X_T and solve for the dollar return on the investment $\frac{dE(X_A)}{dI}$. Since investors maximize the expected utility, the firm must make capital budgeting decisions that guarantees that $\frac{dS}{dI} \geq \frac{dE.F.}{dI}$. This leads to the criterion that the expected marginal internal rate of return of a project is higher than the cut-off rate for the marginal investment, or the cost of capital

$$CC = \frac{(R_F + \lambda_F)}{(1-Z)} + \frac{\gamma}{S_T}\left[\frac{d\sum_k \text{cov}(X_A, X_k)}{dI} - \frac{Z}{(1-Z)}\left[\frac{d\sigma^2(X'_T)}{dI} + \frac{d\,\text{cov}(X'_T, X_A)}{dI}\right]\right]$$
$$+ \frac{Z}{(1-Z)}\left[\frac{dE(X'_T)}{dI} - (R_F + \lambda_m)(1 + \frac{dS'_T}{dI} - \frac{dD}{dI})\right] \quad (23)$$

Neglecting some small terms, the cost of capital is given by

$$CC = (R_F + \lambda_B) + \frac{\gamma}{S_T}\left[\frac{d\sum_k \text{cov}(X_A, X_k)}{dI}\right] \quad (24)$$

Since $\frac{dD}{dI}$ is bounded by one and zero, the term $\frac{Z}{(1-Z)}(R_F + \lambda_m)\frac{dD}{dI}$ can be neglected. The others are assumed to be zero.

2. Interpretation of the cost of capital

The comparison of (24) to (21 a) suggests an interpretation of the cost of capital. When the investment does not increase the adjustment term $\frac{\gamma}{S_T}\sum_k \text{cov}(X_A, X_k)$, then the expected marginal rate of return of this investment must be higher than the riskless rate and the term $\frac{\gamma}{S_T}\left[\frac{d\sum_k \text{cov}(X_A, X_k)}{dI}\right]$ must be zero. The cost of capital corresponds to the riskless rate plus a premium for the risk of a given project. Since

$$\frac{\gamma}{S'_T}\left[\frac{V_A\left[E(R_A) - R_F - \lambda_A\right]}{\sum_k \text{cov}(X_A, X_k)}\right] \quad (25)$$

the risk premium in (24) is

$$\left[E(R_A) - R_F - \lambda_A\right] = \left[\frac{\left[\frac{d\sum_k \mathrm{cov}(X_A, X_k)}{dI}\right]}{\left[\frac{\sum_k \mathrm{cov}(X_A, X_k)}{V_A}\right]}\right] \tag{25a}$$

where $[E(R_A) - R_F - \lambda_A]$ is viewed as the risk premium prior to the acceptance of the project.

The risk premium for a given project corresponds to the product of this premium by the fractional change in the risk per dollar of the capital invested. The extension of the analysis in MM suggests the use of the following cost of capital

$$E(R_A) = R_F + \lambda_A + \gamma\,\mathrm{cov}(R_A, R_m)$$

where the term λ_A corresponds to the information cost paid to get informed about that firm. Using $E(R_A)$ as the cost of capital in the generalization of the MM analysis is appropriate for any investment if the relationship

$$V_A = \frac{E(X_A)}{E(R_A)} = \frac{E(X_A)}{R_F + \lambda_A + (\frac{\gamma}{S_T})(\frac{1}{V_A})\sum_k \mathrm{cov}(X_A, X_k)} \tag{26}$$

is preserved after accepting the investment.

When the terms R_F, γ, λ and St are not affected by capital budgeting decisions in firm A, the type of investment that preserves relation (26) is restricted to one with :[5]

$$\frac{d\sum_k \mathrm{cov}(X_A, X_k)}{dI} = \frac{\sum_k \mathrm{cov}(X_A, X_k)}{V} \tag{27}$$

The right hand side of (27) gives a measure of the existing risk per dollar invested. To show that the cost of capital in a market equilibrium approach will be the same as the MM's result for a non−diversifying project, we substitute (27) in (24). This gives

$$CC = R_F + \lambda_A + \frac{\gamma}{S_T V}\sum_k \mathrm{cov}(X_A, X_k)$$

Or

$$CC = R_F + \lambda_A + \gamma\,\mathrm{cov}(X_A, X_k)$$

F. The Effect of Corporate Taxes on the Cost of Capital

1. Derivation of the cost of capital

The valuation formula is

$$S=\frac{(1-\tau)E(X_A)}{R_F+\lambda_B}-\frac{\frac{\gamma}{S_T}\sum_k \operatorname{cov}(X_A,X_k)}{R_F+\lambda_B}+\frac{(1-\tau)\lambda_B D_B}{R_F+\lambda_B}-(1-\tau)D \qquad (28)$$

