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Value-at-risk and Extreme Returns in Asian Stock Markets

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ABSTRACT

The purpose of this paper is to use the extreme value theory to analyze ten Asian stock markets, identifying which type of extreme value asymptotic distribution better fits historical extreme market events. Understanding the influence of extreme market events is of great importance for risk managers. Our empirical tests indicate that the return distributions are not characterized by normality and that the minima and the maxima of the return series may be satisfactorily modeled within an extreme value framework. The average waiting time for an index to present a daily return below/above a specific threshold is generally larger for Asian major markets than for Asian emerging markets. We also compute VaR estimates using extreme value theory and compare the results with the empirical and normal VaR estimates. The results suggest that the extreme value method of estimating VaR is a more conservative approach to determining capital requirements than traditional methods.

JEL: *G15, G18*

Keywords: *Value-at-risk; Extreme values; Asia*

I. INTRODUCTION

Extreme Value Theory (EVT) has gained much attention lately (Reiss and Thomas (1997), Leadbetter et al (1993), Embrechts et al (1997)) and there have been a number of applications in the field of finance (Longin (1996), Longin (2000), Longin and Solnik (1998), Danielson and De Vries (1997), Danielson and De Vries (1998), Danielson et al (1998), Diebold et al (1999), Emmer et al (1998), McNeil (1998), McNeil and Frey (1998), Ferreira, Mendes and Duarte Jr (2000), Ho, Burridge, Cadle and Theobald (2000), among other studies).

In financial markets, extreme price movements may correspond to market correction during ordinary periods, to stock market crashes, to bond market collapses or to foreign exchange crises during extraordinary periods. Recently, emerging markets have experienced several extreme market events. Examples include the Mexican devaluation (end of 1994), the Brady bond crisis (beginning of 1995), the Asian series of devaluation (during 1997), the Russian crisis (end of 1998), among others. The recent turmoil that has occurred in Asian financial markets provides interesting exploratory opportunities to use the extreme value theory to analyze these markets.

The East Asian financial crises, which erupted in mid-1997, have been one of the most serious and challenging economic events of the 1990s. Although many factors contributed to the Asian currency crises, a few were common to all Asian countries that have experienced trouble: an inflexible exchange rate system, a weak banking system, and external over-borrowing were common to all countries that suffered large declines in their currency values and stock prices. According to the importance of East Asia in the world economy, its crisis had severe regional and world-wide implications, such as considerable depreciations of national currencies and a sharp drop in stock indexes.

Understanding the influence of extreme market events, such as the East Asian financial crises, is of great importance for risk managers (Ewing (1995), Longin (2000)). Since all risk measurement methodologies used to estimate the Value-at-Risk (VaR) of a portfolio assume that the market behavior is stable, extreme market events demand a special approach from risk managers. A more recent methodology to estimating VaR focuses on modeling the tail of the distribution based on extreme value theory (see Longin (2000), Danielson and De Vries (1997), Danielson et al (1998), Diebold et al (1999), McNeil (1998), McNeil and Frey (1998), Ho, Burridge, Cadle and Theobald (2000), Mendes (2000)).

The purpose of this work is to use the extreme value theory to analyze ten Asian financial markets (Hong Kong, India, Indonesia, Japan, Korea, Malaysia, Philippines, Singapore, Taiwan, and Thailand). This paper is organized as follows. The next section reviews the basic concepts of the extreme value theory. In section III, we present the data and methodology used in this paper. Section IV reports the results of the empirical analysis conducted on each Asian stock market. In Section V, a comparative analysis using the extreme value theory in Asian stock markets is summarized. Section VI reports the results of the VaR estimates computed in an extreme value perspective and compared to the empirical VaR measures. Conclusions are presented in Section VII.

II. EXTREME VALUE THEORY

The purpose of this section is to briefly discuss the statistical behavior of extremes. Extreme value theory gives some interesting results about the statistical distribution of extreme returns. In particular, the limiting distribution of extreme returns observed over a long time period is largely independent of the distribution of returns itself.

Let R_1, R_2,..., R_n be a sample of independent and identically distributed returns observed over n basic time-intervals. Extremes are defined as the minimum and the maximum of the n random variables $R_1,R_2,...,R_n$. Let Y_n denote the maximum observed over n trading intervals: $Y_n=Max(R_1,R_2,...,R_n)$ and Z_n the minimum observed over n trading intervals: $Z_n=Min(R_1,R_2,...,R_n)$. The remainder of this section presents theoretical results for the maximum only, since the results for the minimum can be directly deduced from those of the maximum by transforming the random variable R into -R. Assuming R_t is independent with common distribution function F_R, the exact distribution of the maximum Y_n is given by:

$$F_{Y_n}(r) = (F_R(r))^n \tag{1}$$

In practice, the distribution of returns is not precisely know and, therefore, neither is the exact distribution of maximal returns. From the equation above, it can be concluded that the limiting distribution of Y_n obtained by letting n tend to infinity is degenerate (see Embrechts et al (1997)).

In order to find a non-degenerate distribution, the maximum Y_n is reduced with a scale parameter σ_n and a location parameter μ_n such that the distribution of the standardized maxima $(Y_n-\mu_n)/\sigma_n$ is non-degenerate. According to the Fisher and Tippett (1928) Theorem, if a non-degenerate distribution F_Y exists, it must be of the type of one of the three standard extreme value distributions:

1. The Gumbel distribution, defined as

$$F_Y(y) = \exp\{-\exp\{-y\}\} \quad y \in \Re \tag{2}$$

2. The Fréchet (k) distribution, defined for k > 0 as

$$\begin{aligned} F_Y(y;k) &= \exp\{-y^{-k}\} && \text{if } y > 0 \\ &= 0 && \text{if } y < 0 \end{aligned} \tag{3}$$

3. The Weibull (k) distribution, defined for k < 0 as

$$\begin{aligned} F_Y(y;k) &= \exp\{-(-y)^{-k}\} && \text{if } y < 0 \\ &= 0 && \text{if } y > 0 \end{aligned} \tag{4}$$

