CPPI Method with a Conditional Floor

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ABSTRACT

We propose an extension of the CPPI method, which is based on conditional floors. In this framework, we examine in particular the margin based strategies. This method allows to keep part of the past gains and to protect the portfolio value against future high drawdowns of the financial market. However, as for the standard CPPI method, the investor can benefit from potential market rises. To control the risk of such strategies, we introduce the Value-at-Risk (VaR) as risk measure. We show that the conditional floor must be higher than a lower bound. We illustrate these results, for a quite general ARCH type model, including the Egarch(1,1) as a special case.

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I. INTRODUCTION

Portfolio insurance strategies allow the investor to control downside risk, while benefiting from market rises. The two main methods of portfolio insurance are: the OBPI (Option Based Portfolio Insurance) introduced by Leland and Rubinstein (1976); the CPPI (Constant Proportion Portfolio Insurance) proposed by Perold (1986)[1] and further developed by Black and Jones (1987) for equity instruments and Perold (1986, 1988) for fixed-income instruments.[2] The CPPI strategy is based on a dynamic portfolio allocation on two basic assets: a riskless asset (usually a treasury bill) and a risky asset (a financial stock index for example). This strategy depends crucially on the cushion C, which is defined as the difference between the portfolio value V and the floor P. This later one corresponds to a guaranteed amount at any time t of the portfolio management period [0,T]. The key assumption is that the amount invested on the risky asset, called the exposure and denoted by e, is equal at any time to the cushion multiplied by a fixed coefficient m, called the multiple. The floor and the multiple can be determined according to the investor's risk tolerance. The higher the multiple, the more the investor will benefit from increases in stock prices. Nevertheless, the higher the multiple, the higher the risk that the portfolio value becomes smaller than the floor if the risky asset price drops suddenly. As the cushion value is approximately equal to zero, exposure is near zero too. In continuous-time, if asset dynamics have no jump, this keeps portfolio value from falling below the floor. The main advantages of this strategy, compared with other portfolio insurance methods, are its simplicity and its flexibility.[3] However, during financial crises, a very sharp drop in the market may occur before the manager can rebalance the portfolio.

Several extensions of the standard CPPI method can be proposed to improve for instance the guarantee robustness: (1) introduction of a conditional multiple depending on the market evolution; and (2) possibility of increasing the floor according to market conditions (ratchet effect, for example, introduced by Boulier and Kanniganti, 2005).

The main objective of this paper is to present and to analyze various CPPI type methods based on conditional floors and within a rather general parametric model. In this model, the floor can be modified according to market fluctuations and portfolio management goals. These extensions allow the investor to make profit from market performance, while for example keeping part of past gains. In that case, an additional portfolio protection is provided. Floor reappraisal allows also better flexibility for CPPI portfolio management. Our approach is based on quantile conditions. Additionally, we do not assume that all risky returns are i.i.d., but we introduce general Arch type models, in particular GARCH type models. We look at the guarantee and performance of the CPPI portfolio for a constant multiple. We compare this model to the standard CPPI method for different values of the multiple.

The paper is organized as follows. Section II provides a brief overview of main properties of the CPPI method in discrete time. Section III deals with conditional floors and risk management of such portfolios. In Section IV, we analyze various CPPI strategies based on conditional floors through simulations and estimations of the S&P 500, from December 2005 to December 2010. Section V provides numerical illustrations. Finally, Section VI contains the main conclusions.

II. THE CPPI METHOD IN DISCRETE TIME

To reduce transaction costs or because of the nature of the funds, investors in the market can choose to trade at predetermined dates. The constant proportion portfolio insurance strategy is based on allocation among two financial assets: a riskless asset, denoted by B, which allows a cash reserve (riskless interest rate denoted by r); a risky asset, denoted by S, which is usually a stock index. The strategies are self-financing. We suppose that rebalancing times are "discrete" along the whole management period [0,T].

The risky asset S evolves according to:

$$S_{t_j} = S_0 \prod_{k=1}^{j} (1 - X_{t_k})$$

where X_{t_k} denotes the opposite of the arithmetical returns:

$$X_{t_{k+1}} = -\frac{\Delta S_{t_{k+1}}}{S_{t_k}} = \frac{S_{t_k} - S_{t_{k+1}}}{S_{t_k}}$$

Denote by V_{t_k} the portfolio value at time t_k. As a portfolio insurance method, the CPPI strategy must satisfy the two following conditions: (see Poncet and Portait, 1997)

- The portfolio value must be higher than a guaranteed amount.
- The investor must benefit partly from market rises.