where X_τ. denotes the after-tax earnings, $(1-\tau)X$. When the capital budgeting criterion, $\frac{dS}{dI}\geq\frac{dE.F.}{dI}$ is applied to (28) and we use the fact that $\frac{dD}{dI}+\frac{dE.F.}{dI}=1$, this gives

$$(1-\tau)\frac{dE(X_A)}{dI}\geq(R_F+\lambda_B)(1-\tau\frac{dD}{dI})+\frac{\gamma}{S_T}\left[\frac{d\sum_k \operatorname{cov}(X_{\tau,A},X_{\tau,k})}{dI}\right] \qquad (29)$$

The stockholders wealth is maximized when the after-tax expected marginal internal rate of return of an investment (left-hand side of (29) is at least equal to the right-hand side. Hence, the after-tax cost of capital is

$$(R_F+\lambda_B)(1-\tau\frac{dD}{dI})+\frac{\gamma}{S_T}\left[\frac{d\sum_k \operatorname{cov}(X_{\tau,A},X_{\tau,k})}{dI}\right] \qquad (30)$$

Since for a riskless project, $\frac{d\sum_k \operatorname{cov}(X_{\tau,A},X_{\tau,k})}{dI}=0$, the tax subsidy is the product of the dollar interest cost, $R_F=\frac{dD}{dI}$, then the cost of capital for a riskless project is $R_F-\tau R_F\frac{dD}{dI}$.

MM apply the fact that $\frac{dV}{dI}\geq 1$ to a similar equation as our equation (19) to obtain :

$$CC_{MM}=E(R_A)(1-\tau\frac{dD}{dI})$$

In the context of incomplete information, this can be written as:

$$CC_{MM} = (R_F + \lambda_A)(1 - \tau\frac{dD}{dI}) + \frac{\gamma(1 - \tau\frac{dD}{dI})}{S_T S_A}\sum_k cov(X_{\tau,A}, X_{\tau,k}) \tag{31}$$

An investment preserving the linear homogeneity in (19) so that (31) represents the cost of capital must satisfy the following condition

$$\frac{d\sum_k cov(X_{\tau,A}, X_{\tau,k})}{dS_A} = \frac{d\sum_k cov(X_{\tau,A}, X_{\tau,k})}{S_A} \tag{32}$$

The right-hand side of this equation defines the risk per dollar invested in the firm. New investments must have also this ratio for MM's cost of capital to be applied. The MM cost of capital is a special case of our cost of capital. In fact, the relationship between the cost of the investment dI and the effective capital required dS_A allows to rewrite (32) as[6]

$$\frac{d\sum_k cov(X_{\tau,A}, X_{\tau,k})}{DI} = \frac{(1 - \tau\frac{dD}{dI})}{S_A}\sum_k cov(X_{\tau,A}, X_{\tau,k}) \tag{32a}$$

The MM cost of capital is obtained when (32a) is replaced in (30).

2. Suggestions for estimating the cost of capital

The long-run target debt ratio, L^*, corresponds to a mix for all the investments regardless of how any individual project is financed. Hence, $\frac{dD}{dI} = L^*$ and the cost of capital in (30) can be written as :

$$CC = (R_F + \lambda_B)(1 - \tau L^*) + \frac{\gamma}{S_T}\left[\frac{d\sum_k cov(X_{\tau,A}, X_{\tau,k})}{dI}\right] \tag{30a}$$

For major investments, the risk premium is given by

$$\begin{aligned}\frac{\gamma}{S_T}\left[\frac{d\sum_k cov(X_{\tau,A}, X_{\tau,k})}{dI}\right] &= \frac{\gamma}{S_T}\left[\frac{d\,cov(X_{\tau,A}, X_{\tau,k})}{dI}\right] \\ &= \frac{\gamma}{S_T}\left[cov(X_{\tau,A0} + X_{\tau,A1}, X_{\tau,T} + X_{\tau,A1}) - cov(X_{\tau,A0}, X_{\tau,T})\right] \\ &= \gamma\,cov(X_{\tau,A1}, R_m) + \frac{\gamma}{S_T}\left[cov(X_{\tau,A0}, X_{\tau,A1}) + \sigma^2(X_{\tau,A1})\right]\end{aligned} \tag{30b}$$

where $X_{\tau,A0}$, $X_{\tau,A1}$ and $X_{\tau,T}$ correspond to tax-adjusted earning from firm A's assets, from the new investments and from all capital assets in the market.

