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Constant Proportion Portfolio Insurance Effectiveness under Transaction Costs

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ABSTRACT

In this paper, we examine main properties of the *Constant Proportion Portfolio Insurance* (CPPI) strategy, when trading in continuous-time is not allowed. We focus instead on stochastic-time rebalancing. We prove that investor's tolerance determines crucially portfolio performance, in particular when taking transaction costs into account. We illustrate this feature in the geometric Brownian case and we provide some numerical insights in this framework.

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I. INTRODUCTION

Portfolio insurance allows investors to recover, at maturity, a given percentage of their initial investment, in particular when markets are bearish. One of the main standard portfolio insurance methods is the *Constant Proportion Portfolio Insurance* (CPPI). It has been introduced by Perold (1986), and further developed by Black and Jones (1987) for equity instruments and Black and Perold (1992). This dynamic strategy consists in setting a floor equal to the lowest acceptable value of the portfolio then allocating an amount to the risky asset which is determined as follows: this amount (called the exposure) is equal to the product of the cushion (defined as the excess of the portfolio value over the floor) and of a predetermined multiple. Both the floor and the multiple depend on the investor's risk tolerance. Usually, results about CPPI method are established under the assumption of continuous-time rebalancing. In this framework, the investor can modify his portfolio at any time. For example, if the cushion approaches zero, he reduces his exposure drastically, which keeps portfolio value from falling below the floor.

In this paper, we take account of the impossibility of trading truly in continuous-time. We focus on stochastic-time rebalancing. We prove that the impact of investor's tolerance is important, in particular when transaction costs occur. In Section 2, basic properties about CPPI method are recalled. In Section 3, we consider the case of stochastic time rebalancing with a deterministic target multiple. The investor rebalances his portfolio as soon as the ratio "exposure/cushion" reaches a lower or an upper bound. These bounds can be chosen equal to percentages of a fixed multiple (the target multiple). We provide explicit (or quasi-explicit) formulas for the portfolio values and probability distributions of rebalancing times, when asset price dynamics are driven by a Geometric Brownian motion. In Section 4, we illustrate main properties of such portfolio strategy. Simulations also allow the comparison between these different methods by means of the first four moments and some quantiles.[1]

I. CPPI WITH CONTINUOUS-TIME REBALANCING

A. The Standard Financial Model

The portfolio manager is assumed to invest in two basic assets: a money market account, denoted by B, and a portfolio of traded assets such as a composite index, denoted by S. The period of time considered is [0,T]. The strategies are self-financing. The value of the riskless asset B evolves according to:

$$dB_t = B_t r dt,$$

where r is the deterministic interest rate. The dynamics of the risky asset price S are given by a diffusion process:[2]

$$dS_t = S_{t-}\left[\mu(t,S_t)dt + \sigma(t,S_t)dW_t\right],$$

where W is a standard Brownian motion.

B. The Standard CPPI Method

This strategy consists in managing a dynamic portfolio so that its value is above a floor P at any time t of the management period. The value of the floor indicates the dynamic insured amount. It is assumed to evolve according to:

$$dP_t = P_t r dt$$

Obviously, the initial floor P_0 is smaller than the initial portfolio value V_0. The difference (V_0- P_0) is called the cushion. It is denoted by C_0. Its value C_t at any time t in [0,T] is given by:

$$C_t = V_t - P_t$$

Denote e_t as the exposure. It is the total amount invested in the risky asset. The standard CPPI method consists in letting e_t=mC_t where m is a constant called the multiple. The interesting case is when m>1, that is, when the portfolio profile is convex. Thus, the CPPI method is parametrized by P_0 and m. Note that the multiple must not be too high as shown for example in Prigent (2001a) or in Bertrand and Prigent (2002). The cushion value at any time is given by:

$$C_t = C_0 \exp\left[(1-m)rt + m\left[\int_0^t \left(\mu - (1/2)m\sigma^2(s,S_s)\right)ds + \int_0^t \sigma(s,S_s)dW_s\right]\right]$$

Consequently, the guarantee is satisfied since the cushion is always non negative.[3] When μ and σ are constant, the cushion value is given by:

$$C_t = C_0 e^{m\sigma W_t + [r + m(\mu - r) - (m^2\sigma^2)/2]t} \quad \text{with } C_0 = V_0 - P_0$$