For these two purposes, the standard CPPI method is based on:

- The choice of a deterministic floor P_{t_k} : at any time t_k, the value V_{t_k} must be higher than P_{t_k} that represents the guaranteed amount.
- The choice of a dynamic investment on the risky asset defined as follows: the total amount e_{t_k} (called "the exposure") invested on the underlying asset S_{t_k} is equal to mC_{t_k}, where the cushion C_{t_k} is equal to the difference between the portfolio value V_{t_k} and the floor P_{t_k} :

$$C_{t_k} = V_{t_k} - P_{t_k}$$

The higher the multiple m, the higher the amount e_{tk} invested on the risky asset. Therefore, an “aggressive” investor would choose high values for m. Nevertheless, in that case, his portfolio is riskier and, as shown in what follows, his guarantee may no longer hold.

The value of the floor gives the dynamically insured amount. It is assumed to evolve according to:

$$P_{t_{k+1}} - P_{t_k} = P_{t_k} r_{t_{k+1}}$$

We deduce that the portfolio value is solution of:

$$V_{t_{k+1}} = V_{t_k} - e_{t_k} X_{t_{k+1}} + (V_{t_k} - e_{t_k}) r_{t_{k+1}}$$

Therefore, the cushion is given by:

$$C_{t_{k+1}} = C_{t_k}\left[1 - mX_{t_{k+1}} + (1-m)r_{t_{k+1}}\right]$$

In fact, since r_{t_k} is relatively small and the time period is usually short, the previous inequality yields to the following relation:

$$C_{t_{k+1}} \Big/ C_{t_k} \approx 1 - mX_{t_{k+1}} = 1 + \frac{\Delta S_{t_{k+1}}}{S_{t_k}}$$

Denote by $R^m_{t_{k+1}}$ the previous return:

$$R^m_{t_{k+1}} = 1 - mX_{t_{k+1}}$$

From this relation we can determine an upper bound on the multiple.

Proposition 1. *The guarantee $C_{t_k} > 0$ is satisfied at any time t_k of the management period with a probability equal to 1 if and only if:*

$$\forall k \le n, X_{t_k} \le \frac{1}{m} \text{ or equivalently } M_n = Max\left(X_{t_k}\right)_{k \le n} \le \frac{1}{m}$$

If the right end point d of the common distribution F of the variables X_{t_k} is positive, the insurance is perfect along any period [0,T] if and only if m is smaller than (1/d).

Many studies have focused on the determination of the multiple m. When the multiple is assumed to be constant, quantile conditions have been introduced to control the probability that the portfolio value is smaller than the floor[4] for a given probability threshold ε. Hamidi et al. (2009) consider a modified quantile hedging strategy where the multiple is conditional. In what follows, we examine extensions of the CPPI method, when the floor is conditional as suggested by Boulier and Kanniganti (2005).

III. CONDITIONAL FLOOR

In practice, portfolio managers do not rebalance their portfolios in continuous time. Their market timing can be based on fixed transaction times (usually, each month) or driven by market events.

For the CPPI strategy, the risk corresponds to sudden financial drops involving negative cushion values. A conditional floor can potentially provide an additional portfolio protection, such as the TIPP strategy. It can also better take account of market fluctuations.

Several extensions have been used to minimize this gap risk. In what follows, we present a general model with conditional floor, based on quantile conditions. We assume that the risky asset logreturn follows a quite general Arch type model.

A. The Financial Model

We introduce the following notations and assumptions:

$$\nabla P_{t_k} = P_{t_k}^{+} - P_{t_k}^{-}$$

where ΔP_{t_k} represents the variation of the floor at time t_k due to the specific choice of the floor.

