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Portfolio Credit Risk Models and Name Concentration Issues: Theory and Simulations

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ABSTRACT

Credit risk is an important aspect in the activity of commercial banks. Regulators require from banks to measure credit risk within Basel II and also during the Internal Capital Assessment and Adequacy Process (ICAAP) and stress tests. Name concentration in a lending portfolio arises when there are few borrowers in a bank portfolio or when loan amounts are very unequal in distribution. The portfolio credit risk model underpinning the Basel II Internal Ratings-Based (IRB) approach does not account for name concentration. To measure the latter the literature proposes specific concentration indexes such as the Herfindahl-Hirschman index, the Gini index or more general approaches like the granularity adjustment (GA) to calculate the appropriate economic capital needed to cover the risk arising from the potential default of large borrowers. This paper investigates the practical aspects of granularity adjustment, Gini index and Herfindahl index for contribution's quantification of name concentrations to portfolio risk. We try also to extend the upper bound approach of GA developed by Gordy and Lütkebohmert (2007). For many banks, this approach would permit dramatic reductions in data requirements relative to the full GA.

JEL Classifications: *G21, G38*

Keywords: *concentration risk; granularity adjustment; Herfindahl-Hirschman index; Gini index; ICAAP*

I. INTRODUCTION

The global financial crisis highlights the importance of credit risk in the banking portfolios. Credit risk is an important source of profit in the activity of commercial banks. Regulators and in particular BIS and Central Bank regulators and supervisors require from banks to measure credit risk within Basel II at least once a year if not every quarter. Boards of directors require from risk management to provide also quarterly reports at least regarding the credit portfolio. Regulators and supervisors require also during the Internal Capital Assessment and Adequacy Process, ICAAP and stress tests exercise to provide the credit risk information. Historical experience shows that concentration of credit risk in asset portfolios has been one of the major causes of bank distress. This is true both for individual institutions as well as banking systems at large. The failures of large borrowers like Enron, Worldcom and Parmalat were the source of sizeable losses in a number of banks. Concentration of exposures in credit portfolios may arise from two types of imperfect diversification. The first type, name concentration, relates to imperfect diversification of idiosyncratic risk in the portfolio either because of its small size or because of large exposures to specific individual obligors. However the second type, sector concentration, relates to imperfect diversification across systematic components of risk, namely sectorial factors.

The existence of concentration risk violates one or both of two key assumptions of the Asymptotic Single-Risk Factor (ASRF) model that underpins the capital calculations of the internal ratings-based (IRB) approaches of the Basel II Framework.

The ASRF model in the new Basel capital framework does not allow for the explicit measurement of concentration risk. In the risk-factor frameworks that underpin the internal ratings-based (IRB) risk weights of Basel II, credit risk in a portfolio arises from two sources, systematic and idiosyncratic risks. Systematic risk represents the effect of unexpected changes in macroeconomic and financial market conditions on the performance of borrowers. Borrowers may differ in their degree of sensitivity to systematic risk, but few firms are completely indifferent to the wider economic conditions in which they operate. Therefore, the systematic component of portfolio risk is unavoidable and only partly diversifiable. Meanwhile, idiosyncratic risk represents the effects of risks that are particular to individual borrowers. As a portfolio becomes more fine-grained, in the sense that the largest individual exposures account for a smaller share of total portfolio exposure, idiosyncratic risk is diversified away at the portfolio level. This risk is totally eliminated in an infinitely granular portfolio[1].

From the mentioned types of concentration risk, name concentrations are better understood than sector concentrations. The theoretical derivation of the granularity adjustment that accounts for name concentrations was done by Wilde (2001) and improved by Pykhtin and Dev (2002) and Gordy (2003). The adjustment formulas are derived in a more straightforward approach by Martin and Wilde (2002), Rau-Bredow (2002) and Gordy (2004). Furthermore, the adjustment is extended and numerically analyzed in detail by Gürtler, Heithecker, and Hibbeln (2008). A related approach is the granularity adjustment from Gordy and Lütkebohmert (2007), whereas the semi-asymptotic approach from Emmer and Tasche (2005) refers to name concentrations due to a single name while the rest of the portfolio remains infinitely granular, so this can be called “single name concentration”.

On the basis of previous evidences and references, we propose in this paper a comparison between the methods available for measuring name concentration and we try to improve the upper bound of granularity adjustment approach proposed by Gordy and Lütkebohmert (2007). The paper is organized as follows. Section II describes the methods available for measuring concentration risk and presents the method of "upper bound" based on partial information of the portfolio. Section III describes the data set that we used in our empirical studies. The performance of the GA is evaluated in various ways in Section IV.

