

## Corporate Investment Choice and Exchange Option between Production Functions

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### ABSTRACT

This paper examines the strategy in resource allocation of a firm which must choose between several production functions. These latter ones can differ by their respective initial investment amounts, input costs, output levels and prices...Such management problem is often posed when input values increase significantly, as for example energy and commodity prices. We determine the values of exchange options when we have to evaluate all the "Profits and Losses" (P&L) depending on the different production models, to take account of potential switches between projects during the management period. We provide a general valuation formula of the exchange options associated to these P&L by using a family of switching options.

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## I. INTRODUCTION

The theory of real options can potentially provide fundamental tools to analyze the investment values, when dealing with research and development projects, natural-resource investments, technological innovations...Real options have several characteristics in common with usual financial options. For example, they can take account of other investment opportunities such as standard financial assets. Using a substitute asset, a risk-premium can be determined and analyzed.

We refer for instance to the seminal paper by Pindyck (1988) on real options and investment choice, Pindyck (1994), Dixit and Pindyck (1994), Trigeorgis (1993a,b,c; 1996) or Schwartz and Trigeorgis (2001) for general properties about real option analysis. Other references are for example Brennan and Schwartz (1985) for valuation of natural resources investments, Paddock et al. (1988) for the case of offshore petroleum leases, McDonald and Siegel (1986) for the value of time to invest..

In this paper, we compare several production strategies of a firm that can decide either to invest only on one single production mode, either to switch between two production strategies. They can have different initial investment amounts, input costs, output levels and prices...Such investment choice is often encountered when the input costs increase significantly, as for example the energy and commodity prices. To analyze such decision problem, we introduce a general exchange option valuation when facing the choice between two investment projects.

We assume that, in a first step, the investor must invest a fixed amount before starting the production. In a second step, when the production begins, the manager must pay production costs. We study the different solutions of this valuation problem, in particular when the manager searches for the maximization of the expected return of his "Profit and Loss" (P&L). We determine the values of exchange options when we have to evaluate all the P&L depending on the different production models, to take account of potential switches between projects during the management period.

We provide a general valuation formula of the exchange options associated to these P&L by using a family of switching options. We assume that the project manager compares two different ways of production: Products can be identical or differ; the manager can choose either to allocate his whole endowment on only one production mode, or to switch from one production mode to the other one.

In that latter case, he must invest initially the required amount, which corresponds to the sum of both amounts necessary to initialize each of the production mode. We detail several particular cases, for example when output levels are equal.

This paper is organized as follows: Section II recalls main results about the valuation of the exchange options for Lognormal asset prices, first introduced by Margrabe (1978). Section III provides the general model and results about the valuation of exchange options between production functions. Finally, Section VI contains the concluding remarks.<sup>1</sup>

## II. EXCHANGE OPTIONS

In what follows, we briefly recall the main result of Margrabe (1978) about the valuation of the exchange option between assets  $X_1$  and  $X_2$ , in order to get  $\text{Max}(X_1, X_2)$ . The idea of Margrabe (1978) is to use the Black and Scholes (1973) formula by considering asset  $X_2$

as a numeraire (see also Stulz, 1982). Thus the price of  $X_1$  corresponds to  $(X_1/X_2)$  units of asset  $X_2$ . Therefore, the exchange option has strike 1 with underlying  $(X_1/X_2)$ . Assume that both  $X_1$  and  $X_2$  are lognormal distributed:

$$\begin{aligned} X_1 &= \exp[m_{X_1} + \sigma_{X_1} W_1] \\ X_2 &= \exp[m_{X_2} + \sigma_{X_2} W_2] \end{aligned} \quad (1)$$

where  $W_1$  and  $W_2$  are Gaussian random variables with variance 1 and linear correlation  $\rho$ . Then, we get:

$$X_1/X_2 = \exp[m_{X_1} - m_{X_2} + \sigma_{X_1} W_1 - \sigma_{X_2} W_2]$$

The volatility of underlying  $\sigma_{(X_1/X_2)}$  is equal to:

$$\sigma_{(X_1/X_2)} = \sqrt{\sigma_{X_1}^2 + \sigma_{X_2}^2 - 2\rho\sigma_{X_1}\sigma_{X_2}} \quad (2)$$

The riskless rate is equal to 0. Using the Black and Scholes (1973) formula for Call options and multiplying by asset  $X_2$ , we get the following result:

$$C = [X_1 N(d_1) - X_2 N(d_2)] \quad (3)$$

where  $d_1 = -[\ln(X_1/X_2) + (\sigma_{(X_1/X_2)}^2/2)\tau]/(\sigma_{(X_1/X_2)}\sqrt{\tau})$ ,  $d_2 = d_1 - \sigma_{(X_1/X_2)}\sqrt{\tau}$ ,  $\tau$ =time to maturity,  $N$ =standard Gaussian cdf.