- The value $P_{t_k}^{-}$ is equal to the previous floor value chosen at time t_{k-1} for the period $\lfloor t_{k-1}, t_k \lfloor$ and invested on the riskless asset during this time period:

$$P_{t_k}^{-} = P_{t_{k-1}}^{+}\left(1 + r_{t_k}\right)$$

- The value P_{tk}^{+} is chosen at time t_k in order to satisfy the portfolio management objectives at that time. In the same way, we define the variations of the cushion at time t_k :

$$\nabla C_{t_k} = C_{t_k}^{+} - C_{t_k}^{-}$$

We deduce the portfolio value dynamics:[5]

$$V_{t_{k+1}} = \left(V_{t_k} - mC_{t_k}^{+}\right)\frac{B_{t_{k+1}}}{B_{t_k}} + mC_{t_k}^{+}\frac{S_{t_{k+1}}}{S_{t_k}}$$

We also assume as usual that the time scale δ is equal to 1/T and that $r_{t_k} = r\delta$. We have:

$$C_{t_{k+1}}^{-} = V_{t_{k+1}} - P_{t_{k+1}}^{-} = V_{t_{k+1}} - P_{t_k}^{+}\exp(r\delta)$$

In what follows, we assume that the logreturn Y of the risky asset S follows a general GARCH(p,q) model.[6] The logreturn Y satisfies:

$$Y_{t_k} = \log\left(\frac{S_{t_k}}{S_{t_{k-1}}}\right), \text{ or equivalently } \frac{S_{t_k} - S_{t_{k-1}}}{S_{t_{k-1}}} = \exp(Y_{t_k}) - 1.$$

The GARCH model is defined as follows. Consider the system of auto regressive equations:

$$\begin{cases} Y_{t_k} = \alpha_0 + \sum_{i=1}^{p} \alpha_i \times Y_{t_{k-i}} + \sigma_k \times \varepsilon_k \\ \Lambda\left(\sigma_{t_k}\right) = \beta + C_0(\varepsilon_{k-1}) + C_1(\varepsilon_{k-1}) \times \Lambda\left(\sigma_{t_{k-1}}\right), \end{cases}$$

where σ_k denotes the volatility, the sequence $\left(\varepsilon_{t_k}\right)_k$ is i.i.d. with common pdf $f > 0$ and Λ, C_0 (.), and C_1 (.) are deterministic functions. The function Λ: $\mathbb{R}^+ \to \mathbb{R}$ is assumed to be strictly increasing.

The information $F_{t_{k-1}}$ at time t_{k-1} delivered by the observation of risky asset returns is generated by the sequence $\left(\varepsilon_{t_1}, \ldots, \varepsilon_{t_{k-1}}\right)_k$.

B. The Quantile Conditions

For an investment period $[0,T]$, the financial market volatility can have several phases (high or low regimes). It is interesting to can adapt the value of the CPPI hedging parameters to these several phases. Thus, in this paper we determine a floor that is subject to market conditions. The aim of this approach is to divide the time management period into several sub-periods and calibrate the floor to the market fluctuations. To meet this objective, we introduce the following risk control conditions, which correspond to various "local" quantile conditions:

- A^0 : $\forall k, P^{G_{t_k}}\left[C^-_{t_k} < 0\right] \leq \varepsilon$
- A^L , with L>0: $\forall k, P^{G_{t_k}}\left[C^-_{t_k} < -L \middle| R^m_{t_{k+1}} < 0\right] \leq \varepsilon$

where G_{t_k} is the $\sigma-$algebra generated by F_{t_k} and the random event $\left\{C^+_{t_1} > 0, \ldots\ldots, C^+_{t_k} > 0\right\}$.

Proposition 2. The previous local quantile conditions imply the following global quantile:

$$P[C^+_{t_1} > 0, \ldots\ldots, C^+_{t_k} > 0] \geq (1-\varepsilon)^T .$$

IV. CONDITIONAL FLOOR UNDER VAR CONSTRAINT

A. The General Model Based on Quantile Condition

We suppose that the floor can be modified at any time t_k, since the portfolio is rebalanced in discrete time. The multiple m is assumed to be constant over time. We consider the "local" quantile condition[7] A^L , with L>0 to revaluate a floor:

$$\forall k, P^{G_{t_k}}\left[C^-_{t_k} < -L \middle| R^m_{t_{k+1}} < 0\right] \leq \varepsilon$$

Denote $f_{\varepsilon_{t_{k+1}}}$ the pdf of $\varepsilon_{t_{k+1}}$ and introduce

$$\begin{cases} a_{t_k} = \alpha_0 + \sum_{i=1}^{k} \alpha_i \times Y_{t_{k-i}} \\ b_{t_k} = \sigma_k, \end{cases} \text{ and } h_{t_k}(m) = \frac{\ln\left[1 - \frac{1}{m}\right] - a_{t_k}}{b_{t_k}}$$