II. THE MODELS TO MEASURE CREDIT RISK IN SINGLE-NAME CONCENTRATION

The approaches available for measuring single-name concentration can be broken down into model-free and model based methods. The first approach is to adapt indices of concentration such as Herfindahl-Hirschman index proposed by Kwoka (1977) and Gini index suggested by (Gini, 1921). While these indices can be good measures for concentration itself, they do not seem to serve well for concentration risk because they do not take distribution of different quality obligors into account. The second approach is granularity adjustment. In this paper, we present and evaluate the revised GA proposed by Gordy and Lütkebohmert (2007), appropriate for the application under the Pillar 2 of Basel 2. The proposed methodology is similar in form and spirit to adjustment of granularity proposed in the second consultative paper (CP2) of Basel 2 (2001). As in the CP2 version, the data inputs to the revised GA are drawn from quantities already required to calculate the IRB capital requirements.

In the practical application, the data inputs can present the greatest obstacle to effective implementation. When a bank has several exposures to the same underlying borrower, it is important that these multiple exposures are aggregated into a single exposure in order to calculate GA inputs.

To reduce the difficulties associated with exposures aggregation, the revised GA provides for the possibility that banks are allowed to calculate the GA on the basis of the largest exposures in the portfolio and saving them the need to aggregate data on each borrower. To enable this option, regulators must be able to calculate the largest possible GA that is consistent with the incomplete data provided by the bank. The approach proposed by Gordy and Lütkebohmert (2007) is based on an upper bound formula for the GA as a function of data on the m largest exposures of a portfolio of n loans (with $m \leq n$). This methodology takes advantages of theoretical advances that have been made since the time of CP2. In practice, both approaches are used to measure the concentration risk of their portfolio. However the concentration measurement index such as Herfindahl-Hirschman index cannot measure the actual risk accurately, granularity adjustment sometimes overestimates the actual concentration risk of a portfolio.

A. The Use of Indexes to Measure Concentration in Credit Risk: Herfindahl-Hirschman Index

Herfindahl-Hirshman index (HHI) is a commonly used ratio to measure concentrations. The HHI can also be used to calculate portfolio concentration risk. This is a very straight forward measure of concentration. Herfindahl index gives a weight depending on the exposure to the counterparties. HHI measures concentration as the sum of the squares of the relative portfolio shares of all borrowers.

$$HHI = \sum_{i=1}^{n} s_i^2 \tag{1}$$

Where s_i is the portfolio share of borrower i, and n denote the number of positions in the portfolio. It is assumed that exposures have been aggregated so that there is a single borrower for each position.

$$s_i = A_i / \sum_{j=1}^{n} Aj.$$

Where A_i is the exposure at default (EAD_i).

Well-diversified portfolios with a large number of small credits have an HHI value close to zero, whereas heavily concentrated portfolios can have a considerably higher HHI value. In the extreme case where we observe only one credit, the HHI takes the value of 1.

In the context of the measurement of concentration risk, the HHI formula is included as a main component of a number of approaches.

B. The Measure of Concentration Credit Risk and the Gini Index

The Gini index (G) or Lorenz ratio is a standard measure of inequality or concentration of a group distribution. It is defined as a ratio with values between 0 and 1. A low Gini index indicates more equal income or distribution of loan assets with different industries/groups, sectors, etc., while a high Gini index indicates more unequal distribution. 0 corresponds to perfect equality and 1 corresponds to perfect inequality.
More formally, G could be expressed as:

$$G = 1 + 1/n - (2/n^2 \overline{A})(A_1 + 2A_2 + 3A_3 + \ldots + nA_n), \tag{2}$$

Where $A_i, i = 1, \ldots, n$ is the credit amount by each borrower in a specific portfolio in decreasing order of size and $\overline{A}$ is the mean of $A_i, i = 1, \ldots, n,$ equal to $\overline{A} = \frac{1}{n} \sum_{i=1}^{n} A_i$.

G is thus a weighted sum of the shares, with the weights determined by rank order position. A value of Gini index close to zero corresponds to a well-diversified portfolio where all exposures are more or less equally distributed and a value close to one corresponds to a highly concentration portfolio. A Gini index in the range of 0.3 or less indicates substantial equality, Gini >0.3 to 0.4 indicate acceptable normality. However, if Gini index is above 0.4 means concentration is large or inequality is high.