The shape parameter k is related to the weight of the tail of the distribution F_Y. Jenkinson (1955) proposed a one-parameter representation for the three limit distributions. It is called the Generalized Extreme Value (GEV) distribution, given by:

$$F_Y(y;\tau) = \exp\{-(1-\tau y)^{1/\tau}\} \text{ if } \tau \neq 0 \\ = \exp\{-\exp(-y)\} \text{ if } \tau = 0 \quad (5)$$

where $1-\tau y > 0$. The tail index τ is such that $\tau = -1/k$. The Gumbel, the Fréchet and the Weilbull distributions are respectively related to $\tau = 0$, to $\tau < 0$, and to $\tau > 0$. The Gumbel distribution is reached for thin-tailed return distributions such as the normal or log-normal distributions. The Fréchet distribution is obtained for fat-tailed distributions of returns such as the t-Student and stable Paretian distributions. Finally, the Weilbull distribution is obtained when the distribution of returns has no tail.

The GEV distribution was used to analyze the tails of the distributions (minimum and maximum) of the Asian stock market indexes. The extremes were selected as the maximum and the miminum over four disjoint periods of size n: one month (n=21), two months (n=42), three months (n=63) and six months (n=126). We use the L-moments procedure (Greenwood et al (1979), Hosking (1990), Hosking and Wallis (1997)) to obtain the shape parameter τ, the scale parameter σ and the location parameter μ of the GEV distribution of each Asian stock market index.

The L-moments estimates are obtained by equating the sample L-moments to the corresponding population quantiles. They are convenient computationally efficient estimators, and for several distributions they yield closed form expressions. According to Hosking et al (1985) and Hosking and Wallis (1987), for small and moderate sample sizes, the L-moments estimators are more efficient than maximum likelihood. Their exact distribution is difficult to derive, but large-sample approximations can be obtained by asymptotic theory.

III. CHARACTERIZING ASIAN EMERGING MARKETS

Our sample consists of the largest capitalization markets in Asia, divided in 7 emerging markets (India, Indonesia, Korea, Malaysia, Philippines, Taiwan, and Thailand) and 3 major markets (Hong Kong, Japan, and Singapore). The Japanese market, the largest Asian market by the end of 1998, is 5 times smaller than the US market (see Table 1). The total market capitalization of Asian emerging markets was 35% of the emerging market capitalization, and the market capitalization of Asian major markets was 11% of the developed market capitalization according to the IFC (1999). The market capitalization of Asian stock markets (including emerging and developed markets) represented 13% of the world's total market capitalization.

The daily index levels for our sample were obtained from the first day in January 1990 through December 1999. The series of returns were collected from the Datastream database. Specifically, the data consists of the closing daily levels of the Hang Seng (Hong Kong), Bombay Sensitivity Index (India), Jakarta Stock Exchange Composite Index (Indonesia), Nikkei Average (Japan), Seoul Composite Index (Korea), Kuala Lumpur Composite Index (Malaysia), Manila Composite Index (Philippines), Singapore Straits Industrial (Singapore), Taipei Weighted Price Index (Taiwan), and Bangkok S.E.T. Index (Thailand).

Table 1
Asian markets profile as of December 1998

Market	Market Capitalization (in US$ Billion)
Emerging Markets	
India	105.2
Indonesia	22.1
Korea	114.6
Malaysia	98.6
Philippines	35.3
Taiwan	260.0
Thailand	34.9
Major Markets	
Hong Kong	343.4
Japan	2,495.8
Singapore	94.5
US	13,451.0

Source: Emerging Markets Factbook, International Finance Corporation, 1999.

We computed daily logarithmic returns in local currency for each Asian stock market index. Our data is summarized in Table 2. Asian emerging markets are much more volatile than developed markets, as measured by their standard deviations of returns. The statistics for the Asian stock indexes suggest a heavy tailed and slightly skewed to the right distribution, indicating departure from the normal distribution. This is confirmed by the Jarque-Bera test, which rejects the null hypothesis of normality at the 1% level.

IV. RESULT ANALYSIS FOR EACH ASIAN STOCK MARKET INDEX

A. Hong Kong

Table 3 shows the L-moments estimates of the GEV parameters (μ, σ, and τ) for samples of selected minima and maxima grouped monthly, bimonthly, quarterly and semi-annually in Hong Kong. As we can see from Table III, all tail index values (τ) have been estimated as a negative number, suggesting that the minima and maxima should follow a Fréchet distribution, which is consistent with the findings of other studies of financial time series return data (Danielson and De Vries (1997), Longin (1996, 2000), McNeil (1998), Ferreira, Mendes and Duarte Jr. (2000), Ho, Burridge, Cadle Theobald (2000)).

Table 2

Descriptive statistics for daily returns in local currency from January 1990 to December 1999 (%)

Market	Mean	Std Dev	Min	1% Quantile	1st Quartile	Median	3rd Quartile	99% Quartile	Max	Skewness	Kurtosis	Jarque-Bera
Panel A: Asian Emerging Markets												
India	0.07	1.88	-13.66	-4.86	-0.80	0.05	0.94	5.40	18.90	0.36	8.91	0.08*
Indonesia	0.02	1.58	-12.73	-4.62	-0.50	-0.01	0.52	5.49	13.13	0.61	12.36	0.14*
Korea	0.00	1.89	-11.60	-5.50	-0.88	-0.06	0.80	6.05	10.02	0.27	4.06	0.09*
Malaysia	0.01	1.78	-24.15	-4.55	-0.64	0.01	0.65	4.85	20.82	0.52	34.51	0.12*
Philippines	0.03	1.67	-9.74	-4.83	-0.76	0.01	0.80	5.03	9.72	0.06	4.09	0.07*
Taiwan	-0.01	2.17	-16.33	-6.50	-0.84	-0.02	0.89	6.16	20.67	0.31	10.89	0.11*
Thailand	-0.02	1.95	-10.03	-5.69	-0.94	-0.04	0.79	6.06	11.36	0.38	5.00	0.09*
Panel B: Asian Major Markets												
Hong Kong	0.07	1.69	-14.73	-4.92	-0.63	0.06	0.84	4.69	17.25	0.06	11.29	0.09*
Japan	-0.03	1.50	-7.49	-4.09	-0.77	-0.02	0.73	4.22	12.43	0.28	4.29	0.06*
Singapore	0.03	1.33	-9.67	-3.55	-0.55	0.01	0.61	3.85	14.87	0.48	12.73	0.09*

