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Optimal Pricing with Asymmetric Demands of Senders and Receivers

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ABSTRACT

This paper deals with optimal pricing in the network industry. We adopt the approach of two-way calling among individuals who differ in their utility towards sending and receiving calls, and use a very simple and stylized model which enables us to obtain some solid results. Pricing policies of uniform positive pricing on senders only vs. on both senders and receivers are compared to discriminatory policies including negative pricing on receivers, and the profit values as well as the consumer surplus and welfare values are compared for those pricing policies. We develop applicable results that can be derived from these policy comparisons.

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I. INTRODUCTION

The issue of optimal pricing in the network industry has become very prominent over the last decade. Several papers deal with the beneficial external effects of messages to the receiver. For example a recent paper by Hermalin and Katz (2004) discuss the question of who should pay for electronic messages. They conclude that the receiving party should subsidize the sender who is generating a benefit to the receiver in order to maximize welfare and profits of the firm that supplies the connecting service between the two parties. The above conclusion was first established by Kim and Lim ((2001) and (2002)). Different pricing policy options are introduced that yield higher levels of welfare and profits, and all these are examined in the context of calls/messages externalities. In most of the papers (e.g., Kim and Lim (2001), Kim et al (2002)) the issues of positive price sharing by senders and receivers are considered especially in the context of two-way calling where either party initiates a call, such that in case of communication each individual ("party") can be a sender or a receiver. Although in general in the communication industry the initiator, i.e. the sender, is charged for the message there has recently been a trend towards a receiver based payment principle (see Jeon et.al. (2004), as originally named by Doyle and Smith (1998) and Kim and Lim (2001)) using the term receiver pays principle (RPP).

The sender-receiver market issue has been discussed by Rochet and Tirole (2004). They focused on the question of how to deal with a market where buyers and sellers need to be brought together for the market to exist, as well as on what is the nature of the pricing policy that may lead to an efficient solution. For example, the initiator of the call only gains from communication if the other party i.e., the receiver picks up the phone, thus as both parties benefit both should pay for their communication. The question as to what happens in cases where the benefits to the two sides are not positive and/or are not symmetric, as well as the resulting implications as to pricing policy were not addressed in their paper.

Rochet and Tirole (2004) discussed the differences between (pure) usage pricing and membership pricing where the externalities are created from these two different sources (usage vs. membership).

Another paper that deals with asymmetry between parties, i.e., senders and receivers, who may acquire positive or negative externalities, is that of Loder Van Alstyke and Wash (2006). The authors ask how to deal efficiently with communication between parties that arises from unsolicited and unwarranted contact (such as email spam), termed by the authors as "message pollution". Although undesired by some receivers they may be wanted and useful for others.

In their paper they deal with homogeneous senders and receivers while in our paper we discuss the network market with "dual dimension" asymmetry with different attitudes towards any contact between parties as well as the asymmetry of being a sender vs. that of being a receiver.

Lyn Squire (1973) also discusses the pricing policy that should be used in order to capture the positive external benefit of the receiver, a benefit for which the receiver usually does not have to pay. However a negative price "paid" by the caller as well as asymmetry between senders and receivers were both beyond the author's scope.

M. Armstrong (1998) suggested in his concluding remarks to extend the analysis of network competing companies to the case where subscribers are more heterogeneous and "how the outcomes are affected if networks can target high usage subscribers groups".

Carter and Wright (1999) analyzed the interconnection of price determination in the network industries where competing suppliers need to interconnect to utilize the facilities of their rivals to provide services to their final consumers. This is also of importance due to the asymmetry between industries.

The uniqueness of phone calls, mailing, chatting, fax messages and some (but not all) network instruments is precisely that communicating is achieved by the two parties only if the actual connection occurs (Kim et al (2002) call these goods "ping-pong goods" whose values are generated only through joint consumption). The sender is willing to pay an appropriate price for his phone call only if the receiver actually picks up the phone and establishes communication. Thus for the sender to benefit an appropriate response is required on the part of the receiver and the *ideal solution* can only be achieved if the quantity demanded of sent calls is equal to the quantity demanded of received calls. Otherwise, the actual number of phone calls would be determined by the smaller of the two quantities. Based on the demand curves for sent and received calls the phone company or carrier should adopt a pricing policy that coordinates the behavior of the sender and the receiver so as to bring about equilibrium where quantities demanded and quantities of actual contacts are equal. This is not only in the interest of the sender and the receiver but also in the interest of the company that supplies the service since their revenue from both parties is an increasing function of the actual number of successful communications between parties. If the pricing policy were not adjusted to achieve this kind of equilibrium, revenues would not be maximized.

In our paper we also adopt the approach of two-way calling and assume two individuals who communicate with each other, however, those individuals differ in their approaches towards sending and receiving calls from each other. The asymmetry between the individuals is brought about by different attitudes towards sending and receiving calls from each other, and also by the assumption that the party who initiates communication usually derives more benefit from initiating a call than from receiving a call. One example of this kind of asymmetry is the case of communication between parents and children where it is often likely that parents have a strong preference to send and/or receive calls from their children who are located far away in college or travel, whereas children are often likely to have a much weaker liking or preference towards receiving those calls. A telephone company that is aware of this asymmetry should consider a pricing policy that will not only affect its profits but also may affect the consumer surplus as well as the total welfare of all parties concerned. The possibilities of instituting positive as well as negative pricing imposed on either or both senders and receivers are considered, as well as that of comprehensive price discrimination for the sake of increasing the firm's own profits, and which could also bring about an increase in consumer surplus as well as total welfare.

In 2002 Kim and Lim examined the welfare effect of introducing a receivers payers principle using a simplified model with two representative consumers: a receiver and a caller. They show that the calling price under RPP must be lower than the price under CPP. Under RPP the firm can increase its profit since the price burden is shared between the parties (caller and receiver), the social welfare under RPP is always higher although consumers' surplus does not necessarily increase under the RPP. Doyle and

Smith (1998) seem to be the first paper that deals with the RPP, but they do not address the welfare issue.

Commodities like phone calls, mailing and chatting are typical examples of a class of goods called "Ping-Pong Goods" whose values are generated only through joint consumption by two (or more) people. The decision of participant parties to consume a ping-pong good is usually made based on the share of the price he has to pay, i.e., the demand for the good is affected by the price-sharing rule among the participants. Furthermore the actual communication between caller (sender) and receiver is always determined by the minimal number (value) of calls of either the sender and/or the receiver. The price sharing rule should consider the synchronization between the two desires in order to achieve actual contact between the parties so as to increase profits for the firm and/or social welfare for the economic planner. The interesting contribution of our paper is that of allowing heterogeneous consumers with multi-dimension asymmetry (different senders and receivers as well as different attitudes towards sending and receiving messages).