In this case, the cushion value and the portfolio value are independent of the risky asset paths. The insurance is perfect. Their probability distributions are lognormal (up to a translation for the portfolio value) with a volatility equal to mσ. The instantaneous mean rate of return is equal to r+m(μ-r). The multiple m can be viewed as a weight in the volatility and in the excess of return (μ-r). The value V_t of the portfolio is given by:

$$V_t = V_t(m, S_t) = P_0.e^{rt} + \alpha_t.S_t^m, \text{ where}$$

$$\alpha_t = \left(C_0 / \left(S_0^{\ m}\right)\right)\exp[\beta t],$$

and

$$\beta = \left[r - m\left(r - (1/2)\sigma^2\right) - m^2\left(\sigma^2/2\right)\right]$$

Then, the portfolio value has mean and variance, which are respectively given by:

$$E[V_t] = (V_0 - P_0)e^{[r+m(\mu-r)]t} + P_0 e^{rt},$$

$$Var[V_t] = (V_0 - P_0)^2 e^{2[r+m(\mu-r)]t}\left[e^{m^2\sigma^2 t} - 1\right]$$

II. CPPI WITH STOCHASTIC-TIME REBALANCING

In previous section, the investor is assumed to continuously rebalance his portfolio. In practice, this rebalancing cannot be made at any time of the management period and the impact of the market timing has to be analyzed, in particular when there are transaction costs. One of the standard method is to fix a target multiple m and to rebalance the portfolio as soon as the value of the ratio "exposure/cushion" is smaller than m(1-τ) or higher than m(1+τ). This method implies to rebalance the portfolio along a sequence of increasing random times $(T_n)_n$.[4] In what follows, we examine the problem when the target multiple is deterministic.

A. The Model

When the cushion rises, the exposure can reach the maximum level that the investor wants to invest or the minimum level that he requires. While the exposure lies between these two bounds, he does not trade. Otherwise, for example when market fluctuations are significant, he may rebalance his portfolio in order to keep the ratio exposure/cushion within a given set of values. For this purpose, he can define a tolerance to market fluctuations which determines the two bounds on percentages of variations. Introduce the lower bound m and the upper bound m on the multiple m. The investor begins by investing a total amount V_0 and by setting a given initial floor P_0. The share θ_0^S invested on the underlying S and the share θ_0^B invested on the riskless asset B are given by:

$$\theta_0^S = m(V_0 - P_0)/S_0 \text{ and } \theta_0^B = (V_0 - m(V_0 - P_0))/B_0.$$

The portfolio value $V_{T_{n+1}}^-$ (before rebalancing) at each time T_{n+1} is equal to:

$$V_{T_{n+1}}^- = \theta_{T_n}^{B+} B_{T_{n+1}} + \theta_{T_n}^{S+} S_{T_{n+1}}.$$

Note that $\theta_{T_n}^{B+} = \theta_{T_{n+1}}^{B-}$ and $\theta_{T_n}^{S+} = \theta_{T_{n+1}}^{S-}$. Thus, we have also:

$$V_{T_{n+1}}^- = \theta_{T_{n+1}}^{B-} B_{T_{n+1}} + \theta_{T_{n+1}}^{S-} S_{T_{n+1}}.$$

However, the goal of the CPPI strategy is to keep an amount $e_{T_{n+1}}$ of risk exposure that is proportional to the cushion:

$$e_{T_{n+1}}^+ = mC_{T_{n+1}}^+ \text{ with } C_{T_{n+1}}^+ = \left(V_{T_{n+1}}^+ - P_{T_{n+1}}\right).$$

This latter condition allows the determination of the quantities $\theta_{T_{n+1}}^{S+}$ and $\theta_{T_{n+1}}^{B+}$ to invest during the period $]T_{n+1},T_{n+2}[$. The portfolio value $V_{T_{n+1}}^+$ at time T_{n+1} (after rebalancing) is given by:

$$V_{T_{n+1}}^+ = \theta_{T_{n+1}}^{B+} B_{T_{n+1}} + \theta_{T_{n+1}}^{S+} S_{T_{n+1}}.$$