C. The Granularity Adjustment Formula in Credit Risk Measurement

The granularity adjustment (GA) can be applied to any risk-factor model portfolio credit risk. Gordy and Lütkebohmert (2007) follow the treatment of Martin and Wilde (2002) in the mathematical presentation, but the parameterization of the GA formula is different. Let X be the systematic risk factor. We assume that X is unidimensional for consistency with the asymptotic single risk factor framework of Basel 2. Let U_i be the rate of loss on position i and let L_n denote the rate of loss in the portfolio of the first n positions.

$$L_n = \sum_{i=1}^{n} s_i U_i, \tag{3}$$

Let $\alpha_q(Y)$ be the q^{th} percentile of the distribution of a random variable Y. When economic capital is measured as the value at risk (VaR) at the q^{th} percentile, we estimate $\alpha_q(l_n)$. The IRB formula gives us the q^{th} percentile of the conditional expected loss $\alpha_q(E[L_n \mid X])$. The difference $\alpha_q(L_n) - \alpha_q(E[L_n \mid X])$ is the "accurate" adjustment for the effect of undiversified idiosyncratic in the portfolio. Such an adjustment cannot be obtained through an analytical form, but we can construct a Taylor series of approximations in the orders of $1/n$.

The functions $\mu(X) = E[L_n \mid X]$ and $s^2 = V[L_n \mid X]$ are the conditional mean and variance of portfolio losses respectively. Let h denote the probability density function of X. Wilde (2001b) shows that the first order of the granularity adjustment is given by

$$GA = \frac{-1}{2h(\alpha_q(X))} \frac{d}{dx}\left(\frac{\sigma^2(x)h(x)}{\mu'(x)} \right)\Big|_{x=\alpha_q(X)} \tag{4}$$

In the granularity adjustment formula, the terms $\mu(x)$, $\sigma^2(x)$ and $h(x)$ are model-dependent. For the purposes of the GA in a supervisory setting, it is desirable to base the GA on the same model as the one that underpins the IRB capital formula. Unfortunately, this is not feasible because the IRB formula is derived within a single risk factor model mark-to-market Vasicek (2002). The expressions of $\mu(x)$ and $\sigma^2(x)$ in such models are very complex. We base the GA on a model chosen for the tractability of the resulting expressions and we reparameterize the inputs in a way that restores consistency as much as possible. Our chosen model is an extended version of the single factor CreditRisk+ model that take into account for the idiosyncratic risk. As CreditRisk+ is an actuarial loss model, we define the rate of loss as $U_i = LGD_i D_i$ where D_i is an indicator of default equals to 1 if the borrower defaults and 0 otherwise. The systematic risk factor X generates correlation across borrower defaults by varying the probability of default. Conditional on $X = x$. The probability of default is expressed as follows

$$PD_i(x) = PD_i(1 - w_i + w_i x).$$

Where PD_i is the unconditional probability of default.

The factor loading w_i controls the sensitivity of the borrower i to the systematic risk factor. We assume that X is distributed according to a gamma distribution with mean 1 and variance $1/\xi$ for some positive ξ . Finally, to obtain an analytical solution for the model, we approximate in CreditRisk + the distribution of the indicator variable of default as a Poisson distribution.

We define the functions $\mu_i(x) = E[U_i \mid x]$ and $\sigma_i^2(x) = V[U_i \mid x]$. By the assumption of conditional independence, we have

$$\mu(x) = E[L_n \mid x] = \sum_{i=1}^{n} s_i \mu_i(x)$$
$$\sigma^2(x) = V[Ln \mid x] = \sum_{i=1}^{n} s_i^2 \sigma_i^2(x).$$

The function $\mu_i(x)$ is :

$$\mu_i(x) = ELGD_i PD_i(x) = ELGD_i PD_i(1 - w_i + w_i x).$$

For the conditional variance:

$$\begin{aligned} \sigma_i^2(x) &= E[LGD_i^2 D_i^2 \mid x] - ELGD_i^2 PD_i(x)^2 \\ &= E[LGD_i^2]E[D_i^2 \mid x] - \mu_i(x)^2. \end{aligned} \tag{5}$$

As D_i given X, is assumed to be Poisson distributed, we have

$$E[D_i \mid X] = V[D_i \mid X] = PD_i(X)$$

This implies

$$E[D_i^2 \mid X] = PD_i(X) + PD_i(X)^2.$$

For the term $E[LGD_i^2]$ in the conditional variance, we can replace

$$E[LGD_i^2] = V[LGD_i] + E[LGD_i]^2 = VLGD_i^2 + ELGD_i^2$$

This brings us to

$$\sigma_i^2 = (VLGD_i^2 + ELGD_i^2)(PD_i(X) + PD_i(X)^2) - \mu_i(x)^2$$
$$= C_i \mu_i(x_q) + \mu_i(x_q)^2 \frac{VLGD_i^2}{ELGD_i^2}$$

where C_i is defined as

$$Ci \equiv \frac{ELGD_i^2 + VLGD_i^2}{ELGD_i}. \quad (6)$$

We replace the gamma probability density function $h(x)$ and $\mu(x)$ and $\sigma^2(x)$ in equation (4), and evaluate the derivative in this equation at $x = \alpha_q(X)$. The resulting formula depends on the instrument-level parameters PD_i , w_i, $ELGD_i$ and $VLGD_i$.