In two papers of Kim and Lim (2001, 2002) a comparison of three variables is undertaken: (a) the consumer surplus, (b) the profits of the firm that supplies the "ping-pong" product or service and (c) the social welfare (value).

The above values are compared using two different charging methods. The first and most common one is that of only charging the sender/caller (caller pays principle (CPP)) while the other system consists of charging both parties (receiver pays principle RPP). The main conclusion that Kim and Lim (2001) obtain is that under RPP the profits of the firm increased in comparison to CPP, but the consumer surplus changes are ambiguous. In the latter paper they repeat the above conclusion regarding social welfare and definitely say that although the change of consumer surplus as a result of "moving away" from CPP towards RPP is ambiguous, the social welfare always increases under RPP. Those results are based on a simplified assumption of one sender and one receiver (what they call two representative consumers with the same preferences with additively separable utility functions, and the utility from communities of a caller/sender is larger than the utility of the receiver).

We in our model repeat some of the latter assumptions, but go further and assume a more realistic assumption under which individuals are asymmetric in their preferences with respect to sending to each other as well as receiving from each other. This allows us to get different results in comparison between CPP that is introduced in our model as case 2 when only senders of messages pay (sometimes at different prices), and sharing prices (RPP) which we develop in our model as case 4 where receivers also may share and pay (sometimes even at difference prices-- positive or negative) as well.

Based on our assumption the comparison reveals conditions under which (1) profits of case 2 can be either larger or smaller than in case 4, (2) consumer surplus can be larger or smaller in case 2 in comparison to case 4. This conclusion is similar to that of Lim and Kim in both their papers, (3) the social welfare comparison in our model between case 2 and case 4 demonstrate an ambiguous result, sometimes the social welfare in case 2 is larger and sometimes the social welfare is smaller compared to case 4. We use a very simple and stylized model and as a result we obtain some solid results that we believe differs from some of the works mentioned above. We also incorporate the idea that the network industry differs from other markets where the benefit to the consumer from the use of some good or service is independent of the use of other parties. In this sense we may put our discussion also under the externalities umbrella, since "it takes two to tango", i.e., it also takes two to talk on the phone, exchange emails, etc (see Hermalin and Katz 2004). However, the realistic implications of using the above information in applying optimal pricing on receivers and senders takes us further towards some interesting and innovating results.

The structure of the paper is as follows: In the next section we develop a simple model with two individuals who are both senders and receivers in the network market but who behave asymmetrically. Several pricing policies of equal positive pricing on senders only vs. on both senders and receivers are compared to different pricing discrimination policies. In the third section the profit values as well as the consumer surplus and welfare values are compared for those pricing policies that are used in the model. In the concluding section we derive the applicable results that can be derived from the policy comparisons undertaken in the previous sections. We conclude with closing remarks and future research suggestions.

II. THE MODEL

We introduce a simplified case where we have two individuals who differ in their attitude towards sending and receiving calls from each other: As a result we can define four linear demand curves:

$$Q_1^S = S_1 - P_1^S \tag{1}$$

$$Q_2^S = S_2 - P_2^S \tag{2}$$

$$Q_1^R = R_1 - P_1^R \tag{3}$$

$$Q_2^R = R_2 - P_2^R \tag{4}$$

where notations 1 and 2 represent individuals 1 and 2 respectively, R and S represent receiving and sending notations, S_1, S_2 represent the reservation prices of sending calls by individuals 1 and 2 respectively, and R_1, R_2 represent the reservation prices of receiving calls by those individuals. Equilibrium holds:

$$\text{(a)} \quad \text{if } Q_1^S = Q_2^R \text{ and then } P_1^S - P_2^R = S_1 - R_2$$

$$\Rightarrow$$

$$\text{(b)} \quad \text{if } Q_2^S = Q_1^R \text{ and then } P_2^S - R_1^R = S_2 - R_2 > 0$$

The cost function is assumed to be proportional to the number of calls, Q, or $TC = C \cdot Q$, thus $MC = C$.

We assume an asymmetry between individuals based on their different subjective attitudes towards contact with each other. We also assume that individual 1 represents the "agent" who has a greater desire to send as well as to receive more messages in both directions from individual 2, e.g., in a family context individual 1 represents a parent and individual 2 represents a child. The other individual has a lower interest in sending and/or receiving calls to and from individual 1. This context is very realistic in relationships between parents and children and could often also hold in the relationship between married couples, employer and employee, or in the business life between sellers and customers, etc.

Based on the discussion above we assume that:

$$S_1 > S_2, \quad R_1 > R_2 \tag{5}$$

We assume further that:

$$S_1 > R_1 \tag{6}$$

$$\text{and } S_2 > R_2 \tag{7}$$

which says that people usually prefer to initiate calls rather than to receive them (there are times, for example, when it is not convenient for a receiver to be called). From (5) – (7) we also get the condition:

$$S_1 > R_2 \tag{8}$$

However, the relationship between S_2 and R_1 is ambiguous.[1]

In the discussion below we will investigate different pricing policies used by the network/communication company and their effects on its profits as well as on total welfare (calculated by summing the profits of the company plus the consumers' surplus of both senders and receivers).

We will compare the policy of identical prices placed only on senders versus that of adopting a policy of price discrimination (case 1 below). Afterwards we will discuss combined charges placed both on senders and receivers that take the form of either non-discriminating or discriminating pricing (cases 2 and 3 below)

<u>*Case 1:*</u> Only senders pay the same identical price per call.[2]

In this case the maximum calls that are completed can be R_2 for sender 1 and R_1 for sender 2, since both receivers are willing to receive these maximum numbers of calls when they are not paying for received calls.

The question is what is the relationship between Q_1^S at a given price in comparison to R_1, and what is the relationship between Q_2^S in comparison to R_2. The actual number of calls that can be completed and generate revenues to the company network (since we assume that only completed calls are charged and paid for by the senders) is always the minimum between Q_i^S and R_j. Thus, the revenues from individual 1 are:

$$P_1^S \cdot \min\left[S_1 - P_1^S, R_2\right]$$

Since we assume identical pricing, i.e., $P_1^S = P_2^S = P$ and we also assumed above that $S_1 > S_2$ and $R_1 > R_2$ then $Q_1^S > Q_2^S$ for any identical price.