We suppose that there exist transaction costs which are proportional to the risky amount variation (the transaction cost rate is denoted by γ). We assume that these costs are null at time T_0. At each rebalancing time T_{n+1}, the portfolio value $V^{-}_{T_{n+1}}$ is reduced by the amount of transaction costs equal to:

$$\gamma\left|\theta^{S+}_{T_{n+1}} - \theta^{S-}_{T_{n+1}}\right| S_{T_{n+1}}.$$

Therefore, the portfolio value $V^{+}_{T_{n+1}}$ (after rebalancing) is given by:

$$V^{+}_{T_{n+1}} = V^{-}_{T_{n+1}} - \gamma\left|\theta^{S+}_{T_{n+1}} - \theta^{S-}_{T_{n+1}}\right| S_{T_{n+1}}.$$

Proposition 1. The quantity $\theta^{S+}_{T_{n+1}}$ invested on the risky asset, after rebalancing at time T_{n+1}, is determined from a buy/sell condition. We obtain:

$$\text{If we buy: } \theta^{S+}_{T_{n+1}} > \theta^{S-}_{T_{n+1}}, \text{then } \theta^{S+}_{T_{n+1}} = \frac{m(V^{-}_{T_{n+1}} + \gamma\theta^{S+}_{T_n} S_{T_{n+1}} - P_{T_{n+1}})}{(1+\gamma m) S_{T_{n+1}}}.$$

$$\text{If we buy: } \theta^{S+}_{T_{n+1}} < \theta^{S-}_{T_{n+1}}, \text{then } \theta^{S+}_{T_{n+1}} = \frac{m(V^{-}_{T_{n+1}} - \gamma\theta^{S+}_{T_n} S_{T_{n+1}} - P_{T_{n+1}})}{(1-\gamma m) S_{T_{n+1}}}.$$

Proposition 2. (Characterization of the buy/sell condition) Assume that, at time T_{n+1}, we have: $m > 1$, $0 < \gamma < 1/m$ and the cushion value satisfies: $C^{+}_{T_n} > 0$, Then, we deduce the following equivalence:

$$\text{Buy condition: } \theta^{S+}_{T_{n+1}} > \theta^{S-}_{T_{n+1}} \Leftrightarrow \frac{\Delta S_{T_{n+1}}}{S_{T_n}} > \frac{\Delta B_{T_{n+1}}}{B_{T_n}},$$

$$\text{Sell condition: } \theta^{S+}_{T_{n+1}} < \theta^{S-}_{T_{n+1}} \Leftrightarrow \frac{\Delta S_{T_{n+1}}}{S_{T_n}} < \frac{\Delta B_{T_{n+1}}}{B_{T_n}},$$

$$\text{No buy/sell: } \theta^{S+}_{T_{n+1}} = \theta^{S-}_{T_{n+1}} \Leftrightarrow \frac{\Delta S_{T_{n+1}}}{S_{T_n}} = \frac{\Delta B_{T_{n+1}}}{B_{T_n}}.$$

We now determine the probability distribution of the rebalancing times.

B. Rebalancing Times

We begin by determining the first rebalancing time. Since usually the amount $\theta^{B}_{0} B_0$ invested on the riskless asset is smaller than the initial floor P_0, then the rebalancing condition is determined as follows. At time t = 0, we have:

$$S_0\theta^{S}_{0} = m(V_0 - P_0), \theta^{S}_{0} = m(V_0 - P_0)/S_0 \text{ and } \theta^{S}_{0} B_0 + \theta^{S}_{0} S_0 = V_0.$$

Denote T_1 as the first rebalancing time. If $t < T_1$, then the portfolio value, the cushion value, and the exposure are respectively equal to:

$$V_t = \theta_0^B B_t + \theta_0^S S_t,\ C_t = V_t - P_0 e^{rt},\ \text{and}\ e_t = \theta_0^S S_t.$$

The condition that determines the rebalancing time corresponds to the first time T_1 at which the ratio *exposure/cushion* is lower than a lower bound $\underline{m}$ or higher than an upper bound $\overline{m}$:

$$\underline{m} \leq \frac{e_t}{C_t} \leq \overline{m}.$$

This is equivalent to:

$$\underline{m} \leq \left(\theta_0^S S_t\right) / \left(\theta_0^B B_t + \theta_0^S S_t - P_0 e^{rt}\right) \leq \overline{m},$$

which also means:

$$\frac{\overline{m}\left(P_0 - \theta_0^B B_0\right)}{\left(\overline{m}-1\right)\theta_0^S} \leq S_t e^{-rt} \leq \frac{\underline{m}\left(P_0 - \theta_0^B B_0\right)}{\left(\underline{m}-1\right)\theta_0^S}.$$

Setting $X_t=\ln(S_t/S_0)-rt$, we deduce that there exist two constants A and B such that T_1 is equal to the first time at which condition $A \leq X_t \leq B$ is no longer satisfied.