Notes: * Jarque-Bera test significant at the 1% level

Table 3
L-moments estimates and goodness-of-fit statistics for Hong Kong

Period of Time	μ	σ	τ	Sherman Statistic (p-value)	Hosking Statistic (p-value)
A. Minimal Returns					
Monthly	-1.92	1.07	-0.27	-2.37 (99.1%)	-3.91 (<0.1%)
Bimonthly	-2.50	1.35	-0.25	-1.75 (96.1%)	-2.56 (0.5%)
Quarterly	-2.85	1.48	-0.27	0.11 (45.4%)	-2.32 (1.0%)
Semi-annually	-4.10	2.18	-0.12	0.32 (37.5%)	-0.70 (24.1%)
B. Maximal Returns					
Monthly	2.18	0.98	-0.32	-1.17 (88.0%)	-4.61 (<0.1%)
Bimonthly	2.75	1.21	-0.30	0.55 (29.1%)	-3.05 (0.1%)
Quarterly	3.14	1.45	-0.29	0.28 (39.1%)	-2.45 (0.7%)
Semi-annually	3.79	1.55	-0.37	0.97 (16.7%)	-2.20 (1.4%)

The goodness-of-fit of the asymptotic behavior of the distribution of extremes is evaluated using the Sherman (1957) test, which compares the probability given by the asymptotic distribution to the observed frequency used as a proxy of the exact distribution. In the case of Hong Kong, for all data sets, the hypothesis that the data follow a GEV distribution cannot be rejected at the 1% level. These results support the use of the GEV distribution when analyzing extreme market events in Hong Kong.

According to Hosking et al (1985), we computed the Hosking test statistic in order to test if the tail index values (τ) are significantly different from zero. Significant positive values of the Hosking statistic imply rejection of the null hypothesis that the minima and maxima follow a Gumbel distribution (τ=0) in favor of a Weibull distribution. On the other hand, significant negative values of the Hosking statistic imply rejection of the null hypothesis in favor of a Fréchet distribution.

The results, except for the semi-annually data, suggest that the minima and the maxima follow a Fréchet distribution at the 1% significance level. Figure 1 displays the adequacy of the Fréchet distribution for the monthly minima and maxima. The graphical evidence supports the goodness-of-fit results previously obtained. The final conclusion of the analysis of the Hang Seng Index daily returns is that the minima and the maxima follow a Fréchet distribution.

Figure 1
Histogram with the estimated GEV density for the Hang Seng Index (Hong Kong)

Panel A – Minima Monthly Returns Panel B – Maxima Monthly Returns

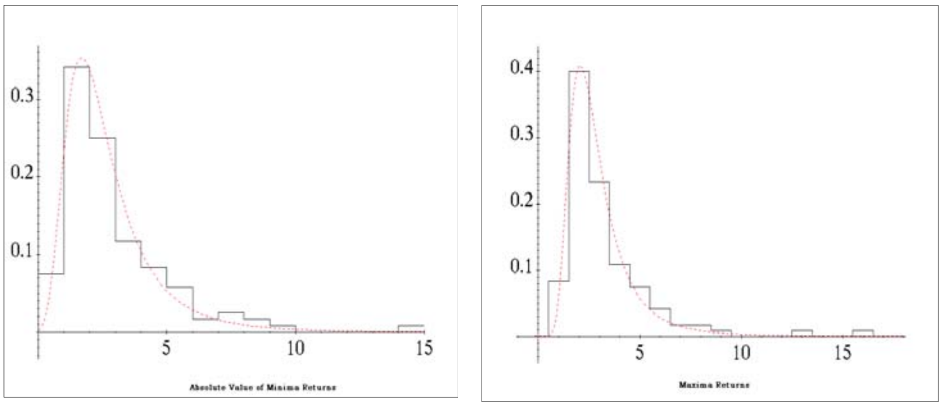


Table 4
L-moments estimates and goodness-of-fit statistics for India

μ	σ	τ	Sherman Statistic (p-value)	Hosking Statistic (p-value)
A. Minimal Returns				
-2.39	1.14	-0.25	0.96 (16.8%)	-3.66 (<0.1%)
-3.19	1.37	-0.24	1.74 (4.1%)	-2.44 (0.7%)
-3.50	1.64	-0.18	0.37 (35.7%)	-1.50 (6.7%)
-4.57	2.28	-0.06	-1.98 (97.6%)	-0.36 (35.8%)
B. Maximal Returns				
2.75	1.28	-0.21	1.34 (9.1%)	-2.99 (0.1%)
3.42	1.43	-0.24	0.86 (19.4%)	-2.48 (0.7%)
3.81	1.48	-0.28	0.37 (35.7%)	-2.38 (0.9%)
4.48	1.84	-0.28	-0.29 (61.5%)	-1.65 (4.9%)