As a result the relationship between R_1 and S_2 is not relevant, thus, we can conclude that if $Q_1^S < R_2$ then $P > S_1 - R_2$ and then $Q_2^S < S_2 - P < S_2 + R_2 - S_1$. The other extreme case is where $Q_1^S = R_2$. Let us first introduce the case where $Q_1^S < R_2$ leads to regular profit maximization:

$$\max_P \pi = (P - C) \cdot [S_1 - P] + (P - C)(S_2 - P) \tag{9}$$

In this case the F.O.C. is

$$\frac{d\pi}{dP} = S_1 - 2P + C + S_2 - 2P + C = 0 \tag{10}$$

Thus, the price at equilibrium is:

$$P = \frac{S_1 + S_2 + 2C}{4} \tag{11}$$

The total number of calls completed for both senders is given by:

$$Q_1^S = S_1 - P = \frac{3S_1 - S_2 - 2C}{4} \tag{12}$$

$$Q_2^S = S_2 - P = \frac{3S_2 - S_1 - 2C}{4} \tag{13}$$

and the total profit earned by the company is:

$$\pi = \frac{(S_1 + S_2 - 2C)^2}{8} \tag{14}$$

The consumer surplus is:

$$\begin{aligned} CS = CS_1^S + CS_2^S + CS_1^R + CS_2^R &= \frac{(3S_1 - S_2 - 2C)^2}{32} + \frac{(3S_2 - S_1 - 2C)^2}{32} + \frac{R_1 \cdot (3S_2 - S_1 - 2C)}{4} - \\ &- \frac{(3S_2 - S_1 - 2C)^2}{32} + \frac{R_2 \cdot (3S_1 - S_2 - 2C)}{4} - \frac{(3S_1 - S_2 - 2C)^2}{32} = \\ &= \frac{R_2 \cdot (3S_1 - S_2 - 2C)}{4} + \frac{R_1 \cdot (3S_2 - S_1 - 2C)}{4} \end{aligned} \tag{15}$$

The second scenario is the extreme case where $Q_1^S = R_2$. In this case the actual price per call charged by the company is: $P = S_1 - R_2$ and the actual number of calls is $Q_1^S = R_2$, while $Q_2^S = S_2 + R_2 - S_1$. Therefore the actual profits earned by the company are:

$$\pi = (S_1 - R_2 - C)\cdot(S_2 + 2R_2 - S_1) \tag{16}$$

and the consumer surplus is:

$$CS = CS_1^S + CS_2^S + CS_1^R + CS_2^R = R_1\cdot(S_2 + R_2 - S_1) + R_2{}^2 \tag{17}$$

Case 2: The second case we are interested in is where only senders are charged but price discrimination is "allowed" or possible. This possibility is not considered by Kim and Lim (2001), since they assume two identical individuals.

In this case we face two decision variables P_1^S and P_2^S, and the general profit function we maximize is:

$$\underset{P_1^S, P_2^S}{\text{Max}}\ \pi = (P_1^S - C)\cdot \min\left[S_1 - P_1^S, R_2\right] + (P_2^S - C)\cdot \min\left[S_2 - P_2^S, R_1\right] \tag{18}$$

Again two cases are considered: (a) The first case is where $Q_1^S < R_2$ and $Q_2^S < R_1$, i.e., the optimal price leads to an internal solution. (b) The second case is where $Q_1^S = R_2$ and $Q_2^S = R_1$ for which the optimal price yields a corner solution. In the first case the profit function is

$$\underset{P_1^S, P_2^S}{\text{Max}}\ \pi = (P_1^S - C)\,(S_1 - P_1^S) + (P_2^S - C)(S_2 - P_2^S) \tag{19}$$

The F.O.C. in this case are

$$\frac{\partial \pi}{\partial P_1^S} = S_1 - 2P_1^S + C = 0 \tag{20}$$

$$\frac{\partial \pi}{\partial P_2^S} = S_2 - 2P_2^S + C = 0 \tag{21}$$

Thus, prices and quantities at equilibrium are:

$$P_1^S = \frac{S_1 + C}{2},\ P_2^S = \frac{S_2 + C}{2},\ Q_1^S = \frac{S_1 - C}{2},\ Q_2^S = \frac{S_2 - C}{2}$$

and the profit the company achieves is: Since $Q_1^S = \frac{S_1 - C}{2} < R_2$ and $Q_2^S = \frac{S_2 - C}{2} < R_1$, we can define the two conditions for an internal solution as follows: (1) $S_1 < 2R_2 + C$

and (2) $S_2 < 2R_1 + C,$ and as a result of these conditions the profit that the company achieves is:

$$\pi = \frac{(S_1 - C)^2}{4} + \frac{(S_2 - C)^2}{4} < R_2^2 + R_1^2 \tag{22}$$

The consumer surplus is:

$$CS = CS_1^S + CS_2^S + CS_1^R + CS_2^R = \frac{R_1 \cdot (S_2 - C)}{2} + \frac{R_2 \cdot (S_1 - C)}{2} \tag{23}$$

In the second case of a corner solution such that $Q_1^S = R_2$ and $Q_2^S = R_1$ The actual prices are: $P_1^S = S_1 - R_2$, $P_2^S = S_2 - R_1$ Therefore the profit of the company is given by:

$$\pi = (S_1 - R_2 - C) \cdot R_2 + (S_2 - R_1 - C) \cdot R_1 \tag{24}$$

and the consumer surplus is:

$$CS = CS_1^S + CS_2^S + CS_1^R + CS_2^R = {R_1}^2 + {R_2}^2 \tag{25}$$

The company has to determine which of the two profit functions (22) or (24) is higher in order to determine which pricing policy should be adopted:
In the case where on the one hand (26) holds:

$$\frac{(S_1 - C)^2}{4} > (S_1 - R_2 - C) \cdot R_2 \quad \text{and this occurs if} \quad R_2 < \frac{S_1 - C}{2} \tag{26}$$

and at the same time (27) holds

$$\frac{(S_2 - C)^2}{4} > (S_2 - R_1 - C) \cdot R_1 \quad \text{which occurs if} \quad R_1 < \frac{S_2 - C}{2} \tag{27}$$

And then internal solution is preferable, otherwise the corner solution exists.

However, it is possible that for individual 1 the internal solution is desirable while the corner solution is adopted for individual 2 and then (28) holds:

$$\pi = \frac{(S_1 - C)^2}{4} + (S_2 - R_1 - C) \cdot R_1 \tag{28}$$

At this stage we compare profit levels for both cases where price discrimination is adopted and compare them to that of simple monopoly pricing:

$$\pi_{\text{No discrimination}} = \frac{(S_1 + S_2 - 2C)^2}{8} \text{ , or} \tag{29}$$

$$\pi_{\text{No discrimination}} = \frac{(S_1 + S_2 - 2C)^2}{8} = \frac{(S_1 - C)^2}{8} + \frac{(S_2 - C)^2}{8} + \frac{(S_1 - C).(S_2 - C)}{4} \tag{29'}$$

whereas under discrimination (30) holds:

$$\pi_{\text{Discrimination}} = \frac{(S_1 - C)^2}{4} + \frac{(S_2 - C)^2}{4} \tag{30}$$

The first two terms of (29') are the same as the first two terms of (30'). The other two terms of (30') are larger than the right-hand term of (29') or (30')

$$\pi_{\text{Discrimination}} = \frac{(S_1 - C)^2}{8} + \frac{(S_2 - C)^2}{8} + \frac{(S_1 - C)^2}{8} + \frac{(S_2 - C)^2}{8} \tag{30'}$$

Thus as expected, $\pi_{\text{Discrimination}} > \pi_{\text{No Discrimination}}$.