We get a characterization of the quantile condition A^L.[8]

Proposition 3. The quantile condition A^L is equivalent to the following conditions on the conditional floor:

- If $\exp\left(\left(F^{G_{t_k}}\right)^{-1}\left[\alpha \times \int_{-\infty}^{h_{t_k}(m)} f_{\varepsilon_{t_{k+1}}}(x)dx\right] \times b_{t_k} + a_{t_k}\right) \leq 1 - \frac{1}{m}$, then $P^+_{t_k}$ must satisfy the following constraint:

$$P^{+}_{t_k} \geq V_{t_k} + \frac{L}{\left(\exp\left(\left(F^{G_{t_k}}\right)^{-1}\left[\alpha \times \int_{-\infty}^{h_{t_k}(m)} f_{\varepsilon_{t_{k+1}}}(x)dx\right] \times b_{t_k} + a_{t_k}\right) - 1\right)m + 1},$$

since we have necessarily $P^{+}_{t_k} \leq V_{t_k}$.

- If $\exp\left(\left(F^{G_{t_k}}\right)^{-1}\left[\alpha \times \int_{-\infty}^{h_{t_k}(m)} f_{\varepsilon_{t_{k+1}}}(x)dx\right] \times b_{t_k} + a_{t_k}\right) \geq 1 - \frac{1}{m}$, we do not make revaluation of the floor and we have $\Delta P_{t_k} = P_{t_k} r_{t_k}$.

Remark. The previous proposition provides a lower bound on the floor at any time of the management period. This is in accordance to portfolio insurance. Indeed, the higher the floor, the lower the cushion and the exposure to risky asset.

Denote: $\theta_{t_{k+1}} = \left(\exp\left(\left(F^{G_{t_k}}\right)^{-1}\left[\varepsilon \times \int_{-\infty}^{L(m)} f_{\varepsilon_{t_{k+1}}}(x)dx\right] \times b_{t_k} + a_{t_k}\right) - 1\right)m + 1.$

We can interpret previous condition on the conditional floor by using the variation ∇p_{t_k}, which corresponds to the change imposed on the floor at each time t_k.

- If $\theta_{t_{k+1}} < 0$, then we have: $P^{+}_{t_k} - P^{-}_{t_k} \geq V_{t_k} + \frac{L}{\theta_{t_{k+1}}} - P^{-}_{t_k}$.

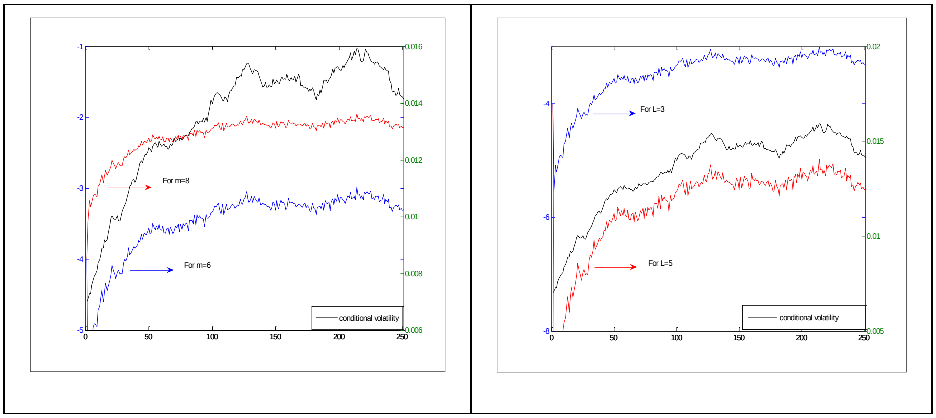
Thus, $\nabla P_{t_k} \leq V_{t_k} - P^{-}_{t_k} + \frac{L}{\theta_{t_{k+1}}}$ and $\nabla P_{t_k} \geq C^{-}_{t_k} + \frac{L}{\theta_{t_{k+1}}}$.