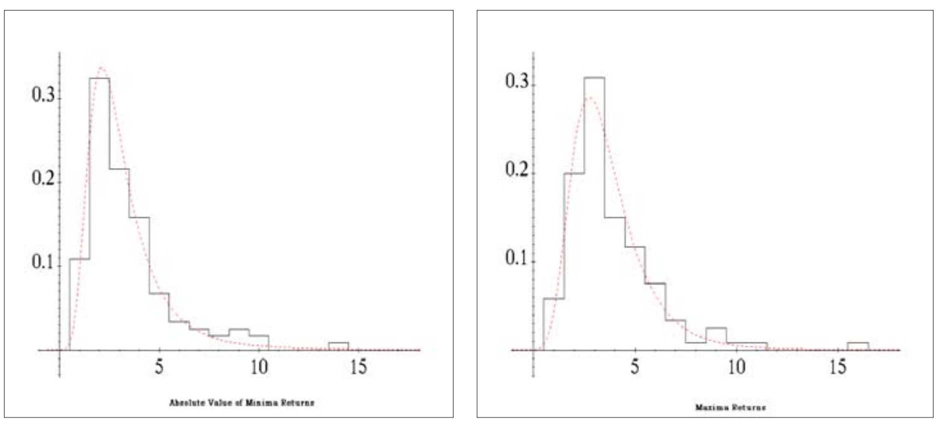
B. India

Table 4 shows the L-moments estimates of the GEV parameters for the minima and the maxima in India. The Sherman statistic indicates that for all data sets the hypothesis that data follow a GEV distribution cannot be rejected at the 1% level. The Hosking statistic show statistical evidence that the monthly and bimonthly minimum returns follow a Fréchet distribution, the quarterly and semi-annually minimum returns follow a Gumbel distribution, and maximal returns, except for the semi-annually data, follow a Fréchet distribution.

Figure 2 displays the adequacy of the Fréchet distribution for the monthly minima and maxima. The graphical evidence supports the goodness-of-fit results previously obtained.

Figure 2
Histogram with the estimated GEV density for the Bombay Sensitivity Index (India**)**

Panel A – Minima Monthly Returns Panel B – Maxima Monthly Returns



C. Indonesia

Table 5 shows the L-moments estimates of the GEV parameters for the minima and the maxima in Indonesia. The Sherman statistic indicates that for all data sets the hypothesis that data follow a GEV distribution cannot be rejected at the 1% level. The Hosking statistic show statistical evidence that the monthly and bimonthly minimum and maximum returns follow a Fréchet distribution, and the quarterly and semi-annually minimum and maximum returns follow a Gumbel distribution. The graphical evidence (for the sake of brevity the results are not reported) supports the goodness-of-fit results previously obtained.

Table 5
L-moments estimates and goodness-of-fit statistics for Indonesia

μ	σ	τ	Sherman Statistic (p-value)	Hosking Statistic (p-value)
A. Minimal Returns				
-1.37	1.08	-0.30	-0.87 (80.7%)	-4.38 (<0.1%)
-1.95	1.45	-0.23	-1.83 (96.7%)	-2.34 (0.9%)
-2.38	1.62	-0.20	-1.62 (94.7%)	-1.71 (4.4%)
-3.06	1.93	-0.16	-1.36 (91.3%)	-0.93 (17.6%)
B. Maximal Returns				
1.53	1.20	-0.35	0.72 (23.5%)	-5.05 (<0.1%)
2.08	1.51	-0.30	1.00 (15.7%)	-3.11 (0.1%)
2.65	1.99	-0.24	1.20 (11.6%)	-1.99 (2.3%)
3.47	2.70	-0.13	0.34 (36.6%)	-0.75 (22.6%)

D. Japan

Table 6 shows the L-moments estimates of the GEV parameters for the minima and the maxima in Japan. The Sherman statistic indicates that for all data sets, the hypothesis that data follow a GEV distribution cannot be rejected at the 1% level. The Hosking statistic show statistical evidence that the minimum and maximum returns, except the monthly maxima, follow a Gumbel distribution.

E. Korea

Table 7 shows the L-moments estimates of the GEV parameters for the minima and the maxima in Korea. The Sherman statistic indicates that for all data sets the hypothesis that data follow a GEV distribution cannot be rejected at the 1% level. The Hosking statistic show statistical evidence that the minima and maxima returns follow a Gumbel distribution.

F. Malaysia

Table 8 shows the L-moments estimates of the GEV parameters for the minima and the maxima in Malaysia. The Sherman statistic indicates that for all data sets the hypothesis that data follow a GEV distribution cannot be rejected at the 1% level. The Hosking statistic show statistical evidence that the minima and maxima returns follow a Fréchet distribution.

Table 6
L-moments estimates and goodness-of-fit statistics for Japan

μ	σ	τ	Sherman Statistic (p-value)	Hosking Statistic (p-value)
A. Minimal Returns				
-2.11	1.10	-0.03	-0.69 (75.4%)	-0.44 (33.1%)
-2.69	1.25	0.03	-3.03 (99.9%)	0.31 (62.2%)
-3.16	1.36	0.10	-2.91 (99.8%)	0.84 (80.0%)
-3.94	1.47	0.19	-1.36 (91.3%)	1.13 (87.1%)
B. Maximal Returns				
2.15	1.07	-0.18	-1.45 (92.6%)	-2.63 (0.4%)
2.85	1.33	-0.12	0.83 (20.4%)	-1.24 (10.8%)
3.55	1.37	-0.08	0.59 (27.7%)	-0.67 (25.0%)
4.40	1.70	-0.02	0.62 (26.6%)	-0.12 (45.3%)

Table 7
L-moments estimates and goodness-of-fit statistics for Korea

μ	σ	τ	Sherman Statistic (p-value)	Hosking Statistic (p-value)
A. Minimal Returns				
-2.25	1.25	-0.16	-0.80 (78.9%)	-2.27 (1.1%)
-2.80	1.54	-0.09	-0.83 (79.6%)	-0.93 (17.6%)
-3.27	1.55	-0.09	-1.50 (93.3%)	-0.76 (22.4%)
-4.22	1.78	-0.06	-1.14 (87.3%)	-0.36 (36.0%)
B. Maximal Returns				
2.58	1.32	-0.14	-0.34 (63.2%)	-2.04 (2.1%)
3.15	1.48	-0.14	0.40 (34.4%)	-1.44 (7.4%)
3.53	1.50	-0.17	1.10 (13.6%)	-1.43 (7.6%)
4.18	1.61	-0.14	-1.68 (95.3%)	-0.83 (20.2%)