Proposition 3. (First rebalancing time) The first rebalancing time corresponds to the first time at which the process X defined by: $X_t = \ln\left(S_t / S_0\right) - rt$ escapes from the corridor [A,B] where A and B are two constants defined from the equivalence:

$$\underline{m} \leq \frac{e_t}{C_t} \leq \overline{m} \Leftrightarrow A \leq X_t \leq B.$$

Suppose that both the lower and the upper bounds on the multiple are determined as follows:

$$\underline{m} = m(1-\tau)\ \text{and}\ \overline{m} = m(1+\tau),$$

where m denotes the target multiple and τ denotes the investor's tolerance with respect to the target multiple. In that case, the two constants A and B are only functions of the target multiple m and the rebalancing tolerance τ. They are respectively given by:

$$A(\tau, m) = \mathrm{Ln}\left[\frac{\overline{m}}{\overline{m}-1}\frac{P_0 - \theta_0^B B_0}{m\left(V_0 - P_0\right)}\right] = \mathrm{Ln}\left[\frac{m-1}{m - \frac{1}{1+\tau}}\right]$$

$$B(\tau, m) = \mathrm{Ln}\left[\frac{\underline{m}}{\underline{m}-1}\frac{P_0 - \theta_0^B B_0}{m\left(V_0 - P_0\right)}\right] = \mathrm{Ln}\left[\frac{m-1}{m - \frac{1}{1-\tau}}\right]$$

Consider now the Geometric Brownian case, where the asset price S is given by:

$$S_t = S_0 \exp((\mu - (1/2)\sigma^2)t + \sigma W_t).$$

Thus, the process X is a Brownian motion with drift, defined by:

$$X_t = (\mu - r - 1/2\sigma^2)t + \sigma W_t.$$

The conditional distribution of time rebalancing is characterized by the property that the Brownian motion with drift goes beyond the corridor [A,B]. This probability can be deduced from the trivariate distribution of the running maximum, minimum and terminal value of the Brownian motion (See Revuz and Yor, 1994) after an appropriate change of probability to eliminate the drift.[5] Recall that the density of this joint law in the presence of a constant drift ρ is defined for all values of x in [A,B] by:

$$g(x, A, B) = \exp[(\rho x / \sigma^2) - (\rho^2 t/(2\sigma^2))] \times \sum_{n=-\infty}^{n=+\infty} \left(\frac{1}{\sigma\sqrt{t}}\right)\phi\left(\frac{x - 2n(B-A)}{\sigma\sqrt{t}}\right) - \phi\left(\frac{x - 2n(B-2A)}{\sigma\sqrt{t}}\right),$$

where ϕ is the probability density function (pdf) of the centered and reduced Gaussian distribution and N is its cumulative distribution function (cdf). If $A < 0$ and $B > 0$, then the distribution of the first passage time T_1 is given by:

$$P[T_1 \le t] = 1 - P\left[\underset{s\le t}{\text{Max}}\, X_s \le B, \underset{s\le t}{\text{Min}}\, X_s \ge A\right],$$

with

$$P[\text{Max}_{s\le t} X_s \le B, \text{Min}_{s\le t} X_s \ge A] = \sum_{n=-\infty}^{+\infty} e^{2n\rho(B-A)/\sigma^2}\left[N\left(\frac{B - \rho t - 2n(B-A)}{\sigma\sqrt{t}}\right) - N\left(\frac{A - \rho t - 2n(B-A)}{\sigma\sqrt{t}}\right)\right] - \sum_{n=-\infty}^{+\infty} e^{2A\rho/\sigma^2}\left[N\left(\frac{B - \rho t - 2n(B-A) - 2A}{\sigma\sqrt{t}}\right) - N\left(\frac{A - \rho t - 2n(B-A) - 2A}{\sigma\sqrt{t}}\right)\right]$$