We now re-parameterize the inputs. Let R_i denote the expected loss reserve requirements, expressed as a share of EAD for instrument i. In CreditRisk +, it is expressed as follows

$$R_i = ELGD_i PD_i.$$

Let K_i denote the unexpected loss of capital requirement as a share of EAD. In CreditRisk +, this is simply

$$K_i = E[U_i \mid X = \alpha_q(X)] = ELGD_i PD_i w_i(\alpha_q(X) - 1) \quad (7)$$

If replace R_i and K_i into the GA of CreditRisk +, we find that the PD_i and w_i inputs can be eliminated. This gives:

$$GA = \frac{1}{2K^*}\sum_{i=1}^{n} s_i^2\left[\left(\delta C_i(K_i+R_i)+\delta(K_i+R_i)^2\frac{VLGD_i^2}{ELGD_i^2}\right)\right.$$
$$\left.-K_i\left(C_i+2(K_i+R_i)\frac{VLGD_i^2}{ELGD_i^2}\right)\right]. \tag{8}$$

Where $K^* = \sum_{i=1}^{n} s_i K_i$ is the required capital per unit of exposure for the entire portfolio and where

$$\delta \equiv (\alpha_q(X)-1)\left(\xi + \frac{1-\xi}{\alpha_q(X)}\right).$$

The variance parameter ξ affects the GA through δ. In the CP2 version, we set $\xi = 0,25$. We assume that $q = 0,999$. This implies $\delta = 4,83$. To avoid the burden of a new data requirement, it seems preferable to impose a regulatory assumption on VLGD. We impose the relationship as found in the CP2 version of the GA:

$$VLGD_i^2 = \gamma ELGD_i(1-ELGD_i) \tag{9}$$

where the regulatory parameter $\gamma = 0,25$.

The formula of the GA can be simplified somewhat. The quantities R_i and K_i are typically small and so the terms that are products of these quantities contribute little to the GA. If these second-order terms are eliminated, we arrive at the following simplified formula:

$$GA_{simplified} = \frac{1}{2K^*}\sum_{i=1}^{n} s_i^2 C_i(\delta(K_i+R_i)-K_i). \tag{10}$$

The accuracy of this approximation to Equation (8) will be evaluated in Section IV.

1. Defining An Upper Bound Based on Incomplete Data

The aggregation of multiple exposures into a single exposure per borrower may be the only real challenge in the implementation of the GA. To reduce these difficulties on banks, Gordy and Lütkebohmert (2007) propose to allow them to calculate the GA based on a subset containing the largest exposures. An upper bound can be calculated for exposures that have been excluded of the computation. The bank can therefore find a trade-off between the cost of data collection and the cost of the additional capital associated with the upper bound. We require information on both the distribution of aggregated positions by EAD and capital contribution. We assume:

1. The bank has identified the m borrowers to whom it has the largest aggregated exposures measured by their capital contribution A_iK_i . We denote this set of borrowers by Ω . For each borrower $i \in \Omega$, the bank knows (s_i , K_i , R_i).
2. For the $n-m$ exposures unreported, the bank sets an upper bound on share (denoted s') such that $s_i \le s'$ for all i in the unreported set.
3. The bank knows K^* and R^* for the portfolio as a whole.

A bank can easily determine s' if, for example, internal risk management systems report on the borrowers to which the bank has the largest one exposure in EAD[2]. Denote this set by Λ and let λ be the smallest s_i in this set. So s' is either the greatest of the s_i which is in Λ but not in Ω or simply λ, (if this set is empty).

$$s' = \max\{s_i \ : \ s_i \in \Lambda \,|\, \Omega \cup \{\lambda\}\}$$

We generalize K^* and R^* such that

$$K_k^* = \sum_{i=1}^{k} s_i K_i$$
$$R_k^* = \sum_{i=1}^{k} s_i R_i,$$

where K_k^* and R_k^* are partial weighted sums of K_i and R_i , respectively. Finally, we define

$$Q_i \equiv \delta(K_i + R_i) - K_i.$$

Using the above notation, the GA can be reformulated as follows

$$\begin{aligned} GA_{simplified} &= \frac{1}{2K^*} \sum_{i=1}^{n} s_i^2 C_i (\delta(K_i + R_i) - K_i) \\ &= \frac{1}{2K^*} \left(\sum_{i=1}^{m} s_i^2 C_i Q_i + \sum_{i=m+1}^{n} s_i^2 C_i Q_i \right). \end{aligned} \tag{11}$$