Consider now the continuous-time valuation. Asset prices are assumed to follow:

$$dX_{i,t} = X_{i,t} [\alpha_i dt + \sigma_i dt + \sigma_i dW_{i,t}] \quad (i=1,2),$$

where the processes  $W_i$  are standard Brownian motions with correlation  $\rho_{1,2}$ . Then, the exchange option:

$$V(X_{1,T}; X_{2,T}; T) = \text{Max}[0, X_{1,T} - X_{2,T}]$$

is valued by using the Black and Scholes (1973) formula:

$$V(X_{1,T}; X_{2,T}; t) = X_{1,t} N(d_1(t)) - X_{2,t} N(d_2(t)), \quad (4)$$

with

$$\begin{aligned} d_1(t) &= -[\ln(X_{1,t}/X_{2,t}) + (\sigma^2(T-t)/2)]/(\sigma\sqrt{T-t}), \quad d_2(t) = d_1(t) - \sigma\sqrt{T-t}, \\ \sigma^2 &= \sigma_1^2 - 2\sigma_1\sigma_2\rho_{1,2} + \sigma_2^2. \end{aligned}$$

### III. THE GENERAL MODEL AND RESULTS

In this section, we first present the formulation of the investment choice problem. Then, we provide valuation formula to determine the best investment strategy. Contrary to the standard exchange option valuation of Margrabe (1978), the payoff options are no longer written on lognormal underlyings but for example written on sum of lognormal random variables, which is a more involved problem. Our problem is linked to the valuation of flexibility option, as illustrated by Trigeorgis and Mason (1987), Triantis and Hodder (1990), Kulatilaka (1988, 1993), Kulatilaka and Trigeorgis (1994), Laughton (1998), Brennan and Trigeorgis (2000), and Duckworth and Zervos (2001).

#### A. The Model

**Notations.** Denote by  $i \in \{1,2\}$  the two basic production modes. The fixed initial cost is denoted by  $C_0^{(i)}$ . The proportional costs (the wages for example) and the input costs are respectively denoted by  $C_t^{(i)}$  and  $P_t^{(i)}$ . The global cost on the whole management period  $[0,T]$  corresponds to  $C_T^{(i)}$ . The variable  $q_t^{(i)}$  is equal to the level of output  $i$  at time  $t$ . Its market price is denoted  $S_t^{(i)}$ . The time period between the beginning of the investment process and the beginning of production is equal to  $T^{(i)}$ .  $T$  denotes the management horizon.

The global cost is defined by:

$$\begin{aligned} C_T^{(i)} &= C_0^{(i)} + \sum_{t=T^{(i)}}^T q_t^{(i)} (c_t^{(i)} [q_t^{(i)}] + P_t^{(i)}) \text{ for } t \geq T^{(i)}, \\ &= C_0^{(i)}, \text{ for } t < T^{(i)}. \end{aligned}$$

Therefore, the P&L associated to mode  $i$  is given by the following for  $t < T^{(i)}$ ,

$$\begin{aligned} P \& L_T^{(i)} &= \sum_{t=T^{(i)}}^T q_t^{(i)} S_t^{(i)} - [c_0^{(i)} + \sum_{t=T^{(i)}}^T q_t^{(i)} (c_t^{(i)} q_t^{(i)} + P_t^{(i)})], \\ &= -C_0^{(i)}, \text{ for } t < T^{(i)}. \end{aligned}$$

We assume that prices follow the dynamics given by:

$$\begin{aligned} dS_t^{(i)} &= S_t^{(i)} [\alpha^{(i)} dt + \sigma^{S^{(i)}} dW_t^{(i)}], \\ dP_t^{(i)} &= P_t^{(i)} [\mu^{(i)} dt + \sigma^{P^{(i)}} dW_t^{(i)}], \end{aligned}$$

where  $\alpha^{(i)}$ ,  $\mu^{(i)}$ ,  $\sigma^{S^{(i)}}$  and  $\sigma^{P^{(i)}}$  are constant parameters,  $W = (W^{(1)}, W^{(2)})$  is a

Brownian motion with  $\langle W^{(1)}, W^{(2)} \rangle_t = \rho t$  ( $\rho$  denotes the correlation coefficient).