<u>*Case 3*</u>: All senders and receivers pay the same prices (which are either positive or negative).

At this stage we investigate the case where both senders pay the same identical fee and possibly a different but identical fee is charged to both receivers. Let us start with the simple case where $Q_1^S = Q_2^R$ and $Q_2^S = Q_1^R$. In this case we get $P^S - P^R = S_1 - R_2 > 0$ and $P^S - P^R = S_2 - R_1 > 0$, thus, $S_2 - R_1 = S_1 - R_2$. This indicates that all Q_i^j *for* $i \neq j$ cannot be equal.

In such a case we face contradictory results. Although the relationship between S_2 and R_1 is ambiguous, still it is obvious that $S_1 > S_2$ and $R_1 > R_2$. The immediate result derived from the above is that $S_1 - R_2 >> S_2 - R_1$ and this contradicts the above conclusion that $S_2 - R_1 = S_1 - R_2$.

Therefore, we have to assume different relationships that lead to the existence of a stable equilibrium. Either

$$Q_1^S = Q_2^R \text{ and } Q_2^S \neq Q_1^R \quad \text{or} \quad Q_2^S = Q_1^R \quad \text{but} \quad Q_1^S \neq Q_2^R$$

If $Q_1^S = Q_2^R$ then $P^R = P^S - S_1 + R_2$.
In such a case the profit function is

$$\underset{P^S, P^R}{\text{Max}} \, \Pi = (P^S + P^R - C) \cdot Q_1^S + (P^S + P^R - C) \cdot \min[Q_2^S, Q_1^R] \tag{31}$$

For $Q_2^S > Q_1^R$ we determine P^R in (31) as a function of P^S. Thus, the appropriate profit function (31') is defined as follows:

$$\pi = \left(2P^S - S_1 + R_2 - C\right) \cdot \left[\left(S_1 - P^S\right) + \left(R_1 - P^S + S_1 - R_2\right)\right] \tag{31'}$$

The F.O.C. is derived with respect to P^S as follows:

$$\frac{d\pi}{dP^S} = 2 \cdot \left(2S_1 + R_1 - R_2 - 2P^S\right) - 2 \cdot \left(2P^S - S_1 + R_2 - C\right) = 0 \tag{32}$$

which leads to the following prices and quantities at equilibrium:

$$P^S = \frac{3S_1 + R_1 - R_2 + C}{4} \quad \text{and} \quad P^R = P^S - S_1 + R_2 = \frac{3R_2 + R_1 - S_1 + C}{4}$$

$$Q_1^S = Q_2^R = \frac{S_1 - R_1 + R_2 - C}{4} \quad \text{and} \quad Q_1^R = \frac{3R_1 - 3R_2 + S_1 - C}{4}$$

Since $P^R = P^S - S_1 + R_2$, we can conclude that P^R can be negative when $I: S_1 > 3R_2 + R_1 + C$.

The total price, *TP*, that the monopoly charges from both parties, sender and receiver, for the same message is:

$$TP = P^S + P^R = \frac{S_1 + R_1 + R_2 + C}{2}$$

Thus the total new profit function is:

$$\pi = (TP - C) \cdot \left(Q_1^S + Q_1^R\right) = \frac{\left(S_1 + R_1 + R_2 - C\right) \cdot \left(S_1 + R_1 - R_2 - C\right)}{4} \tag{33}$$

while the consumer surplus is:

$$CS = CS_1^S + CS_2^S + CS_1^R + CS_2^R = \frac{\left(S_1 - R_1 + R_2 - C\right)^2}{8} + \frac{\left(2S_2 + R_1 - R_2 - S_1 - C\right) \cdot \left(3R_1 - 3R_2 + S_1 - C\right)}{8} \tag{34}$$

Another possibility that should be considered although it is less likely to occur is the case where $Q_2^S < Q_1^R$. In this case the profit function is:

$$\Pi = \left(2P^S - S_1 + R_2 - C\right) \cdot \left[\left(S_1 - P^S\right) + \left(S_2 - P^S\right)\right] \tag{35}$$

The F.O.C. with respect to variable P^S is:

$$\frac{d\pi}{dP^S} = 2\cdot\left(S_1 + S_2 - 2P^S\right) - 2\cdot\left(2P^S - S_1 + R_2 - C\right) = 0 \qquad (36)$$

The prices and quantities of equilibrium are:

$$P^S = \frac{2S_1 + S_2 + C}{4} \qquad (37)$$

$$P^R = \frac{S_2 - 2S_1 + 4R_2 + C}{4} \qquad (38)$$

$$Q_1^S = Q_2^R = \frac{2S_1 - S_2 - C}{4} \qquad (39)$$

$$Q_2^S = \frac{3S_2 - 2S_1 - C}{4} \qquad (40)$$

The total price that is charged by the monopoly from both sender and receiver is:

$$TP = P^S + P^R = \frac{S_2 + 2R_2 + C}{2} \qquad (41)$$

and therefore the profit function in this case is:

$$\pi = (TP - C)\cdot\left(Q_1^S + Q_2^S\right) = \frac{\left(S_2 + 2R_2 - C\right)\cdot\left(S_2 - C\right)}{4} \qquad (42)$$

The consumer surplus is:

$$CS = CS_1^S + CS_2^S + CS_1^R + CS_2^R = \frac{(2S_1 - S_2 - C)^2}{8} + \frac{\left(S_2 + 2R_1 - 2R_2 - C\right)\cdot\left(3S_2 - 2S_1 - C\right)}{8} \qquad (43)$$

At this stage we would like to raise again the possibility that the receiver "pays" a negative price, i.e., $P^R < 0$ for $Q_2^S > Q_1^R$. We show above at condition I that $P^R < 0$ if $3R_2 + R_1 + C < S_1$. This may occur when individual 1 likes to send a very large number of calls, S_i in comparison to the desire to receive calls by both individuals and in order to encourage receiver 2 to accept these messages a subsidy may be called for. Another case that we consider is that of $Q_2^S < Q_1^R$. We find from (38) that $P^R < 0$ if $2R_2 + 0.5(S_2+C) < S_1$. Since $2R_2 + 0.5(S_2+C) < 3R_2+R_1+C$[3] we find that a sufficient condition for $P^R < 0$ is $3R_2+R_1+C < S_1$. This condition guarantees a negative price for all receivers thus encouraging more calls between senders and receivers.