The summation over 1 to m is known by assumption 1. By assumption 2 we know that $s' \ge s_i$ for $i = m+1, \ldots\ldots, n$. The assumption on VLGD in Equation (9) is sufficient to ensure that $C_i \le 1$. Therefore,

$$\sum_{i=m+1}^{n} s_i^2 C_i Q_i \le s' \sum_{i=m+1}^{n} s_i Q_i = s'\left(\delta \sum_{i=m+1}^{n} s_i (K_i + R_i) - \sum_{i=m+1}^{n} s_i K_i\right).$$

We also know that

$$\sum_{i=m+1}^{n} s_i K_i = K^* - K_m^*$$
$$\sum_{i=m+1}^{n} s_i R_i = R^* - R_m^*.$$

Assumption 1 implies that K_m^* and R_m^* are known by the bank. Thus we obtain:

$$\sum_{i=m+1}^{n} s_i^2 C_i Q_i \le s'((\delta - 1)(K^* - K_m^*) + \delta(R^* - R_m^*)). \tag{12}$$

We obtain the following upper bound

$$GA_{\text{simplified}}^{\text{upper}} = \frac{1}{2K^*}(\sum_{i=1}^{m} s_i^2 C_i Q_i + s'((\delta - 1) \\ (K^* - K_m^*) + \delta(R^* - R_m^*))). \tag{13}$$

2. A Modified Upper Bound

We reincorporate C_i in the second term of Equation (13) and we obtain the upper bound modified as follows

$$\begin{aligned} GA_{\text{simplified}}^{\text{upper(modified)}} &= \frac{1}{2K^*}(\sum_{i=1}^{m} s_i^2 C_i Q_i + s' C_i (\delta \sum_{i=m+1}^{n} s_i (K_i + R_i) \\ &\quad - \sum_{i=m+1}^{n} s_i K_i)) \\ &= \frac{1}{2K^*}(\sum_{i=1}^{m} s_i^2 C_i Q_i + s' C_i (\delta(\sum_{i=m+1}^{n} s_i C_i K_i + \\ &\quad \sum_{i=m+1}^{n} s_i C_i R_i) - \sum_{i=m+1}^{n} s_i K_i)). \end{aligned}$$

We define Z^* and T^* so that

$$Z_k^* = \sum_{i=1}^{k} s_i C_i K_i$$

$$T_k^* = \sum_{i=1}^{k} s_i C_i R_i,$$

where Z_k^* and T_k^* are partial weighted sums of the $(C_i K_i)$ and $(C_i R_i)$ sequences, respectively. Next we observe that

$$\sum_{i=m+1}^{n} s_i C_i K_i = Z^* - Z_m^*$$

$$\sum_{i=m+1}^{n} s_i C_i R_i = T^* - T_m^*.$$

Then, we obtain our modified upper bound

$$GA_{\text{simplified}}^{\text{upper(modified)}} = \frac{1}{2K^*}\Big(\sum_{i=1}^{m} s_i^2 C_i Q_i + s'((\delta-1)(Z^* - Z_m^*) + \delta(T^* - T_m^*))\Big). \tag{14}$$