Quantities  $q_t^{(i)}$  depend on production modes and respective demands of the two products. They depend also on manager's strategies. They follow:

$$dq_t^{(i)} = q_t^{(i)} [v^{(i)} dt + \sigma^{q^{(i)}} dW_t^{(i)}].$$

### B. The Valuation of Exchange Options between Production Functions

In what follows, we consider a risk-neutral manager. He tries to maximize the expectation of his P&L for a fixed maturity  $T$ .

**Proposition 1 - P&L Expectations.** The P&L expectations are given by:<sup>2</sup>

(1) For  $t \geq T^{(i)}$ ,

$$\begin{aligned} E[P \& L_T^{(i)}] &= \sum_{t=T^{(i)}}^T q_0^{(i)} S_0^{(i)} \exp\left[(\alpha^{(i)} + V^{(i)} + \sigma^{S^{(i)}} \sigma^{q^{(i)}})t\right] \\ &- \sum_{t=T^{(i)}}^T q_0^{(i)} P_0^{(i)} \exp\left[(\mu^{(i)} + v^{(i)} + \sigma^{P^{(i)}} \sigma^{q^{(i)}})t\right] - \sum_{t=T^{(i)}}^T E[q_t^{(i)} c_t^{(i)}(q_t^{(i)})] - C_0^{(i)} \end{aligned}$$

(2) For  $t < T^{(i)}$ ,

$$E[P \& L_T^{(i)}] = -C_0^{(i)}.$$

**Remark 1.** If the cost production  $c_t^{(i)}(q_t^{(i)})$  does not depend on quantity  $q_t^{(i)}$ , we get:

$$\sum_{t=T^{(i)}}^T E[q_t^{(i)} c_t^{(i)}(q_t^{(i)})] = \sum_{t=T^{(i)}}^T q_0^{(i)} \exp(v_t^{(i)}) c_t^{(i)}.$$

Recall that the production function must be concave, which implies that the global cost function  $C$  must be convex.

When for example  $C(q) = qc(q)$  is quadratic, the cost function  $c_t^{(i)}(q_t^{(i)})$  must be linear:  $c_t^{(i)}(q_t^{(i)}) = \psi q_t^{(i)}$ , with  $\psi > 0$ .

In that case, we have:

$$\sum_{t=T^{(i)}}^T E[q_t^{(i)} c_t^{(i)}(q_t^{(i)})] = \psi \sum_{t=T^{(i)}}^T (q_0^{(i)})^2 \exp\left[\left(2v_t^{(i)} + (\sigma^{q^{(i)}})^2\right)t\right].$$

We examine now the valuation of flexibility, i.e., the valuation of the exchange option between the two production functions:

- If the manager chooses initially the mode 1, and wants to get the maximum between the P&L, he must "buy" the option:

$$\text{Max}\left(P \& L_T^{(1)}, P \& L_T^{(2)}\right) - P \& L_T^{(1)} = \text{Max}\left(0, P \& L_T^{(2)} - P \& L_T^{(1)}\right)$$

- If the manager chooses initially the mode 2, and wants to get the maximum between the P&L, he must "buy" the option:

$$\text{Max}\left(P \& L_T^{(1)}, P \& L_T^{(2)}\right) - P \& L_T^{(2)} = \text{Max}\left(0, P \& L_T^{(1)} - P \& L_T^{(2)}\right)$$

Denote respectively by  $V_0^{(i)}$  and  $MG^{(i)}$  the present values of the P&L and of the exchange options.

**Proposition 2 – Parity.** Without friction, the present values of the two production modes satisfy the parity relation:

$$MG^2 - MG^1 = V_0^{(1)} - V_0^{(2)}.$$

**Notation.** In what follows, we denote the two projects by  $i$  and  $i'$  both in  $\{1, 2\}$ .

**Proposition 3 - Exchange Option Valuation.** When production mode  $i$  is selected, the value of the option to get the maximum  $\text{Max}\left(P \& L_T^{(i)}, P \& L_T^{(i')}\right)$  of the two project values  $i$  and  $i'$  is given at any time  $t$  by:

$$MG_t^{(i)} = E_{t,Q}[\text{Max}(0, P \& L_T^{(i)} - P \& L_T^{(i')})],$$

which is also equal to:

$$MG_t^{(i)} = P \& L_T^{(i)} \left[ E_{t,Q} \left[ \text{Max} \left( \frac{P \& L_T^{(i')}}{P \& L_T^{(i)}}, 1 \right) \right] - 1 \right].$$

**Remark 2.** These options can also be priced by using valuation of "basket options".

### C. Basic Examples

Assume that costs  $C_t^{(i)}[q_t^{(i)}]$  are not random:  $C_t^{(i)}[q_t^{(i)}] = C_t^{(i)}$  does not depend on  $q_t^{(i)}$ .