Table 8
L-moments estimates and goodness-of-fit statistics for Malaysia

Period of Time	μ	σ	τ	Sherman Statistic (p-value)	Hosking Statistic (p-value)
A. Minimal Returns					
Monthly	-1.59	1.01	-0.36	1.41 (7.9%)	-5.18 (<0.1%)
Bimonthly	-2.06	1.19	-0.39	-1.63 (94.9%)	-4.06 (<0.1%)
Quarterly	-2.34	1.18	-0.45	-0.54 (70.6%)	-3.82 (<0.1%)
Semi-annually	-3.13	1.47	-0.51	-1.16 (87.7%)	-3.02 (0.1%)
B. Maximal Returns					
Monthly	1.88	1.15	-0.36	1.95 (2.5%)	-5.18 (<0.1%)
Bimonthly	2.45	1.31	-0.41	-0.55 (70.8%)	-4.25 (<0.1%)
Quarterly	2.77	1.59	-0.41	-0.30 (61.8%)	-3.49 (<0.1%)
Semi-annually	3.29	1.91	-0.47	-1.09 (86.3%)	-2.81 (0.3%)

Table 9
L-moments estimates and goodness-of-fit statistics for Philippines

Period of Time	μ	σ	τ	Sherman Statistic (p-value)	Hosking Statistic (p-value)
A. Minimal Returns					
Monthly	-2.01	1.24	-0.14	0.36 (36.1%)	-2.03 (2.1%)
Bimonthly	-2.75	1.47	-0.10	-0.01 (50.2%)	-1.04 (14.9%)
Quarterly	-3.14	1.79	-0.02	-0.88 (81.1%)	-0.13 (44.6%)
Semi-annually	-4.28	1.99	0.08	-1.34 (91.0%)	0.48 (68.5%)
B. Maximal Returns					
Monthly	2.29	1.23	-0.12	-0.40 (65.4%)	-1.81 (3.5%)
Bimonthly	2.98	1.36	-0.10	-0.76 (77.6%)	-0.99 (16.1%)
Quarterly	3.49	1.50	-0.08	-1.27 (89.7%)	-0.67 (25.3%)
Semi-annually	4.33	1.86	0.02	-0.19 (57.4%)	0.12 (54.7%)

G. Philippines

Table 9 shows the L-moments estimates of the GEV parameters for the minima and the maxima in Philippines. The Sherman statistic indicates that for all data sets the hypothesis that data follow a GEV distribution cannot be rejected at the 1% level. The Hosking statistic show statistical evidence that the minima and maxima returns follow a Gumbel distribution.

H. Singapore

Table 10 shows the L-moments estimates of the GEV parameters for the minima and the maxima in Singapore. The Sherman statistic indicates that for all data sets the hypothesis that data follow a GEV distribution cannot be rejected at the 1% level. The Hosking statistic show statistical evidence that the minima (except the semi-annually data) and the maxima follow a Fréchet distribution.

I. Taiwan

Table 11 shows the L-moments estimates of the GEV parameters for the minima and the maxima in Taiwan. The Sherman statistic indicates that for all data sets the hypothesis that data follow a GEV distribution cannot be rejected at the 1% level. The Hosking statistic show statistical evidence that the minima follow a Gumbel distribution, and the maxima (except the semi-annually data) follow a Fréchet distribution.

J. Thailand

Table 12 shows the L-moments estimates of the GEV parameters for the minima and the maxima in Thailand. The Sherman statistic indicates that for all data sets, except the monthly maximal returns, the hypothesis that data follow a GEV distribution cannot be rejected at the 1% level. The Hosking statistic show statistical evidence that the minimum and maximum returns (except the monthly maxima) follow a Gumbel distribution.

V. COMPARATIVE ANALYSIS OF EXTREME MARKET EVENTS IN ASIAN STOCK MARKETS

In order to compare the results obtained for the ten Asian stock markets, we considered two questions: (a) what is the expected waiting time to observe a daily return below/above a given threshold value? (b) for a fixed period of time, what is the probability of observing at least one daily return below/above a specific threshold?

Table 10
L-moments estimates and goodness-of-fit statistics for Singapore

μ	σ	τ	Sherman Statistic (p-value)	Hosking Statistic (p-value)
A. Minimal Returns				
-1.44	0.77	-0.30	0.99 (16.1%)	-4.33 (<0.1%)
-1.89	0.96	-0.29	0.84 (20.1%)	-2.99 (0.1%)
-2.27	1.13	-0.27	-1.14 (87.2%)	-2.31 (1.0%)
-2.97	1.73	-0.12	-0.93 (82.3%)	-0.70 (24.1%)
B. Maximal Returns				
1.61	0.79	-0.36	-1.23 (89.0%)	-5.18 (<0.1%)
2.11	0.95	-0.33	-0.95 (83.0%)	-3.42 (<0.1%)
2.40	1.06	-0.35	-0.81 (79.0%)	-2.93 (0.2%)
2.86	1.32	-0.39	-2.23 (98.7%)	-2.32 (1.0%)

Table 11
L-moments estimates and goodness-of-fit statistics for Taiwan

Period of Time	μ	σ	τ	Sherman Statistic (p-value)	Hosking Statistic (p-value)
A. Minimal Returns					
Monthly	-2.72	1.52	-0.12	0.44 (33.1%)	-1.69 (4.5%)
Bimonthly	-3.55	1.65	-0.11	-0.93 (82.3%)	-1.12 (13.0%)
Quarterly	-4.00	1.64	-0.15	0.46 (32.4%)	-1.28 (10.0%)
Semi-annually	-4.84	1.53	-0.28	-0.50 (69.2%)	-1.64 (5.0%)
B. Maximal Returns					
Monthly	2.74	1.18	-0.28	-1.28 (89.9%)	-4.15 (<0.1%)
Bimonthly	3.38	1.23	-0.35	-0.95 (82.8%)	-3.64 (<0.1%)
Quarterly	3.70	1.39	-0.36	-1.52 (93.6%)	-3.00 (0.1%)
Semi-annually	4.78	2.00	-0.30	-1.53 (93.7%)	-1.78 (3.8%)