Merton's (1987) model can be used to estimate the project's major risk component $\gamma cov(X_{\tau,A1},R_m)$ using a good "proxy" for R_m and the systematic risk in $X_{\tau,A1}$ can be explained by a linear relationship

$$X_{\tau,A1} = a + b(R_m + \lambda_m) + \xi$$

where a and b are parameters and $E(\xi) = cov(R_m, \xi)$. When (33) is applied to the definition of the covariance, we have

$$\gamma\, cov(X_{\tau,A1}, R_m) = \gamma E\left[b(R_m + \lambda_m) + \xi - bE(R_m)\right](R_m + \lambda_m)$$

$$\gamma b\sigma^2(R_m) = b[E(R_m) - R_F - \lambda_m]$$

so that b and $E(R_m)$ are all that must be estimated.

IV. SUMMARY

The financing and investment decisions are considered in the framework of Merton (1987) simple model of capital market equilibrium with incomplete information. The famous Modigliani and Miller Propositions I and II hold in this context with and without taxes. Following the work of Hamada (1969), I use a switching operation in place of arbitrage. The analysis covers also the cost of capital and the minimum required rate of return for individual projects within the firm. MM's cost of capital appears as a special case (for non diversifying investments) of the context proposed here. As it appears in the work of Modigliani and Miller and Hamada, the propositions hold in the standard portfolio model under market equilibrium conditions and should be regarded as a tribute to the Modigliani and Miller partial equilibrium concept of the homogeneous risk-class. These results are all generalized to account for the effects of information uncertainty in the spirit of Merton (1987) and Bellalah and Zhen (2002) models of capital market equilibrium with incomplete information.

ENDNOTES

1. See the models in Bellalah and Jacquillat (1995) and Bellalah (1999).
2. The following definition used in Hamada (1969) can also be applied: $R_m = \sum_{k=1}^{T} \frac{S_k}{S_T} R_k = \frac{1}{S_T}\sum_{k=1}^{T} X_k$ where S_T stands for the market value of all assets, k outstanding. The term T indicates the number of risky assets. In this case,

$$Cov(X_A, R_m) = \frac{1}{S_T} \sum_{k=1}^{T} cov(X_A, X_k) \quad (6a)$$

When (6a) is substituted into (6) and (7), we have

$$Cov(R_A, R_m) = \frac{1}{S_A S_T} \sum_{k=1}^{T} cov(X_A, X_k) \quad (6b)$$

and

$$Cov(R_B, R_m) = \frac{1}{S_B S_T} \sum_{k=1}^{T} cov(X_A, X_k) \quad (7a)$$

3. The criterion in MM is $\frac{dV}{dI} \geq 1$. This is equivalent to the criterion in (20).
4. Since by definition, we have: $\gamma = \frac{[E(R_m) - R_F - \lambda_m]}{\sigma^2(R_m)}$, and substituting $R_m = \sum_{k=1}^{T} \frac{S_k}{S_T} \frac{X_k}{S_k} = \frac{X_T}{S_T}$. Then $\gamma = S_T \frac{[E(X_T) - (R_F + \lambda_m) S_T]}{\sigma^2(X_T)}$, and since,

   $cov(R_a, R_m) = \frac{1}{S_a S_t} \sum_{k=1}^{t} cov(X_a, X_k)$, then we have (21).
5. The investment preserving the linear homogeneity of (26),so that $E(R_A)$ is the correct cost of capital. It is obtained by differentiating the right hand side of (26) with respect to dI. Starting with equation (19), we can write :

   $$S = \frac{(1-\tau)E(X_A)}{R_F}\left[\frac{R_F + \lambda_A}{E(R_A)}\right] - \frac{\lambda_B S_B}{R_F} + \tau D - D = \frac{(1-\tau)E(X_A)}{R_F}\left[\frac{(R_F + \lambda_A)}{R_F + \lambda_A + \gamma\, cov(R_A, R_m)}\right] - \frac{\lambda_B S_B}{R_F} + \tau D - D$$

   where A refers to the firm with no debt and R's is expressed after-tax.

   Since $cov(R_A, R_m) = \frac{1}{S_A S_T} \sum_k cov(X_{\tau,A}, X_{\tau,k})$ and $\frac{(R_F + \lambda_A) S_A - \lambda_B S_B}{R_F} = V - \tau D$

   we get equation (28).
6. Since dI must be financed, then dI = dS + dD = dV. From (18), $dS_A = dV - \tau dD$ so that $dS_A = dI - \tau\, dD = dI(1 - \tau \frac{dD}{dI})$. When we replace this expression in (32), this gives (32 a).

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