Suppose also that outputs are identical and quantities are equal ( $q_t^{(i)} = q_0$ ,  $v^{(i)} = v$ ).

Due to the risk-neutrality, the manager's objective is to minimize the expectation of the cumulated costs until maturity  $T$ .

We consider five main cases:

- \* option (1): investment only on production mode 1;
- \* option (2): investment only on production mode 2;
- \* option (3): switching between the two production functions but initial investment equal to  $c_0^{(1)} + c_0^{(2)}$  in order to be able to use the two modes. In that case, at any time  $t$ , he can choose the production function that minimizes the production cost;
- \* option (4): choice of mode 1 while buying the option  $MG^{(1)}$  to switch to production 2, so that he gets the maximum of the two P&L at maturity;
- \* option (5): choice of mode 2 while buying the option  $MG^{(2)}$  to switch to production 1 so that he gets the maximum of the two P&L at maturity.

In what follows, we analyze these five cases:

- 1) If he invests only on one mode  $i$ , we get:

$$E[C_T^{(i)}] = C_0^{(i)} + \sum_{t=T^{(i)}}^T q_0 [P_0^{(i)} \exp[(\mu^{(i)} + v)t] + c_t^{(i)} \exp[vt]] \text{ for } t \geq T^{(i)},$$

$$= C_0^{(i)}, \text{ for } t < T^{(i)}.$$

- 2) If he invests on the two modes, he can produce 100% with each mode at any time period  $[t; t+1]$  and choose the mode with minimal production cost during this time period. Denote:

$$T^{(i)} = \text{Min}[T^{(1)}, T^{(2)}] \text{ and } T^{(i')} = \text{Max}[T^{(1)}, T^{(2)}]$$

Therefore, we have:

- (1) For  $T < T^{(i)}$ ,

$$E[C_T^{(3)}] = (C_0^{(1)} + C_0^{(2)}).$$

- (2) For  $T^{(i)} \leq T < T^{(i')}$ ,

$$E[C_T^{(3)}] = (C_0^{(1)} + C_0^{(2)}) + \sum_{t=T^{(i)}}^T q_0 [P_0^{(i)} \exp[(\mu^{(i)} + v)t] + c_t^{(i)} \exp[vt]]$$

- (3) For  $T > T^{(i')}$ ,

$$E[C_T^{(3)}] = (C_0^{(1)} + C_0^{(2)}) + \sum_{t=T^{(i')}}^{T^{(i'-1)}} q_0 [P_0^{(i)} \exp[(\mu^{(i)} + v)t] + c_t^{(i)} \exp[vt]]$$

$$+ \sum_{t=T^{(i')}}^T E[q_t \text{Min}_{i \in \{1,2\}} (C_t^{(i)} + P_t^{(i)})]$$

For this latter case, if additionally,  $q_t$  is deterministic we have to evaluate the expectations:

$$E[\text{Min}_{i \in \{1,2\}}(C_t^{(i)} + P_t^{(i)})].$$

This problem is as follows:

$$E[\text{Min}(a^{(1)} + X^{(1)}, a^{(2)} + X^{(2)})],$$

with

$$X^{(1)} = \exp[m_{X_1} + \sigma_{X_1} W_1], \text{ and } X^{(2)} = \exp[m_{X_2} + \sigma_{X_2} W_2].$$

Using the relation  $\text{Min}(x, y) = y - \text{Max}(0, y - x)$ , we are led to Margrabe options.

**Proposition 4.** If proportional costs are equal ( $C_t^{(i)} = c_t$  does not depend on  $i$ ), we get:

$$E[\text{Min}_{i \in \{1,2\}}(C_t^{(i)} + P_t^{(i)})] = c_t + E[P_t^{(1)}] - V(P_t^{(1)}, P_t^{(2)}, t),$$

where

$$V(P_t^{(1)}, P_t^{(2)}, t) = E[\text{Max}(0, P_t^{(1)} - P_t^{(2)})],$$

which satisfies:

$$V(P_t^{(1)}, P_t^{(2)}, t) = P_t^{(1)} N(d_1(t)) - P_t^{(2)} N(d_2(t)),$$

with  $d_1(t) = [\ln(P_t^{(1)} / P_t^{(2)}) + (\sigma^2(t - T) / 2)] / (\sigma\sqrt{T - t})$ ,  $d_2(t) = d_1(t) - \sigma\sqrt{T - t}$ , and  $\sigma^2 = (\sigma^{P^{(1)}})^2 - 2\sigma^{P^{(1)}} \sigma^{P^{(2)}} \rho + (\sigma^{P^{(2)}})^2$ .