III. NUMERICAL ILLUSTRATIONS

In this section, first we examine some properties of the portfolio returns, and then we analyze the distributions of the rebalancing times. Our numerical base case is as follows:

$$r = 3\%, T = 1(\text{year}), \sigma = 20\%, \mu = 10\%, p = 95\%.$$

Table 1 provides the first four moments of the portfolio return. We indicate the expectation and standard deviation of the return, and the skewness and kurtosis of the log return (to better illustrate the comparison with the Gaussian distribution). We analyze how these moments depend on both the tolerance and the transaction cost rate.

Table 1
First four moments of returns

	μ=10% and m=6					μ=10% and m=4				
	Sensitivities of Mean portfolio return (%)									
τ\γ (%)	0	1	2	3	4	0	1	2	3	4
0	7,092	-4,900	-4,990	-5,000	-5,000	5,598	-3,440	-4,770	-4 ,960	-4,990
5	7,087	-0,820	-3,590	-4,540	-4 ,860	5,590	3,109	1,196	-0,280	-1,440
10	7 ,071	1,936	-1,050	-2,810	-3,830	5,581	4,246	3,072	2,031	1,103
15	7,014	3,324	0,728	-1,110	-2,430	5,539	4,649	3,829	3,068	2,357
20	6,917	4 ,109	1,928	0,211	-1,150	5 ,519	4,879	4,275	3,701	3,150
25	6,867	4,674	2,858	1,331	0,033	5,460	4,975	4,511	4,063	3,627
30	6,799	5,064	3,562	2,242	1,065	5,403	5,033	4,674	4,324	3,979
35	6,693	5,310	4,072	2,945	1,905	5,298	5,007	4,722	4,441	4,161
40	6,554	5,452	4,440	3,495	2,598	5,253	5,033	4,814	4 ,596	4,377
	Standard deviation of portfolio return									
0	0,202	0,002	0,000	0,000	0,000	0,098	0,014	0,002	0,000	0,000
5	0,202	0,071	0,025	0,008	0,003	0,098	0,075	0,058	0,044	0,034
10	0,199	0,116	0,067	0,039	0,022	0,096	0,084	0,074	0,065	0,057
15	0,193	0,134	0,093	0,065	0,045	0,094	0,086	0,079	0,073	0,067
20	0,187	0,144	0,111	0,086	0,067	0,092	0,087	0,082	0,077	0,073
25	0,181	0,148	0,122	0,101	0,084	0,089	0,085	0,081	0,078	0,075
30	0,173	0,149	0,128	0,112	0,098	0,085	0,082	0,079	0,077	0,075
35	0,163	0,144	0,128	0,115	0,104	0,081	0,079	0,077	0,075	0,073
40	0,153	0,139	0,127	0,117	0 ,108	0,078	0,076	0,074	0,073	0,072
	Skewness of portfolio logreturn									
0	3,452	6,751	7,253	7,816	8,591	2,302	2,976	3,197	3,294	3,387
5	3,455	4,749	6,257	7,892	9,657	2,290	2,415	2,541	2,669	2,803
10	3,421	4,047	4,761	5,587	6,560	2,246	2,294	2,347	2,407	2,475
15	3,316	3,648	4,016	4 ,447	4,983	2,197	2,219	2,247	2,282	2,326
20	3,277	3,527	3,823	4,196	4,690	2,114	2,113	2 ,118	2,129	2,146
25	3,181	3,346	3,540	3,781	4,087	2,024	2,010	2,001	1,996	1,996
30	3,079	3,192	3,333	3,514	3,751	1 ,885	1,856	1,830	1,808	1,790
35	2,911	2,967	3,042	3,143	3 ,279	1,765	1,729	1,696	1,666	1,639
40	2,805	2,832	2,872	2,931	3,012	1,626	1,585	1,547	1,511	1,479
	Kurtosis of portfolio logreturn									
0	21,29	85,23	98,960	114,80	137,70	11,55	18 ,61	21,53	22,93	24,27
5	21,43	41,25	75,229	123,80	186,20	11,44	12,54	13,71	14,97	16,36
10	21,03	29,33	40,905	56,95	79,31	11,11	11,52	11,97	12,49	13,11
15	19,82	23,87	28,995	35,84	45,66	10,79	11,00	11,25	11,56	11,95
20	19,64	22,77	26,912	32 ,71	41,42	10,18	10,19	10,23	10 ,31	10,43
25	18,66	20,61	23,063	26,25	30,58	9,623	9,541	9,481	9,445	9,436
30	17,67	18,97	20,625	22,79	25,73	8,614	8,412	8,228	8,064	7,918
35	16,13	16,77	17,584	18,65	20,07	7,929	7,702	7,489	7,290	7,105
40	15,34	15,62	15,999	16,50	17,17	7,210	6,982	6,766	6,561	6,366