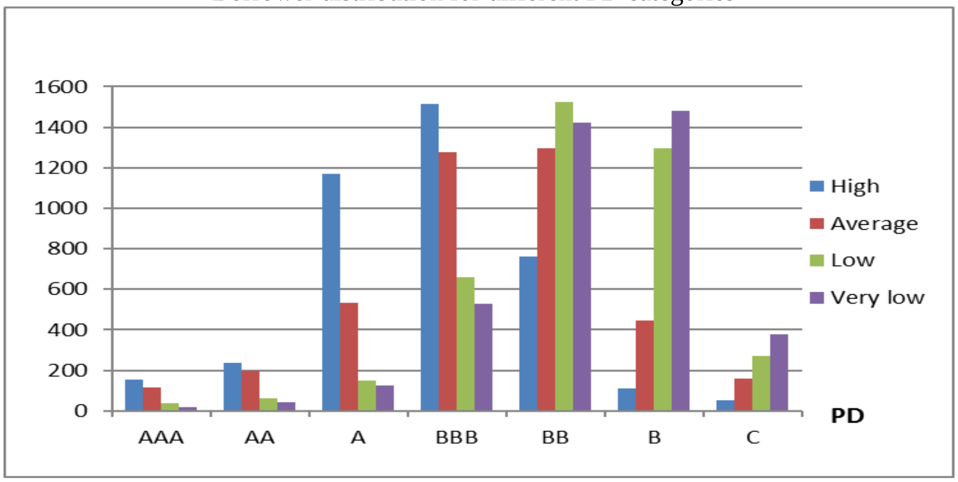
III. DATA AND EMPIRICAL TESTS

To measure Gini index and HHI index and to show the impact of the granularity adjustment on Economic Capital, we apply theses indices on banks portfolios. We use simulated data similar to Gordy and Lütkebohmert (2007) on data from the German credit register. The number of loans in our portfolio varies between 250 and 4000. The amount of each loan is greater or equal to 1 Million euro. We group the banks in large, medium, small and very small banks where large refers to those with 4000 exposures, the medium refers to one with 1000 exposures, small refers to a bank with 500 exposures and very small to a bank with 250 exposures. The mean of the loan size distribution is 4 million Euros. Figure 1 shows the borrower distribution for different PD categories. The PD ranges for each rating grade are listed in Table 1 below.

IV. NUMERICAL RESULTS

In Table 2, we present the granularity adjustments calculated on high quality portfolios varying in size and degree of heterogeneity with ELGD = 45%. As expected, the GA is always small (4 to 18 basis points) for larger portfolios, but can be more important (up to 228 basis points) for the smallest. The table shows the strong correlation between the Herfindahl index and the GA across these portfolios, even if the correspondence is not exact since the GA is sensitive to credit quality. We remark also that portfolio size and credit quality are not taken into consideration when we use the Gini index. In addition, we set three values for the variable ELGD 15%, 45% and 85%.

Figure 1
Borrower distribution for different PD categories



The average probabilities of default are 0.65%, 1.72%, 3.33% and 4.05% for portfolios of high, medium, low and very low quality. They are all higher than the average PD portfolio used by Gordy and Lütkebohmert (2007), about 0.43%.

Table 1
PD ranges associated with rating buckets

Rating Grade	PD (%)
AAA	PD≤0.02
AA	0.02≤PD≤0.06
A	0.06≤PD≤0.18
BBB	0.18≤PD≤0.106
BB	0.106≤ PD ≤4.94
B	4.94≤PD≤19.14
C	PD≤19.14

Table 2
GA, Gini index and HHI index for different portfolios

Portfolio	Number of Exposures	GA (in %)	HHI	G
Large	4000	0.0003-0.0014	0.04-0.18	0.15-0.65
Medium	1000	0.0012-0.0056	0.15-0.67	0.15-0.65
Small	500	0.0025-0.0113	0.3-1.33	0.15-0.65
Very Small	250	0.005-0.0227	0.61-2.28	0.15-0.65

Table 3 presents the relative add-on for the adjustment of granularity on the Risk Weighted Assets (RWA) of Basel II for the very small, small, medium and large sizes of average quality portfolios with ELGD = 45%. For the largest portfolio (benchmark) with 4000 exposures, the most homogeneous in terms of size of exposure, the GA is about 0.0440% and the IRB capital requirements is 5.853%. Thus, the add-on due to the granularity is approximately 0.747% and the economic capital to capture both the systematic risk and risk from single name concentration is 5.897% of the total portfolio exposure. For the rest of the portfolios, the add-on for the GA is higher than for the reference portfolio, but it is still small for some medium-sized portfolios. For smaller portfolios (less than 1000 exposures), the add-on for the GA is more significant. We also show in Figure 2 that the add-on of the GA (red part) is more significant for smaller portfolios.

Table 3
GA as percentage add-on to RWA

Portfolio	Number of Exposures	Relative Add-On For RWA
Large	4000	0.7-3.3
Medium	1000	3-12.51
Small	500	5.56-21.02
Very Small	250	10.62-35.91