**Proposition 5.** If initial dates are equal ( $T^{(1)} = T^{(2)}$  denoted by  $T_d$ ), all the five options provide the same output value  $\sum_{t=T_d}^T q_t S_t$ . Thus, we have just to compare:

- (1)  $E[C_T^{(1)}] = C_0^{(1)} + \sum_{t=T_d}^T q_0 [P_0^{(1)} \exp[(\mu^{(1)} + v)t] + e^{vt} c_t]$
- (2)  $E[C_T^{(2)}] = C_0^{(2)} + \sum_{t=T_d}^T q_0 [P_0^{(2)} \exp[(\mu^{(2)} + v)t] + e^{vt} c_t]$
- (3)  $C_0^{(1)} + C_0^{(2)} + \sum_{t=T_d}^T q_0 [e^{vt} E[P_t^{(i)} - \text{Max}(0, P_t^{(i)} - P_t^{(i')})]] + e^{vt} c_t]$
- (4)  $E[\text{Min}(C_T^{(1)}, C_T^{(2)})] + MG^{(1)}$ ;
- (5)  $E[\text{Min}(C_T^{(1)}, C_T^{(2)})] + MG^{(2)}$ .

**Remark 3.** The previous relations allow the comparison of the five production modes. For example, option 4 is preferred to option 1 if and only if

$$E[\text{Min}(0; C_T^{(2)} - C_T^{(1)})] + MG^{(1)} \geq 0.$$

The option 5 is preferred to option 2 if and only if (see Bouasker (2010) for more details).

$$E[\text{Min}(0; C_T^{(1)} - C_T^{(2)})] + MG^{(2)} \geq 0.$$

#### D. Numerical Illustration

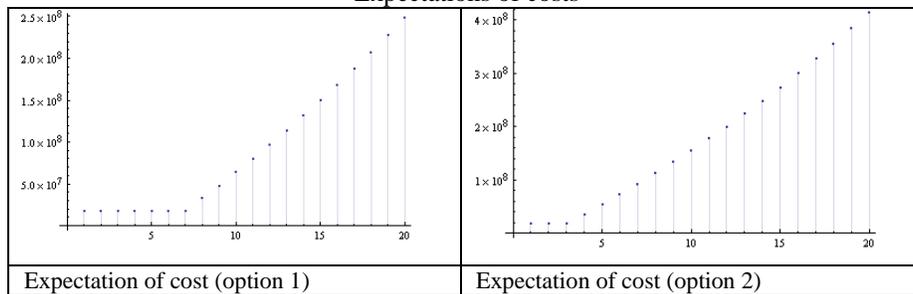
We consider the problem of a firm that produces tires and must choose between two modes of rubber production: the first one is natural (NR); the second one is synthetic (SR).<sup>3</sup>

- The characteristics of the natural rubber production are given by: NR Price (per ton)=1260 US; Cost (per ton)=800 US \$; volatility=24%; initial investment=18 M\$; quantity=15000 tons; time period before production = 7 years.

- The characteristics of the synthetic rubber production (polyisopren) are given by: SR Price (per ton)=1260 US; Cost (per ton)=1060 US \$; volatility=8%; initial investment=18 M\$; quantity=15000 tons; time period before production = 4 years. The volatility NR/SR is equal to 14.5%.

The following figure illustrates the variation of costs expectations with respect to time horizon for options (1) and (2). For a short horizon, option (4) is the best option. For longer horizon, the option (3) can be preferred since the flexibility option is more efficient and can compensate the initial cost. For example, for T=10 years, the flexibility option is about US\$ 0.39/kg.

**Figure 1**  
Expectations of costs



#### IV. CONCLUSION

In this paper, we examine the choice problem between two production functions. The manager can invest only on one project or invest in order to can switch from one project to the other one. We propose a model to evaluate the cost of such option by extending the Margrabe's approach which takes more account of random input prices. Investment costs are divided into an initial cost necessary to start the project and production costs when the production begins. We analyze three main options: investment on only one project; investment to benefit from the switching option at any time during the management period, and finally investment on a given project plus purchase of the option to get the maximum of the P&L at maturity. For sufficiently long time horizon, the options to switch are generally better than the investments on only one project. Further extensions would consider manager with risk aversion, multiple outputs and more complex investment structure to deal with some debt issues.

#### ENDNOTES

1. All proofs are available upon request.
2. Recall that for a lognormal random variable:  $X = e^Y$  where  $Y$  has a Gaussian distribution  $N(M, \sigma)$ , we have  $E[X] = \exp[m + \sigma^2 / 2]$ .
3. Note that demand for rubber is very volatile: for example, 6 million tons during 1996 and 21 million tons in 2006.

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