Case 4: Price Discrimination for Sent and Received Calls

In this case we allow a maximum degree of freedom to the monopoly who supplies the communication services in the sense that any price of any level, positive or negative, can be imposed on the two senders or receivers. In this case the appropriate policy can be such that $Q_1^S = Q_2^R$ and $Q_2^S = Q_1^R$, i.e., the pricing policy guarantees full adjustment between calls of senders and receivers.[4] Every message from senders will receive an appropriate response on the part of receivers. The conditions that are required for this purpose are:

$$\text{Condition II:}\quad P_1^S - P_2^R = S_1 - R_2 > 0$$

$$\text{Condition III:}\quad P_2^S - P_1^R = S_2 - R_1 \geq 0$$

The profit function is:

$$\text{Max}\,\Pi = \left(P_1^S + P_2^R\right)\left(S_1 - P_1^S\right) + \left(P_2^S + P_1^R\right)\left(S_2 - P_2^S\right) - C\left(S_1 + S_2 - P_1^S - P_2^S\right) \tag{44}$$

Using conditions II and III above in (44) we can rewrite the profit function in terms of two decision variables P_1^S and P_2^S as follows:

$$\underset{P_1^S, P_2^S}{\text{Max}} = \left(2P_1^S - S_1 + R_2\right)\left(S_1 - P_1^S\right) + \left(2P_2^S - S_2 + R_1\right)\left(S_2 - P_2^S\right) - C\left(S_1 + S_2 - P_1^S - P_2^S\right) \tag{45}$$

The F.O.C. for maximization is derived as follows:

$$\frac{\partial \pi}{\partial P_1^S} = 3S_1 - R_2 - 4P_1^S + C = 0 \tag{46}$$

Therefore we get P_1^S of equilibrium as $P_1^S = \dfrac{3S_1 - R_2 + C}{4}$ since $P_1^S - P_2^R = S_1 - R_2$ we get P_2^R of equilibrium as $P_2^R = \dfrac{3R_2 - S_1 + C}{4}$ which leads to the equilibrium quantities:

$$Q_1^S = Q_2^R = \frac{S_1 + R_2 - C}{4}$$

The second condition is:

$$\frac{\partial \pi}{\partial P_1^S} = S_1 - 2P_1^S + C = 0$$

$$\frac{\partial \Pi}{\partial P_2^S} = 3S_2 - R_2 - 4P_2^S + C = 0 \tag{47}$$

Therefore we get P_2^S of equilibrium as $P_2^S = \dfrac{3S_2 - R_1 + C}{4}$ since $P_2^S - P_1^R = S_2 - R_1$, we get P_1^R of equilibrium as $P_1^R = \dfrac{3R_1 - S_2 + C}{4}$ which leads to the equilibrium quantities:

$$Q_2^S = Q_1^R = \frac{S_2 + R_1 - C}{4}.$$

From the above results we can find the total price charged on messages/calls that were sent from individual 1 to individual 2 as TP^1 and from individual 2 to individual 1 as TP^2.

$$TP^1 = P_1^S + P_2^R \quad \text{or} \quad TP^1 = \frac{S_1 + R_2 + C}{2}$$

The same applies to TP^2 - price for message/call from 2 to 1 is:

$$TP^2 = P_2^S + P_1^R$$
$$TP^2 = \frac{S_2 + R_1 + C}{2}$$

The profit from messages from sender 1 to receiver 2 is Π_1 while that of the messages from sender to receiver 1 is Π_2. The relationship between the two values is ambiguous, i.e.,

$$\Pi_1 = \frac{(S_1 + R_2)^2 - C^2}{8} \overset{?}{\gtrless} \Pi_2 = \frac{(S_2 + R_1)^2 - C^2}{8} \tag{48}$$

The total profit $\Sigma\Pi$ from discriminating pricing on receivers as well as senders is:

$$\text{Total profit:} \quad \sum\Pi = \frac{(S_1 + R_2 - C)^2}{8} + \frac{(S_2 + R_1 - C)^2}{8} \tag{49}$$

and the consumer surplus is

$$CS = CS_1^S + CS_2^S + CS_1^R + CS_2^R = \frac{(S_1 + R_2 - C)^2}{16} + \frac{(S_2 + R_1 - C)^2}{16} \tag{50}$$

At this stage we want to compare the results of case 2 (where only the sender is charged) to case 4 (where both sender and receiver are charged, either by positive or negative pricing).

In case 4 $TP^1 = \dfrac{S_1 + R_2 + C}{2}$ and is distributed between sender and receiver, as follows:

$$P_1^S(\text{case } 4) = \frac{3S_1 - R_2 + C}{4} \text{ and } P_2^R(\text{case } 4) = \frac{3R_2 - S_1 + C}{4}$$

However, (a) $P_1^S(\text{case } 2) = \frac{S_1 + C}{2} = \frac{2R_2 + C - \varepsilon + C}{2} = R_2 + C - \frac{\varepsilon}{2}$ and (b) $P_1^S(\text{case } 4) = \frac{6R_2 + 3C - 3\,\varepsilon - R_2 + C}{4} = \frac{5}{4}R_2 + C - \frac{3}{4}\varepsilon$, where $\varepsilon = 2R_2 + C - S_1$ is the exact difference between S_1 and $2R_2 + C$ since we found above that an internal solution holds if $S_1 < 2R_2 + C$.

Comparing (a) and (b) yields an ambiguous result as to the price burden on senders in both cases, and as a result we also do not know the difference in quantities of messages that are completed in case 4 and case 2. However TP^1 is definitely higher than P_1^S (case 2). The same conclusions can be shown for P_2^S (case 2) vs. P_2^S (case 4) and Q_2^S (case 2) vs. Q_2^S (case 4). However,

$$TP^2 = \frac{S_2 + R_1 + C}{2} \text{ is larger than } P_2^S(\text{case 2}) = \frac{S_2 + C}{2}$$

The final question we wish to ask is about the level of profit in each case. In case 2, the total profit is according to (22) above:

$$\text{II (case 2)} = \frac{(S_1 - C)^2}{4} + \frac{(S_2 - C)^2}{4} = \frac{(2R_2 + C - \varepsilon - C)^2}{4} + \frac{(2R_1 + C - \varepsilon - C)^2}{4}$$
$$= R_2^2 - \frac{\varepsilon^2}{4} + R_1^2 - \frac{\varepsilon^2}{4} \qquad (22')$$

While in case 4 the profit that is generated is as follows:

$$\text{II (case 4)} = \frac{(S_1 + R_2 - C)^2}{8} + \frac{(S_2 + R_1 - C)^2}{8} > \frac{(S_1 + R_2 - C)^2}{9} + \frac{(S_2 + R_1 - C)^2}{9}$$
$$= \frac{(2R_2 + C - \varepsilon + R_2 - C)^2}{9} + \frac{(2R_1 + C - \varepsilon + R_1 - C)^2}{9} =$$
$$= R_2^2 - \frac{\varepsilon^2}{9} + R_1^2 - \frac{\varepsilon^2}{9} \qquad (49')$$

Since $\frac{\varepsilon^2}{4} > \frac{\varepsilon^2}{9}$, we can conclude that II (case 4) >>II (case 2). Q.E.D. However, if $\varepsilon \to 0$ then, II (case 4) approaches II (case 2).