Table 12
L-moments estimates and goodness-of-fit statistics for Thailand

Period of Time	μ	σ	τ	Sherman Statistic (p-value)	Hosking Statistic (p-value)
A. Minimal Returns					
Monthly	-2.43	1.40	-0.12	0.51 (30.7%)	-1.68 (4.7%)
Bimonthly	-3.17	1.65	-0.08	-0.11 (54.2%)	-0.83 (20.5%)
Quarterly	-3.77	1.84	-0.01	-0.18 (57.0%)	-0.06 (47.6%)
Semi-annually	-5.42	2.09	0.20	0.18 (42.9%)	1.19 (88.2%)
B. Maximal Returns					
Monthly	2.63	1.42	-0.21	2.91 (0.2%)	-3.12 (0.1%)
Bimonthly	3.48	1.81	-0.11	0.83 (20.4%)	-1.09 (13.7%)
Quarterly	4.10	2.15	-0.02	0.12 (45.1%)	-0.16 (43.6%)
Semi-annually	5.23	2.42	0.09	-0.67 (74.7%)	0.55 (70.8%)

Table 13
Expected number of months for a daily return above/below the threshold

	Below -5%	Below -10%	Above 5%	Above 10%
Hong Kong	9.0	63.3	8.3	54.3
India	10.8	59.8	7.9	36.3
Indonesia	6.6	51.3	5.0	43.5
Japan	13.0	663.3	9.3	107.9
Korea	7.1	74.7	5.6	63.7
Malaysia	9.6	48.2	7.2	34.3
Philippines	8.5	99.9	7.6	105.3
Singapore	19.1	138.7	14.0	81.5
Taiwan	4.5	45.6	5.1	35.9
Thailand	5.8	68.5	4.7	33.4

Consider the sequence of independent and identically distributed maxima (or absolute value of minima) with distribution F_Y and let $u > 0$ represent a threshold value. To calculate the expected waiting time to observe a daily return below/above a given threshold value, consider the sequence of i.i.d Bernoulli random variables with probability of success $\Pr\{Y_i>u\}=F^c{}_Y(u)=1-F_Y(u)$. Let $L(u)$ be a random variable representing how much time one must wait until a stock index presents a daily gain greater than u. The expected time for an index to exceed a threshold u is $E[L(u)] = 1/F^c{}_Y(u)$. We have presented our analysis using only monthly data. Therefore, $E[L(u)]$ represents the expected number of months for a stock index to exceed a threshold u.

Table 13 shows the expected number of months until an Asian stock market index presents a daily return above/below the thresholds 5% and 10%. For example, the average number of months for the Singapore Straits Industrial (Singapore) present a daily return below –5% is 19 months, whereas for the Taipei Weighted Price Index (Taiwan) is 4.5 months. The average number of months for the Nikkei Average (Japan) present a daily return above 10% is 108 months, whereas for the Bangkok S.E.T. Index (Thailand) is 33 months. The results indicate that the average waiting time for an index to present a daily return above/below the threshold is larger for major markets (Singapore and Japan) than for emerging markets (Taiwan and Thailand).

The second question is related to the probability of observing at least one daily return below/above a given threshold value for a fixed period of time. The probability p_j that the index daily return will violate the threshold u at least once before time j is given by $p_j = \Pr\{L(u)\leq j\} = 1-(1-F^c{}_Y(u))^j$.

Table 14 shows the probabilities p_j that Asian market indexes present a daily return above/below the thresholds 5% and 10%. For example, the probability that the Nikkei Average (Japan) presents at least one daily return below –5% within the next 12 months is approximately 62%, whereas for the Taipei Weighted Price Index (Taiwan) this probability is 95%. The probability that the Nikkei Average (Japan) present at least one daily return above 10% within the next 12 months is 10%, whereas for the Bangkok S.E.T. Index (Thailand) this probability is 31%. We note that the probability of observing at least one daily return above/below the thresholds for a fixed period of time is larger for emerging markets than for major markets.

Finally, in order to conclude our comparisons, we computed the t-month event, which is an extreme value which we expect to observe at least once in t months. Since $E[L(u)]=1/F^c{}_Y(u)$, the t-month event is such that $u_t = F^{-1}{}_Y(1 - 1/E[L(u)])$, where $F^{-1}{}_Y$ represents the inverse distribution of F_Y. For example, the 6-month event is the value $u_6 = F^{-1}{}_Y(1\text{-}1/6) = F^{-1}{}_Y(0.833)$. Table 15 reports the 6-, 12- and 24-month events for the Asian stock market index minimum returns. For example, we expect a daily return for the Singapore Straits Industrial (Singapore) to be smaller than –5.43%, on average, only in one month out of every 24, whereas for the Taipei Weighted Price Index (Taiwan) the 24-month event is –8.50%. We note that, in general, major markets have lower (in absolute terms) t-month events when compared to emerging markets.

Table 16 reports the 6-, 12- and 24-month events for the Asian stock market index maximum returns. We expect a daily return for the Nikkei Average (Japan) to be higher than 6.73%, on average, only in one month every 2 years, while for the Bangkok S.E.T. Index (Thailand) the 24-month event is 9.00%. As expected, major markets tend to have lower t-month events when compared to emerging markets.

Table 14
Probability of a daily return above/below the threshold

Panel A: Below –5%	Next Month	Next 3 Months	Next 6 Months	Next 12 Months
Hong Kong	11.1%	29.8%	50.6%	75.6%
India	9.3%	25.3%	44.3%	68.9%
Indonesia	15.0%	38.7%	62.4%	85.9%
Japan	7.7%	21.3%	38.0%	61.6%
Korea	14.1%	36.6%	59.8%	83.8%
Malaysia	10.4%	28.0%	48.2%	73.1%
Philippines	11.8%	31.3%	52.8%	77.8%
Singapore	5.2%	14.9%	27.6%	47.6%
Taiwan	22.2%	52.9%	77.8%	95.1%
Thailand	17.1%	43.0%	67.5%	89.5%
Panel B: Below –10%				
Hong Kong	1.6%	4.7%	9.1%	17.4%
India	1.7%	4.9%	9.6%	18.3%
Indonesia	1.9%	5.7%	11.1%	21.0%
Japan	0.2%	0.5%	0.9%	1.8%
Korea	1.3%	4.0%	7.8%	14.9%
Malaysia	2.1%	6.1%	11.8%	22.2%
Philippines	1.0%	3.0%	5.9%	11.4%
Singapore	0.7%	2.1%	4.2%	8.3%
Taiwan	2.2%	6.4%	12.5%	23.4%
Thailand	1.5%	4.3%	8.5%	16.2%
Panel C: Above 5%				
Hong Kong	12.1%	32.1%	53.9%	78.8%
India	12.6%	33.2%	55.4%	80.1%
Indonesia	20.0%	48.7%	73.7%	93.1%
Japan	10.7%	28.9%	49.4%	74.4%
Korea	17.8%	44.4%	69.1%	90.4%
Malaysia	13.9%	36.2%	59.3%	83.4%
Philippines	13.2%	34.6%	57.3%	81.7%
Singapore	7.1%	19.9%	35.9%	58.9%
Taiwan	19.4%	47.7%	72.6%	92.5%
Thailand	21.3%	51.3%	76.3%	94.4%