- If $\theta_{t_{k+1}} > 0$, the floor evolves according to the risk-free rate:

$$\nabla P_{t_k} = P^{+}_{t_k} - P^{-}_{t_k} = 0 \quad \text{and} \quad \nabla P_{t_k} = r_{t_k} P_{t_k}.$$

At each time where the constraint $\theta_{t_{k+1}} < 0$ is met, we anticipate that the market will go down, which may induce that the portfolio value becomes smaller than the floor. Then, the change we impose on the conditional floor will imply to increase it, by selling risky assets and buying riskless asset. Within this model, we keep past gains. In the following figures, we illustrate a path of portfolio return and its conditional volatility according to Garch process. We show how the ratio $L/\theta_{t_{k+1}}$ can evolve for the same path during one year and for daily rebalancing. At each time, the value of $\theta_{t_{k+1}}$ depends on the multiple m, on the probability level ε, on the conditional market volatility, and on the threshold of loss that we have imposed. In this section, we obtain a general model for a floor revaluation. Within this model, we conduct revaluation at each rebalancing time to limit exposure. We propose to combine this model based on quantile condition with CPPI strategies based on floor revaluation. For example, we can consider the model of the margin or the model of floor revaluation depending of the portfolio performance previously detailed. In next section, we examine main properties of such models.

Figure 1
Ratio value for several multiple and threshold values



B. Floor Revaluation within the Margin Strategy

This model is associated to a speculative strategy of portfolio management. Indeed, by reducing the margin, we want to increase portfolio exposure to capture any increase in the market. We start from the general result:

$$V_{t_{k+1}} + \frac{L}{\theta_{t_{k+1}}} \le P^{+}_{t_{k+1}}$$

At each time that the cushion is smaller than to some threshold C^*, we reduce the margin according to the proportion γ_{t_k} that we determine.

Proposition 4. (Margin case under VaR condition)

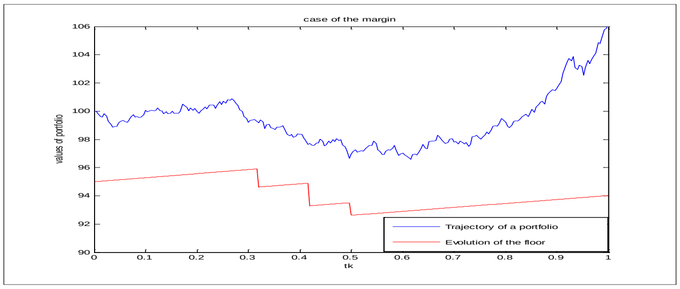
- If $C_{t_k} \le C^*$ then the new value $P^{+}_{t_{k+1}}$ of the floor is equal to the initial floor plus the margin at time t_k. We have: $P^{+}_{t_{k+1}} = \hat{P}_0 \exp\left(r_{t_k}\right) + M_0 \exp\left(r_{t_k}\right)\gamma_{t_k} \ge V_{t_{k+1}} + \frac{L}{\theta_{t_{k+1}}}$. The cushion is equal to: $C^{+}_{t_{k+1}} = V_{t_{k+1}} - P^{+}_{t_{k+1}} = V_{t_{k+1}} - \hat{P}_0 \exp\left(r_{t_k}\right) + M_0 \exp\left(r_{t_k}\right)\gamma_{t_k}$, where $\hat{P}_0$ and M_0 are respectively equal to the initial floor and margin. They both evolve according to the risk-free rate.

$$\gamma_{t_k} \ge \frac{V_{t_{k+1}} - \hat{P}_0 \exp\left(r_{t_k}\right) + \frac{L}{\theta_{t_{k+1}}}}{M_0 \exp\left(r_{t_k}\right)}$$

- If $C_{t_k} \ge C^*$ then: $P^{+}_{t_{k+1}} = P^{-}_{t_{k+1}} \times \exp(r\delta)$. The cushion is equal to: $C^{+}_{t_{k+1}} = V_{t_{k+1}} - P^{-}_{t_{k+1}} \times \exp(r\delta)$.

In the original case of the margin introduced by Boulier and Kanniganti (2005), this proportion is assumed to be constant. This assumption allows the portfolio to not remain sticking to the floor. In this framework, this proportion is variable and based on the quantile condition depending on the values of several variables m, L, C_{t_k} and $\hat{P}_0$ at each time.

Figure 2
Margin case



The use of the margin model can limit the revaluations for cases where exposure is too small. This property allows the portfolio value to take advantage of market rises.