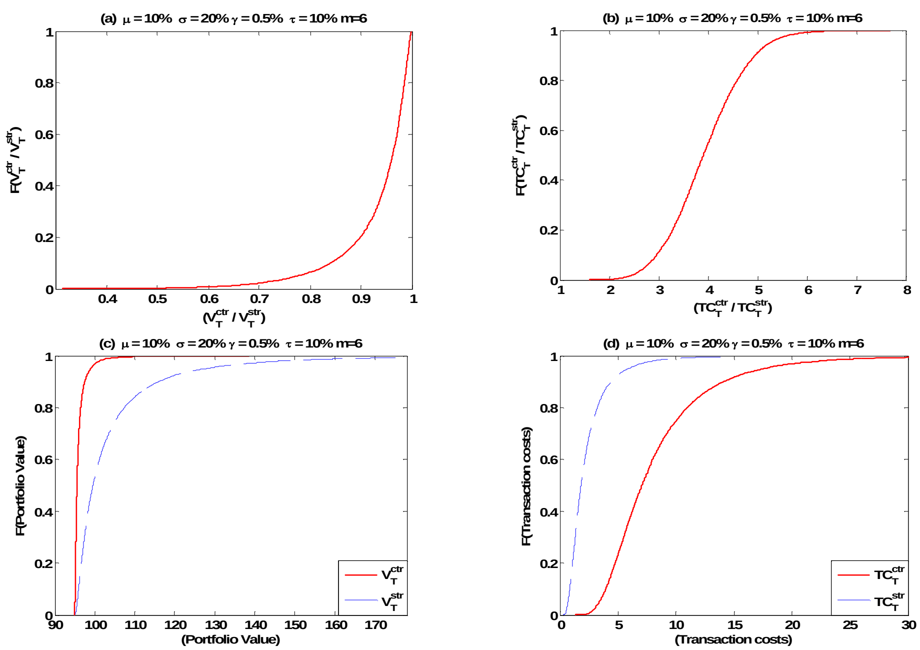
We note that, when there is no transaction cost, the expected return is decreasing w.r.t. the tolerance and increasing w.r.t. the multiple. For a transaction cost rate equal to or higher than 1%, it is the converse. The skewness of logreturn is always positive and the values of the kurtosis show that the logreturn distribution is not Gaussian (they are all higher than 3).

In Table 1, both skewness and kurtosis are increasing w.r.t. the multiple and are decreasing w.r.t. tolerance. We compare now the returns of both the continuous-time rebalancing portfolio value V_T^{ctr} (which corresponds to $\tau = 0\%$) and the stochastic-time rebalancing portfolio value V_T^{str}. We examine the distribution of the ratio V_T^{ctr} / V_T^{str}. Note that its cdf depends on γ and m:

$$F_{(\gamma,m)}(x) = P\left[\frac{V_T^{ctr}(\gamma,m)}{V_T^{str}(\gamma,m)} \leq x\right].$$

Figures 1(c,d) show that there exists a stochastic dominance at the first order between portfolios values V_T^{ctr} and V_T^{str} (with transaction costs), and also between TC_T^{ctr} and TC_T^{str}.