Conclusion: In case 4 where the seller has more degrees of freedom in pricing policy (including negative pricing) he can generate more profits than in case 2. The various equilibrium values generated in the above four cases are summarized in Table1. Based on the results presented in Table 1 above, we now derive some implications and results with to those four basic pricing policies. These four policies consist of: (a) uniform pricing on senders only, (b) discriminatory pricing on senders only, (c) uniform pricing on senders along with a different but uniform price on receivers, (d) discriminatory pricing on senders and receivers.

The comparison between case 1 and case 2 does not reveal anything new that has gone unnoticed in basic microeconomic theory: Uniform price on senders calls (case 1) is lower than the price charged under discrimination (case 2) to sender, whose demand is inelastic and is higher than that of sender 2 whose demand is elastic (since $S_1 > S_2$). The total quantities in both cases are equal and the profit under discrimination is higher than under uniform pricing. The total consumer surplus of all senders and receivers is higher under identical pricing in comparison to price discrimination.

Introducing a policy of charging receivers reveals some interesting results that we discuss below.

Let us start with pricing policies in the case of discrimination, i.e., cases 2 and 4 above. Since $S_1 > S_2$ and $R_1 > R_2$, $S_1 > R_1$ and $S_2 > R_2$ while the relationship between S_2 and R_1 is ambiguous, we can say that when allowing the receiver as well as the sender to be charged, P_1^S *can be* higher when it is imposed only on the sender in comparison to case 4 where it is imposed in a discriminatory fashion on both sender and receiver. The total charge on a call (message) from individual 1 to individual 2 is definitely higher than P_1^S in case 2, i.e., $TP_1 > P_1^S$ of case 2, but the monopoly may reduce the pricing burden on the sender and increase it on the receiver. By allowing both sender and receiver to pay for a call from sender 1 to receiver 2 the supplier of the call prefers to distribute the price burden on both sender and receiver who benefit from the communication between each other. By charging the receiver for his benefit the supplier may reduce the price burden on the sender in comparison to the price the sender is charged in case 2. This encourages the sender to call more often to the receiver which may increase even further the potential profits of the suppliers. It should be emphasized that the possibility that P_1^S of case 2 will be larger than P_1^S of case 4 is more likely to occur when the cost per call is relatively small, which is actually the case. The same conclusion can be reached with regard to $P_2^S < TP_2$ of case 2.

The result of the last series of conclusions is that equilibrium quantities in case 2 are larger for individual 1, but does not necessarily hold for individual 2.

$$Q_1^S \text{ of case } 2 = \frac{S_1 - C}{2} > \frac{S_1 + R_2 - C}{4} = Q_1^S \text{ of case 4, and}$$

$$Q_2^S \text{ of case } 2 = \frac{S_2 - C}{2} \mathrel{\substack{<\\=\\>}} \frac{S_2 + R_1 - C}{4} \text{ since } R_1 \mathrel{\substack{<\\=\\>}} S_2$$

Table 1
Results of pricing policy in 4 cases

Values	Case 1	Case 2	Case 3 for $Q_2^S > Q_1^R$	Case 3 for $Q_2^S < Q_1^R$	Case 4
	Only senders pay the same identical price per call.	only senders are charged and price discrimination is possible	All senders and receivers pay similar prices	All senders and receivers pay similar prices	senders and receivers are charged discriminated prices
P_1^S	$\frac{S_1+S_2+2C}{4}$	$\frac{S_1+C}{2}$	$\frac{3S_1+R_1-R_2+C}{4}$	$\frac{2S_1+S_2+C}{4}$	$\frac{3S_1-R_2+C}{4}$
P_1^R			$\frac{3R_2+R_1-S_1+C}{4}$	$\frac{S_2-2S_1+4R_2+C}{4}$	$\frac{3R_1-S_2+C}{4}$
P_2^S	$\frac{S_1+S_2+2C}{4}$	$\frac{S_2+C}{2}$	$\frac{3S_1+R_1-R_2+C}{4}$	$\frac{2S_1+S_2+C}{4}$	$\frac{3S_2-R_1+C}{4}$
P_2^R			$\frac{3R_2+R_1-S_1+C}{4}$	$\frac{S_2-2S_1+4R_2+C}{4}$	$\frac{3R_2-S_1+C}{4}$
TP_1			$\frac{S_1+R_1+R_2+C}{2}$	$\frac{S_2+2R_2+C}{2}$	$\frac{S_1+R_2+C}{2}$
TP_2			$\frac{S_1+R_1+R_2+C}{2}$	$\frac{S_2+2R_2+C}{2}$	$\frac{S_2+R_1+C}{2}$
Q_1^S	$\frac{3S_1-S_2-2C}{4}$	$\frac{S_1-C}{2}$	$\frac{S_1-R_1+R_2-C}{4}$	$\frac{2S_1-S_2-C}{4}$	$\frac{S_1+R_2-C}{4}$
Q_1^R	$\frac{3S_2-S_1-2C}{4}$	$\frac{S_2-C}{2}$	$\frac{3R_1-3R_2+S_1-C}{4}$	$\frac{3S_2-2S_1-C}{4}$	$\frac{S_2+R_1-C}{4}$
Q_2^S	$\frac{3S_2-S_1-2C}{4}$	$\frac{S_2-C}{2}$	$\frac{3R_1-3R_2+S_1-C}{4}$	$\frac{3S_2-2S_1-C}{4}$	$\frac{S_2+R_1-C}{4}$
Q_2^R	$\frac{3S_1-S_2-2C}{4}$	$\frac{S_1-C}{2}$	$\frac{S_1-R_1+R_2-C}{4}$	$\frac{2S_1-S_2-C}{4}$	$\frac{S_1+R_2-C}{4}$
π	$\frac{(S_1+S_2-2C)^2}{8}$	$\frac{(S_1-C)^2}{4}+\frac{(S_2-C)^2}{4}$	$\frac{(S_1+R_1+R_2-C)\cdot(S_1+R_1-R_2-C)}{4}$	$\frac{(S_2+2R_2-C)\cdot(S_2-C)}{4}$	$\frac{(S_1+R_2-C)^2}{8}+\frac{(S_2+R_1-C)^2}{8}$
CS	$\frac{R_1\cdot(3S_2-S_1-2C)}{4}+\frac{R_2\cdot(3S_1-S_2-2C)}{4}$	$\frac{R_1\cdot(S_2-C)}{2}+\frac{R_2\cdot(S_1-C)}{2}$	$\frac{(S_1-R_1+R_2-C)^2}{8}+$ $+\frac{(2S_2+R_1-R_2-S_1-C)\cdot(3R_1-3R_2+S_1-C)}{8}$	$\frac{(2S_1-S_2-C)^2}{8}+$ $+\frac{(S_2+2R_1-2R_2-C)\cdot(3S_2-2S_1}{8}$	$\frac{(S_1+R_2-C)^2}{16}+\frac{(S_2+R_1-C}{16}$

We wish to further extend our comparisons between the total welfares, *w*, which are defined as the summation of consumers' surplus, CS, plus profits, $\prod$, of cases 2 and 4.