V. NUMERICAL ILLUSTRATIONS

A. The Conditional Volatility Model

Many previous studies have shown the importance of the asymmetric model Garch to estimate the conditional volatility. The asymmetric Garch implies that negative chocks induce greater volatility than positive chocks. Poon and Granger (2003) have analyzed and compared several Garch type models. They conclude that, in general, the asymmetric volatility performs better than Garch. Heynen et al. (1994) conclude that exponential Garch provides the best description of asset prices according to the Akaike information criterion. Chen et al. (2002) test several model to determine the conditional volatility. According to the modified CCK test of Chen (2001), this test can detect asymmetric volatility. Only the Egarch model is accepted for several index prices. Engel and Ng (1993) have shown that the model Egarch can capture most of the asymmetry of the time series but the model presents high conditional variance. Awartani and Corradi (2005) study the daily observation of S&P 500 composite index. They conclude that the asymmetric model Garch gives the best estimation and can capture the leverage effect. From previous empirical observations, we consider the Egarch (1,1) model with parameter values such as in Nelson (1991).

$$y_t = X_t\beta + \varepsilon_t$$
$$\varepsilon_t = Z_t \times \sqrt{h_t},\ Z_t \text{ standard Gaussian iid,}$$

where h_t is the conditional volatility and with

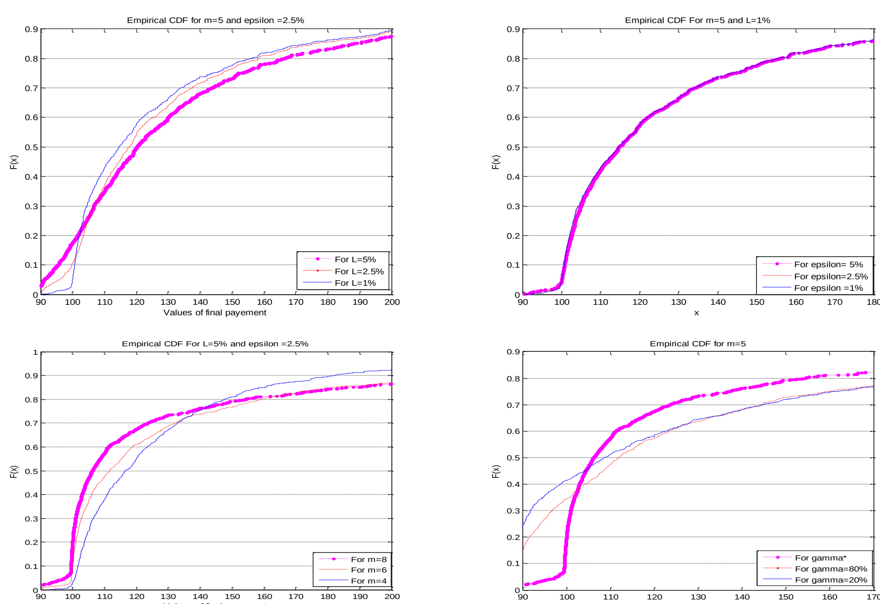
$$\log(h_t) = \alpha_0 + \sum_{i=1}^{q} \alpha_i g(Z_{t-i}) + \sum_{i=1}^{p} \theta_i \log(h_{t-i})$$
$$g(Z_{t-i}) = \theta Z_{t-i} + \gamma\left(|Z_{t-i}| - E|Z_{t-i}|\right)$$

We use this model to estimate and simulate the conditional volatility.

B. The CPPI Margin Strategy under VaR Criterion

In what follows, we illustrate the empirical distribution functions of the portfolio value for a horizon T=5 years, and for several parameter values of the model. There is no stochastic dominance at the first order. With this model, we get higher performances than for the general model. Because revaluations are subject to market conditions, the choice of the parameters L, α and of the multiple m depends on risk aversion of each investor. For $L = 1\% C_0$, we have fixed a binding threshold. On the interval [90,105], this model behaves better than the other ones with less restriction. On the interval [105,200], the other models have better performances. For the other parameters m and α, we conclude in the same manner. Then, there exists a trade-off between the maximization of the gains and the minimization of the risk. In Figure 3, we provide a comparison between the margin case with proportion depending on the market condition that we call γ^* and another margin case with constant proportion. For γ^* we take less risk than for other CPPI models. We note that the CPPI margin strategy yields to better results than an unconditional γ.