Table 14 (continued)

Panel D: Above 10%	Next Month	Next 3 Months	Next 6 Months	Next 12 Months
Hong Kong	1.8%	5.4%	10.6%	20.0%
India	2.8%	8.0%	15.4%	28.5%
Indonesia	2.3%	6.7%	13.0%	24.4%
Japan	0.6%	2.5%	5.1%	10.3%
Korea	1.6%	4.6%	9.1%	17.3%
Malaysia	2.9%	8.5%	16.3%	29.9%
Philippines	1.0%	2.8%	5.6%	10.8%
Singapore	1.2%	3.6%	7.1%	13.8%
Taiwan	2.8%	8.1%	15.6%	28.8%
Thailand	3.0%	8.7%	16.7%	30.6%

Table 15

T-month event using the monthly minima (%)

	6-month event	12-month event	24-month event
	$F^{-1}(0.833)$	$F^{-1}(0.916)$	$F^{-1}(0.958)$
Hong Kong	-4.22	-5.59	-7.19
India	-4.80	-6.21	-7.86
Indonesia	-3.76	-5.24	-7.03
Japan	-4.03	-4.89	-5.75
Korea	-4.70	-6.00	-7.41
Malaysia	-3.96	-5.51	-7.48
Philippines	-4.39	-5.62	-6.91
Singapore	-3.13	-4.17	-5.43
Taiwan	-5.58	-7.00	-8.50
Thailand	-5.04	-6.35	-7.73

Table 16
T-month event using the monthly maxima (%)

	6-month event	12-month event	24-month event
	$F^{-1}(0.833)$	$F^{-1}(0.916)$	$F^{-1}(0.958)$
Hong Kong	4.39	5.78	7.44
India	5.35	6.79	8.41
Indonesia	4.30	6.12	8.33
Japan	4.29	5.43	6.73
Korea	5.12	6.44	7.86
Malaysia	4.57	6.36	8.61
Philippines	4.61	5.78	7.00
Singapore	3.46	4.68	6.18
Taiwan	5.31	6.86	8.72
Thailand	5.54	7.16	9.00

Table 17
Average extremal indexes

	Minimal Returns	Maximal Returns
Hong Kong	0.41	0.48
India	0.49	0.54
Indonesia	0.36	0.37
Japan	0.48	0.50
Korea	0.40	0.43
Malaysia	0.38	0.49
Philippines	0.46	0.48
Singapore	0.39	0.44
Taiwan	0.42	0.45
Thailand	0.48	0.47

VI. CALCULATING VAR USING EXTREME VALUE THEORY

The recent turmoil that has occurred in Asian stock markets provides interesting opportunities to estimate and compare empirical VaRs with values estimated by extreme value theory. We computed VaR measures using extreme value theory (EVT-VaR) and compared the results with the empirical and normal VaR measures. We calculated EVT-VaR using block lengths of a month for both the minimum returns (long position) and maximal returns (short position). We estimated VaR for different confidence levels or probability values (95%, 99% and 99.9%).

Expressing VaR in percentage terms and setting $p=Prob(y_n \geq -VaR^{long})$ gives the probability that the minimum return will not exceed a threhold value equal to VaR^{long}. Similarly for the short position, $p=Prob(y_n \geq VaR^{short})$ is the probability that the maximal return will be above the threshold VaR^{short}.

We also computed extreme value theory VaR adjusted with the extremal index (EVT-θ-VaR). The extremal index θ is the reciprocal value of the mean cluster size of a cluster of exceedances, asymptotically. So, θ^{-1} is the average number of observations in a cluster of extreme values. The index θ can assume values in [0,1]. In the classical case of iid random sequences $\theta=1$, that is, no clustering of exceedances occurs. θ being close to zero indicates a high degree of clustering (long-range dependence). In dependent sequences we might have $0<\theta\leq 1$. This index is estimated by $\theta=Z/N$ where Z is the number of clusters of exceedances above the threshold ν and N is the number of exceedances of a threshold ν. Table 17 shows θ calculated for each Asian stock market index.

The VaR values determined by the extreme value theory adjusted and not adjusted with the extremal index (EVT-θ-VaR and EVT-VaR, respectively) and the empirical and normal VaRs at the 95%, 99% and 99.9% levels are presented in Table XVIII.

The VaR values for the minima series are slightly lower than the corresponding values in the maxima series. At the 95% and 99% levels, the EVT-θ-VaR overestimates a little the empirical VaR, while the EVT-VaR underestimates it. At the 99.9% level, both the EVT-θ-VaR and EVT-VaR overestimate the empirical VaR. However, EVT-VaR provides more accurate measures at the 99.9% level than the EVT-θ-VaR, because selecting a rather large confidence level means that there are only very few exceedances and the estimation of θ is impossible if no exceedance is observed. In this case, the EVT-VaR, where $\theta=1$ (no dependence), provides more accurate measures. Normal distributions provide satisfactory 95% VaR estimates, but underestimate the 99% and 99.9% VaR.