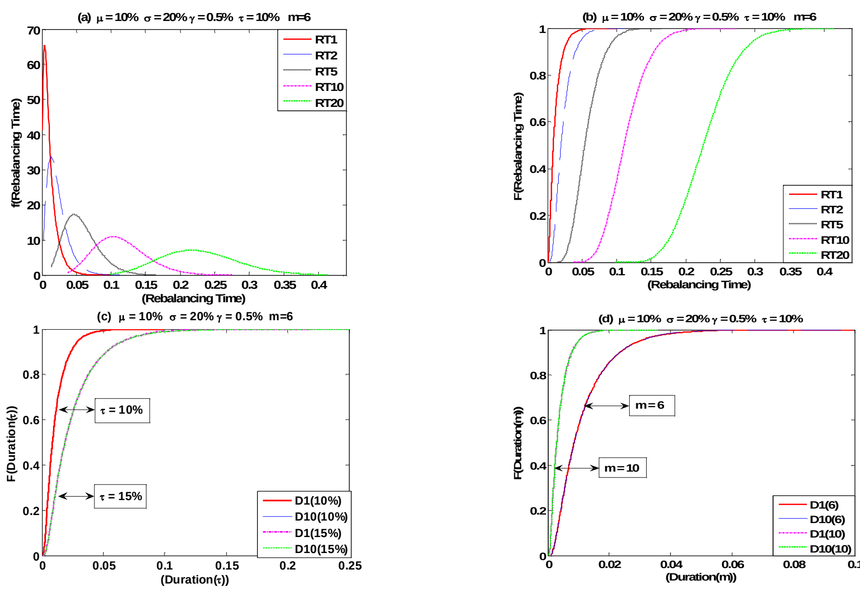
Figure 1
Portfolio values and transaction costs



In particular, we search for the quantile at the level (1/2) and the value of $F_{(\gamma,m)}(1)$. We note that the probability that V_T^{str} is higher than V_T^{ctr} is about 100% and the range of ratio V_T^{ctr}/V_T^{str} is in [0.6;1]. This proves that we must introduce portfolio rebalancing according to the tolerance level. Additionally, the ratio TC_T^{ctr}/TC_T^{str} of the cumulative amounts of transaction costs is always higher than 160%. We find also that TC_T^{str} is smaller than 5% of the initial investment V_0 with a probability equal to 92.36% (see Figure 1 (d)), whereas, for TC_T^{ctr}, this probability is equal to 23%. Note that TC_T^{ctr} can reach 50% of the initial investment V_0. We illustrate now in Figure 2 the impact of the multiple on rebalancing times and durations (pdf denoted by f and cdf denoted by F).

Figure 2

Stochastic time and duration probability distributions



For any fixed tolerance rate τ, the higher the multiple m, the lower the rebalancing times and durations, since the corridor [A,B] is decreasing with respect to the multiple m. Note that the duration associated to the multiple m_1 stochastically dominates any duration associated to the multiple m_2, as soon as $m_1 < m_2$. For example, for m_1=6, the probability to rebalance during one week is about 60% whereas, for m_2=10, this probability is about 85%. The duration associated to a tolerance rate τ_1 stochastically dominates any duration associated to a tolerance rate τ_2, as soon as $\tau_1 > \tau_2$. For example, for $\tau_1 = 10\%$, the probability to rebalance during one month is about 100% whereas, for $\tau_2 = 15\%$, this happens approximately for two months.

IV. CONCLUSION

In this paper, we have examined the CPPI method when portfolio is rebalanced according to investor's tolerance with respect to the target multiple. This strategy is used by practitioners to limit exposure and to reduce global transaction costs. Using various criteria, we have shown that tolerance to the target multiple must be carefully chosen according to the transaction cost level, since this latter one penalizes portfolio performance. We have also compared this stochastic time rebalancing CPPI strategy with the standard one (tolerance equal to zero), when transaction costs occur. Clearly, it dominates the standard strategy. As a by-product, we have provided quasi explicit formula for cumulative distribution function of rebalancing times.

ENDNOTES

1. For more details about proofs of propositions, see Mkaouar (2009).
2. The functions $\mu(.)$, and $\sigma(.)$ satisfy the usual conditions to guarantee the existence, uniqueness and positivity of the solution of this stochastic differential equation.
3. When the risky asset S has jumps, which are greater than a non positive constant d, then condition $0 \leq m \leq -1/d$ implies positivity of the cushion. For example, if d is equal to -10%, then taking $m \leq 10$ allows portfolio to be guaranteed.
4. See Prigent (2001b) for results of option pricing, when such kind of times $(T_n)_n$ are considered.
5. See also Geman and Yor (1994).

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