Figure 3
Empirical CDF for margin case under VaR criterion

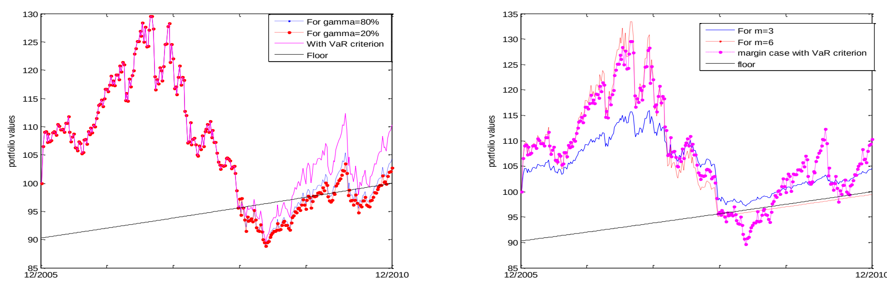


C. Back Testing Model from S&P 500 Data

We estimate the conditional volatility from the Egarch(1,1). In the previous section, we have developed the choice criteria. We choose a very volatile period to test the models: from 12/2005 to 12/2010. We adopt a weekly portfolio rebalancing. In the next graphs, we compare the revaluation floor for the margin case with conditional quantile and expected shortfall criterion to the margin case with fixed proportion and to the standard CPPI model. The margin case is part of speculative strategy of the portfolio insurance, where we adopt an aggressive exposure. However, if the market is bearish, we suffer from large losses. In practice, this strategy is used to avoid finishing with a monetized portfolio. For the margin case with a conditional quantile, market volatility, values of the portfolio and cushion are decisive criteria to select the proportion of the margin to be played again. Interest of this model is that the proportion is not fixed arbitrarily. Beginning the management period with stable market, the proportion determined by the model is high, thus achieving better performance than the margin models with fixed proportion. But this strategy has suffered from higher losses at the end of period. The comparison with standard CPPI shows that this speculative strategy allows better performance than the other models, when the market is bullish. At the end of the period, we take more risk to boost the portfolio by reducing the floor.

Figure 4

Back-testing for the margin case: M_0=8, initial floor $P^{+}_{t_0} = 90$ and m=6.



VI. CONCLUSION

We have proposed an extension of the CPPI method based on local quantile conditions: a model of floor revaluation depending on the market performance and on stock price volatility. This approach allows guaranteeing the portfolio against a potential high volatility market. We have provided an explicit model of portfolio management, which can be easily implemented. It is also possible to involve other state variables. To take the dependence of the yields on the market into account, we have simulated and estimated an asymmetric EGARCH(1,1) model for the conditional volatility and the weekly log-returns. The rebalancing times take place in discrete time. Then, we have compared this model to some standard CPPI models. Before the financial crisis period, margin case portfolio with proportion depending on market volatility performs better than all other models. But, during the crisis, this portfolio falls below the original floor.

The difference between the maximum yields for the two strategies is due to the insurance cost for revaluation strategy depending on portfolio performance. To test our model, we have generated yield paths from Arch type models. However, other processes can be introduced, for example diffusion processes with jumps.

ENDNOTES

1. See Poncet and Portait (1997), Prigent (2007) for more details on these methods.
2. See also Black and Rouhani (1989), Black and Perold (1992).
3. See de Vitry and Moulin (1994), Black and Rouhani (1987) and Boulier and Sikorav (1992).
4. See Prigent (2001) in the Lévy process case and Bertrand and Prigent (2002), using extreme value theory.
5. We suppose that transaction costs are relatively small, so that they can be neglected.
6. The ARCH (Autoregressive Conditionally Heteroscedastic) models, introduced by Engle (1982), are specific non-linear time series models. They can describe quite exhaustive set of risky asset dynamics. They have been widely applied in financial modeling and statistical theory.
7. L is a level guaranteed. It may have several forms. We can take a constant proportion of the initial cushion value (V_0-P_0). Alternatively, we can introduce a variable proportion q_{t_k} of the current cushion value $\left(V_{t_k} - P_{t_k}^{+}\right)$.
8. See Ben Ameur (2009 for detailed proof.

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