These results suggest that the extreme value method of estimating VaR is a more conservative approach to determining capital requirements than traditional/historical methods. A similar result was found by Danielson et al (1998), and Ho, Burridge, Cadle and Theobald (2000). We can also conclude that we should use the extremal index θ to adjust EVT-VaR measures when we calculate VaR at lower confidence levels, such as 95%, 97.5% and 99% levels.

Table 18

Empirical, normal, EVT and EVT/extremal index (θ) VaR

Panel A: Min – Long Position (expressed as a % of the position)

	95% VaR				99% VaR				99.9% VaR		
	EVT + θ	EVT	Normal	Empirical	EVT + θ	EVT	Normal	Empirical	EVT + θ	EVT	Normal
Hong Kong	2.84	1.80	2.78	2.53	5.50	3.90	3.93	4.73	11.93	8.93	5.23
India	3.12	2.25	3.09	2.63	5.78	4.47	4.37	4.86	11.69	9.61	5.80
Indonesia	2.49	1.24	2.59	2.14	5.46	3.41	3.67	4.62	12.61	8.96	4.87
Japan	2.79	1.98	3.12	2.45	4.65	3.80	4.41	4.06	7.31	6.52	5.85
Korea	3.30	2.10	2.92	2.89	5.91	4.37	4.14	5.27	10.92	8.79	5.49
Malaysia	2.61	1.48	2.75	2.44	5.63	3.61	3.88	4.50	14.43	9.68	5.16
Philippines	2.87	1.87	2.19	2.62	5.29	4.09	3.10	4.73	9.70	8.22	4.12
Singapore	2.14	1.35	3.57	1.91	4.16	2.90	5.05	3.52	9.13	6.81	6.70
Taiwan	3.91	2.54	3.21	3.46	6.87	5.22	4.53	6.50	12.16	9.91	6.02
Thailand	3.32	2.26	2.47	2.94	5.98	4.71	3.49	5.53	10.60	9.00	4.63

Panel B: Max – Short Position (expressed as a % of the position)

	95% VaR				99% VaR				99.9% VaR		
	EVT + θ	EVT	Normal	Empirical	EVT + θ	EVT	Normal	Empirical	EVT + θ	EVT	Normal
Hong Kong	2.85	2.07	2.78	2.49	5.38	4.06	3.93	4.66	11.65	9.36	5.23
India	3.43	2.60	3.09	2.91	6.15	5.01	4.37	5.40	11.80	10.11	5.80
Indonesia	2.76	1.39	2.59	2.27	6.27	3.90	3.67	5.47	16.27	10.86	4.87
Japan	2.78	2.02	3.12	2.30	5.02	4.00	4.41	4.20	9.47	7.99	5.85
Korea	3.58	2.42	2.92	3.10	6.23	4.80	4.14	5.89	11.04	9.13	5.49
Malaysia	2.63	1.74	2.75	2.29	5.76	4.17	3.88	4.80	14.68	11.13	5.16
Philippines	3.07	2.15	2.19	2.64	5.45	4.32	3.10	5.02	9.63	8.18	4.12
Singapore	2.22	1.51	3.57	1.96	4.45	3.19	5.05	3.84	10.73	7.94	6.70
Taiwan	3.61	2.60	3.21	3.23	6.54	4.96	4.53	6.13	13.59	10.77	6.02
Thailand	3.61	2.46	2.47	2.99	6.78	5.15	3.49	6.02	13.22	10.95	4.63

VII. CONCLUSIONS

The purpose of this work was to use the extreme value theory to analyze ten Asian stock markets: three major markets (Hong Kong, Japan, and Singapore), and seven emerging markets (India, Indonesia, Korea, Malaysia, Philippines, Taiwan, and Thailand). The main objective was to identify which type of extreme value asymptotic distribution better fitted historical extreme market events in Asia. Understanding the influence of extreme market events is of great importance for risk managers. Most of the risk measurement methodologies used to estimate the value at risk of a portfolio assume that the market behavior is stable. By modeling the extreme market events we are able to obtain the VaR.

Our empirical tests indicate that the return distributions are not characterized by normality and that the minima and the maxima of the return series were found to be satisfactorily modeled within an extreme value framework. The Fréchet distribution turned out to be the distribution that best explained minimum and maximum returns in Hong Kong, Malaysia, and Singapore. The Gumbel distribution explained best the minimum and maximum returns in Japan, Korea, Philippines, and Thailand. In India, the maximum returns and the monthly and bimonthly minimum returns were best modeled by a Fréchet distribution, whereas the quarterly and semi-annually minimum returns were best explained by a Gumbel distribution. In Indonesia, the monthly and bimonthly minimum and maximum returns were best explained by a Fréchet distribution, while the Gumbel distribution explained better the quarterly and semi-annually minimum and maximum returns. In Taiwan, the minimum returns were found to follow a Gumbel distribution, while the maximum returns follow a Fréchet distribution.

We have also performed a comparative analysis in order to answer two questions: (a) what is the expected waiting time to observe a daily return below/above a given threshold value? (b) for a fixed period of time, what is the probability of observing at least one daily return below/above a specific threshold? Our results indicate that the average waiting time for an index to present a daily return below/above a specific threshold is generally larger for Asian major markets than for Asian emerging markets, and that the probability of observing at least one daily return below/above a given threshold for a fixed period of time is larger for emerging markets than for major markets. In general, major markets have lower (in absolute terms) t-month events when compared to emerging markets.

The recent turmoil that has occurred in Asian stock markets provides interesting opportunities to estimate and compare empirical VaRs with values estimated by extreme value theory. We computed VaR measures using extreme value theory, adjusted and not adjusted with the extremal index θ, and compared the results with the empirical VaR measures. At the 95% and 99% levels, the EVT-θ-VaR overestimates a little the empirical VaR, while the EVT-VaR underestimates it. At the 99.9% level, both the EVT-θ-VaR and EVT-VaR overestimate the empirical VaR. However, EVT-VaR provides more accurate measures at the 99.9% level than the EVT-θ-VaR. These results suggest that the extreme value method of estimating VaR is a more conservative approach to determining capital requirements than traditional/historical methods.

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