Figure 2
Adjusted economic capital

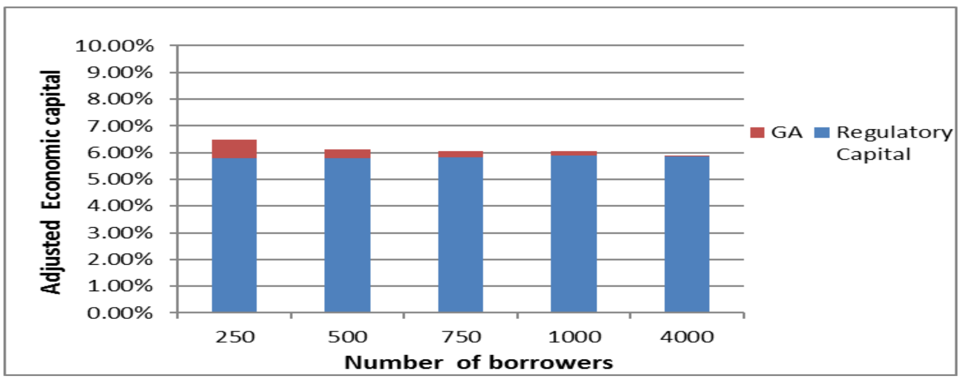


Figure 3 shows the dependence of the granularity adjustment to the loss given default. The GA was estimated on the basis of medium sized (1000 borrowers per portfolio) with different qualities portfolios. We observe that the GA is sensitive to the loss given default for different types of portfolio. Figure 4 shows the sensitivity of the GA on the probability of default. Each point on the curve represents a homogeneous portfolio of n = 250 borrowers of a given PD. The sensitivity on the quality of the portfolio is non-negligible, particularly for lower-quality portfolios. Such dependence cannot be taken into account accurately by the HHI and Gini Index.

Figure 3
Effect of loss given default on GA

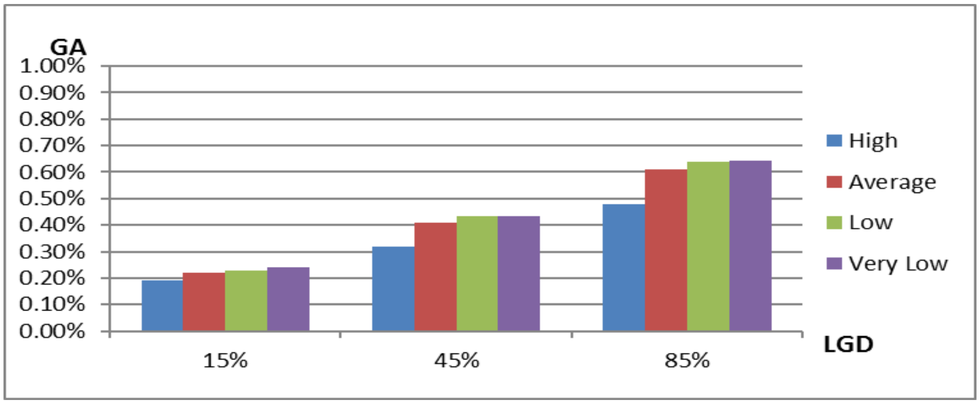
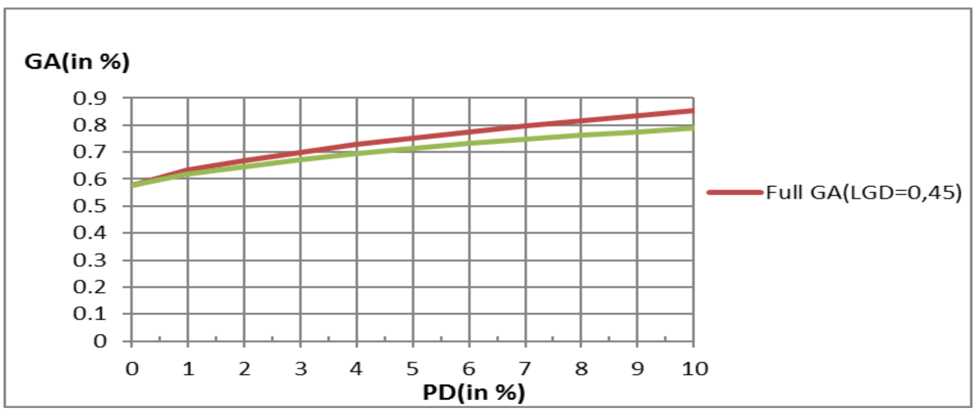


Figure 4
Effect of credit quality on simplified GA



Then, we check for the accuracy of the simplified adjustment of granularity as an approximation to the "full" GA of Equation (8). We construct four portfolios of different degrees of exposure concentrations. Each portfolio consists of n = 250 exposures and has the same PD and ELGD fixed at 45%. Portfolio P1 is completely homogeneous, whereas portfolio P4 is highly concentrated. The values for the full GA and the simplified GA for each of these portfolios are listed in Table 4. We find that the error increases with the degree of concentration and with PD but is still negligible. For example, in the case of portfolio P1 and PD = 4%, the error is only 3 basis points.