Based on Table 1 we can add up consumer surplus and total profit, CS+II, and find that the total welfare in cases 2 and 4 are as follows:

$$W_2 = \frac{4}{16}\left[(S_1 + 2R_2 - C)(S_1 - C) + (S_2 + 2R_1 - C)(S_2 - C)\right] \tag{51}$$

$$W_4 = \frac{3}{16}\left[(S_1 + R_2 - C)^2 + (S_2 + R_1 - C)^2\right] \tag{52}$$

For internal solutions: $\frac{S_2 - C}{2} < R_1$ and $\frac{S_1 - C}{2} < R_2$. Thus, $S_2 - C + \varepsilon = 2R_1$ and $S_1 - C + \delta = 2R_2$, or

$$\begin{aligned} W_2 &= \frac{4}{16}\left[(4R_2 - \delta)(2R_2 - \delta) + (4R_1 - \varepsilon)(2R_1 - \varepsilon)\right] \\ &= \frac{4}{16}\left[8R_2^2 - 6R_2\delta + \delta^2 + 8R_2^2 - 6R_1\varepsilon + \varepsilon^2\right] \\ &= 2R_2^2 + R_1^2 - 1.5(R_2\delta + R_1\varepsilon) + 0.25\left(\delta^2 + \varepsilon^2\right) \end{aligned} \tag{51'}$$

and

$$\begin{aligned} W_4 &= \frac{3}{16}\left[(3R_2 - \delta)^2(3R_1 - \varepsilon)^2\right] \\ &= \frac{3}{16}\left[9R_2^2 - 6R_2\delta + \delta^2 + 9R_1^2 - 6R_1\varepsilon + \varepsilon^2\right] \\ &= \frac{27}{16}R_2^2 + \frac{27}{16}R_1^2 - \frac{18}{16}(R_2\delta + R_1\varepsilon) + \frac{3}{16}\left(\delta^2 + \varepsilon^2\right) \end{aligned} \tag{52'}$$

Since $\frac{27}{16} < 2, \left|-\frac{18}{16}\right| < |-1.5|$ and $\frac{3}{16} < 0.25$

We cannot conclude whether W_2 is larger or smaller than W_4. This is in spite of the fact that $\Pi_2 < \Pi_4$. The main reason for this is that under Case 2, Consumer Surplus can be larger than the consumer surplus under case 4 where the monopoly has larger degrees of freedom in setting a discriminatory pricing policy that extracts the consumer surpluses of both senders and receivers.

Let us compare the consumers' surpluses in case 2, CS_2, and case 4, CS_4, based on the above relationships guaranteeing internal solutions, i.e., $S_2 - C = 2R_1 - \varepsilon$ and $S_1 - C = 2R_2 - \delta$.

$$CS_2 = \frac{R_1(S_2 - C)}{2} + \frac{R_2(S_1 - C)}{2} = \frac{R_1(2R_1 - \varepsilon)}{2} + \frac{R_2(2R_2 - \delta)}{2}$$
$$= (R_1^2 + R_2^2) - \frac{1}{2}(\varepsilon R_1 + \delta R_2)$$
$$CS_4 = \frac{(S_1 + R_2 - C)^2}{16} + \frac{(S_2 + R_1 - C)}{16} = \frac{(3R_2 - \delta)^2}{16} + \frac{(3R_1 - \varepsilon)}{16}$$
$$= \frac{9}{16}(R_1^2 + R_2^2) - \frac{3}{8}(\varepsilon R_1 + \delta R_2) + \left(\frac{\delta^2 + \varepsilon^2}{16}\right)$$

Since $\frac{9}{16} < 1$, while $\left|-\frac{3}{8}\right| < \left|-\frac{1}{2}\right|$, and since the third term of CS_4 is positive we cannot determine the relationship between CS_2 and CS_4. In the case of a large asymmetry between the desire to send and receive messages, it is more likely that $CS_4 > CS_2$.

Turning back to the comparison between W_2 and W_4 we conclude: Since $2(S_1 + R_2 + S_2 + R_1) = S_1 + (2R_2 + S_1) + S_2 + 2R_1 + S_2$, the bracket value in (51) is smaller than the bracket value in (52). However, the bracket values of (51) and (52) are multiplied by $\frac{4}{16}$ and $\frac{3}{16}$ respectively, therefore we can find different values under which W_2 is larger or equal or smaller than W_4. Because we could not make analytical comparisons between welfare and consumer surpluses of cases 2 and 4, we took various combinations of S_1, S_2, R_1 and R_2, and found by simulations (through the use of Excel) the above comparisons.[5] Based on the analysis and simulations above we introduce below four propositions:

Proposition 1: A large degree of asymmetry between individuals as well as between the desire to send and the desire to receive messages, i.e. for large $(S_1 - R_2)$ and small, or even negative, $(S_2 - R_1)$. An increase in the above asymmetry further increases the profit gap differential between cases 2 and 4 (in favor of case 4).

Proposition 2: A large asymmetry between a strong desire to send and a weak desire to receive messages of both individuals leads to a higher consumer surplus in case 4 than in case 2. When the asymmetries are less significant for both parties more consumer surplus is likely to be gained by charging senders only (case 2) instead of charging both senders and receivers (case 4).

Proposition 3: When we face a close symmetry between individuals 1 and 2 in terms of sending and receiving as well i.e., small gap between S_1 and S_2 and between R_1 and R_2, the gap between the CS of case 2 and case 4 is large ($CS_2 >> CS_4$). Therefore, although the profits of case 4 are always larger than of case 2, nevertheless the total welfare in case 2 is larger than in case 4.

Proposition 4: When the value (S_1 - R_2) is large and the value (S_2 - R_1) is small or negative the welfare conclusion is the reverse of proposition 3, i.e., $W_2 < W_4$. This holds true even if $CS_2 > CS_4$ due to the large difference in profits ($\pi_4 >> \pi_2$).

This reflects the case where individuals are very asymmetric in attitudes: individual 1 has a strong desire to send and to receive from individual 2, while in the reverse direction S_2 and R_2 are small. However, in the very extreme cases discussed above with very strongly asymmetric individuals, both $CS_4 > CS_2$ and $\pi_4 >> \pi_2$, and thus the welfare gap is very large i.e., $W_4 >> W_2$.