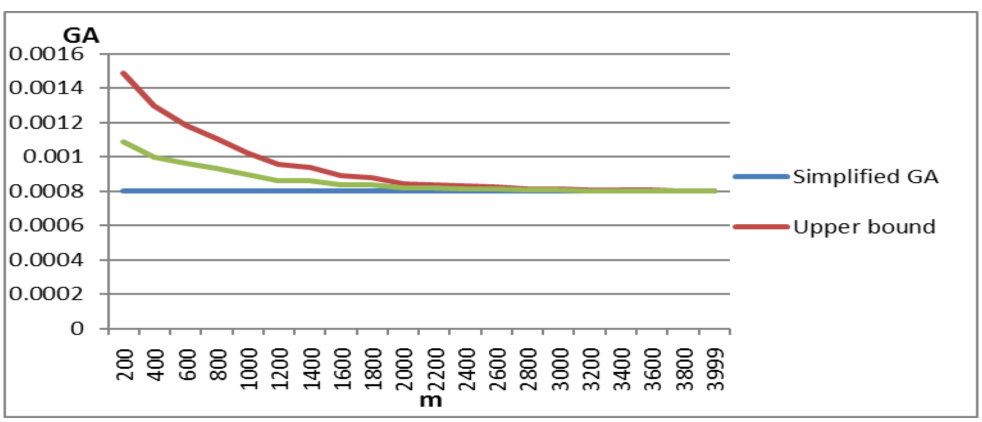
Table 4
Approximation error of the simplified GA

Portfolio	P1	P2	P3	P4
PD = 1%				
EAD (10% of borrowers)	10%	25%	50%	75%
Simplified GA (in %)	0.493	0.616	1.371	2.810
Full GA (in %)	0.506	0.633	1.406	2.883
PD = 4%				
EAD (10% of borrowers)	10%	25%	50%	75%
Simplified GA (in %)	0.555	0.694	1.542	3.161
Full GA (in %)	0.581	0.727	1.616	3.313

The error is 15 basis points in the extreme example of P4 with PD = 4%, but even this remains small compared to the size of the granularity adjustment.

Finally, we use the largest portfolio with 4000 exposures to show the effectiveness of the upper bound (red curve) presented in Section II. In Figure 5, we show how the gap between the upper bound and the "whole portfolio" GA shrinks as m (the number of loans included in the calculation) increases with 25% of exposures included**,** this gap is only 2 basis points (27.6% of GA added). For 50% of the exposures included, the gap is reduced to 0.4 basis point (9.6% of GA added). Within this framework we show also the effectiveness of our modified upper bound (green curve). The latter reduces significantly the gap between the simplified GA and the simplified GA with upper bound. With 25% and 50% of exposures included in the calculation the gap fell respectively to 0.9 and 0.19 basis point (11.7% and 2.3% of the granularity adjustment added). From these results, we can conclude that the upper bound approach performs quite well.

Figure 5
Tightness of the upper bound



V. CONCLUSION

We have examined, in this paper, numerical behavior of granularity adjustment, Gini index and Herfindahl-Hirschman index in several types of portfolios and we have studied its robustness to model parameters. We have also provided an extension to the methodology of upper bound of Gordy and Lütkebohmert (2007) that enabled us to improve the performance of the latter.

A potential source of inaccuracy must be considered. The GA formula is itself an asymptotic approximation, and might not work well on very small portfolios. However, the great advantage of the analytical model is its tractability. This tractability allows us to extend a useful upper bound methodology. We also show that unlike GA approach, HHI and Gini index does not take into account the quality of the loan portfolio. In a further work it would be useful to combine this GA model with a multifactorial concentration risk model. This will allow us to get eventually a more accurate model for diversified portfolios through different sectors and regions.

APPENDIX

Calibration of factor loading w_i

The factor loading w_i is obtained by linking the unexpected losses of capital requirements between the CreditMetrics and CreditRisk models:

$$K_i^{CR^+} = ELGD_i PD_i w_i (\alpha_q(X) - 1)$$

$$K_i^{CM} = \Phi\left(\sqrt{\frac{1}{1-\rho_i}}\Phi^{-1}(PD_i) + \Phi^{-1}(q)\sqrt{\frac{\rho_i}{1-\rho_i}}\right)$$

Thus

$$w_i = \frac{\Phi\left(\sqrt{\frac{1}{1-\rho}}\Phi^{-1}(PD_i) + \Phi^{-1}(q)\sqrt{\frac{\rho}{1-\rho}}\right) - PD_i}{PD_i(\alpha_q(X) - 1)} \quad (15)$$

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