Based on our previous assumptions and propositions with regard to the values S_1, R_1, S_2 and R_2, we can derive several additional important conclusions:

(a) The total price, TP_1, charged to the sender and receiver on calls that are initiated by individual 1 is higher than the price charged to sender 1 of case 2. However, the gap between P_1^S charged to sender 1in case 4 and P_1^S charged to sender 1 in case 2 can be even higher if individual 2 is subsidized for calls he receives from individual 1. If this takes place then the number of actual calls between the two individuals in case 4 are more likely to be larger than in case 2.

(b) The same conclusion regarding $TP_2 > P_2^S$ of case 2 holds true as in the previous result that $TP_1 > P_1^S$ of case 2. However, the possibility that P_1^R can be negative is very slim in spite of the fact that the charges on both call senders in case 4 are definitely positive. Similarly, we can say that in spite of the fact that receiving calls is not as important as sending calls, nevertheless the probability that individual 1 will receive the subsidy for receiving calls is very slim whereas such a policy would more often be desirable for calls received by individual 2.

We can claim the following highlights and interesting conclusions:

(c) Not only that $TP_2 > P_2^S$ of case 2 but at the same time P_2^S of case 4 can be negative while P_2^S of case 2 is always positive. This elucidates the possibility that individual 1 who is eager to receive calls from individual 2 is ready to pay a high price of P_1^R such that the company supplying the service can encourage individual 2 to call individual 1.

(d) Although in case 2 (where we apply price discrimination on senders only) it always holds that $P_1^S > P_2^S$ since individual 1 has a higher demand for sent calls, nevertheless in case 4 (where comprehensive price discrimination on senders and receivers hold) the total charge imposed on individual 1 can be lower than the total charge on calls initiated by individual 2, i.e., $TP_2 > TP_1 > 0$. Whether it is higher or lower depends not only on the actual numerical values of R_i's and S_i's, but also on the gap between $(S_1 - S_2)$ and $(R_1 - R_2)$ which measures the attitude gaps of individuals as to sending and receiving messages.

III. CONCLUSIONS

In the concluding section we review and highlight the most important conclusions of the paper. Based on Table 1 we see that the possibility of imposing more comprehensive price discrimination on senders and receivers in some cases can be preferable from the point of view of consumers and producers and social welfare planners. The kind of price discrimination policy that case 4 represents is very flexible in the sense that it permits subsidizing (negative price) the receiver and even possibly the sender in order to promote more profitable and more efficient market solutions. We have mentioned above the possibility of asymmetry in individuals' behavior that may lead to scenarios under which some individuals pay for sending or receiving calls while others are paid either upon receiving calls, or even more surprisingly, on their readiness to send calls.

This may occur for high levels of R_1 combined with low levels of S_2. By doing so we may simultaneously gain higher social welfare, more profit, and larger consumer surplus for both individuals. This situation would justify the policy presented in case 4 which is preferable to that of regular price discrimination on the sender only that was discussed in case 2.

We reach these results by assuming heterogeneity of consumers (which are explained and discussed above) and are adapted to realistic scenarios of our daily life such as asymmetric attitude to send and receive messages between parents and children, and between advertising/commercial senders and receivers, etc. The latter may be more appropriate to a communication market and to where the attitude towards ping-pong games between senders and receivers is asymmetric. These arguments have not appeared, to the best of our knowledge, in the literature of telecommunication pricing and this is our modest contribution to this literature.

These, we believe, are important conclusions especially to industrial and organizational economists and social planners that should be considered as very applicable nowadays to the network industry. This is the case since in order to apply those conclusions the asymmetric behavior of individuals should be measured properly and accurately. Indeed these days it is possible for the network service providers to collect information such as who sends or receives calls along with the prices individuals are willing to pay under different scenarios of location, timing and circumstance. Several more research extensions to our work can be envisioned under which the synchronization between senders and receivers will be more efficient and where discriminatory pricing can be beneficial. For example, using cellular rather than landline phones may affect the asymmetry between individuals because it may affect the availability and the exposure of the parties to more communication activity. Similarly, allowing recorded messages may also affect the amount of communication between the parties as well as decreasing the gaps between the benefits of senders and receivers from actual connections and unscheduled hiccups, etc. This we leave for future research.

APPENDIX A

Since $\begin{array}{l} Q_1^S = S_1 - P_1^S \\ Q_2^R = R_2 - P_2^R \end{array}$, we can conclude that $Q_1^S = Q_2^R$ if $P_1^S - P_2^R = S_1 - R_2$

Since $\begin{array}{l} Q_2^S = S_2 - P_2^S \\ Q_1^R = R_1 - P_1^R \end{array}$, we can conclude that $Q_2^S = Q_1^R$ if $P_2^S - P_1^R = S_2 - R_1$

APPENDIX B

The profit maximization in case 4 is as follows:

$$\underset{\substack{P_1^S, P_2^S \\ P_2^R, P_1^R}}{\mathrm{Max}\Pi} = \left[\begin{array}{l} \left(P_1^S + P_2^R - C\right)\cdot \mathrm{Min}\left[\left(S_1 - P_1^S; R_2 - P_2^R\right)\right] + \\ + \left(P_2^S + P_1^R - C\right)\cdot \mathrm{Min}\left[\left(S_2 - P_2^S; R_1 - P_1^R\right)\right] \end{array}\right. \tag{B.1}$$

In case that $Q_2^S > Q_1^R$ as well as $Q_1^S > Q_2^R$ the derivative of Π with respect to the four prices respectively are:

$$\frac{d\Pi}{dP_1^S} = R_2 - P_2^R \tag{B.2}$$

$$\frac{d\Pi}{dP_2^R} = R_2 - 2P_2^R - P_1^S + C = 0 \tag{B.3}$$

$$\frac{d\Pi}{dP_2^S} = R_1 - P_1^R \tag{B.4}$$

$$\frac{d\Pi}{dP_1^R} = R_1 - 2P_1^R - P_2^S + C = 0 \tag{B.5}$$

From (B.2) and (B.4) we find that both values are positive which indicates that price increases on both senders should increases as long as $Q_1^S > Q_2^R$ and $Q_2^S > Q_1^R$ respectively. These attitudes continue until we approach to $Q_1^S = Q_2^R$, i.e., Q_1^S is reduced and approach to Q_2^R and the same with quantity reduction of Q_2^S towards Q_1^R. Q.E.D.

ENDNOTES

1. We differ from Kim and Lim (2001) who assume two identical consumers. In their view we should assume $S_1 = S_2, R_1 = R_2$ and $S_1 = S_2 > R_1 = R_2$.
2. Caller pays principle in terms of Kim and Lim (2001).
3. See Appendix A for proof.
4. We prove in Appendix B that indeed full adjustment is optimal for the case where $Q_1^S < Q_2^R$ and $Q_2^S < Q_1^R$ (other cases can be demonstrated upon request).
5. These simulations are available upon request.

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