

Economics of Deals and Optimal Pricing Policy

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ABSTRACT

We expand on the work of Kahneman and Tversky and further develop the optimal pricing policy given a reference price that generates psychological effects. We suggest the possibility of the existence of an **inter-temporal substitution effect**, challenging the standard substitution effect which always points to a reverse relationship between current and future quantity demanded and price. The optimal pricing trajectories are developed for various possible interrelationships between the reference price and the actual price and their possible influence on customer behavior.

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I. INTRODUCTION

The literature concerning behavioral economics dates back almost thirty years with the publication of the ground breaking research of Kahneman and Tversky (1979). They showed that individuals tend to be swayed by psychological factors when deciding on a purchase of a good or service. In the intervening years much further research has been undertaken. Today we use the term "Framing Effect" (see also Tversky and Kahneman (1981)) to describe the fact that individuals' purchasing decisions are often sharply influenced by the wording of the seller's offer although the different formulations of the offer are in effect completely identical. In other words, re-wording the exact same offer or deal could change the decision as to a purchase or as to accepting a certain level of risk in undertaking a project or an investment.

Another aspect of this interesting phenomenon was brought to light in papers by Simonson et al. (1994) and Raghbir (1998) [followed by similar papers by her (2004)] in which give-aways are studied. The idea of the give-away is that when you purchase a unit of product A, a unit of product B is given to you free of charge. Raghbir's results show that being exposed to this give-away brings about a decline in the price that customers are willing to pay for good B when good B is being sold under normal (non give-away) market conditions. Clearly having become aware that good B was involved in a give-away cheapens it in the eyes of the potential consumer. He internalizes (perhaps only subconsciously) that the good is probably cheap to produce, is of low quality, etc. Giving away product B with a price tag attached listing its full price reduces this image problem but does not eliminate it.

We utilize these results but in the opposite direction, and thus add another dimension to these behavioral economics discussions. Our analysis is based on the empirical observation that many large (and small for that matter) department stores advertise large price reductions (on clothing, shoes, household items, etc.) immediately after major holidays or with the approaching change of seasons. Often these reduced prices are reduced even further in the following weeks, and eventually some of those items might altogether disappear from the shelves. Frequently however a unique method of promoting the price reduction is undertaken. The original price is attached to the discounted price with a big X drawn across it to show the less informed customer just how dramatic a price reduction he is being offered. The empirical evidence indicates that sellers consider this tool to be effective, since the deal-prone customers are not only affected by a low price, but they also compare the low price to the original price. A larger gap between the original price and the sale price motivates the deal-prone to buy more. This is probably the result of customers taking the high reference price as an indicator of high quality, and therefore the new lower price is considered a better deal than would be the case had the initial price opened low and continued to remain low. Of course, the less independent information a customer has about a given product, the more likely he is to use price as a predictor of quality. The purpose of this paper is to investigate this popular pricing policy and to develop the optimal price trajectory based on the interdependency between current purchases and previous and current prices.

An intentional policy of making the product expensive and/or hard to get in the initial period can in an inter-temporal framework result in higher income and profits for the seller. Often firms introduce a new product at a sharply reduced price in order to get

the public to try it and develop a taste for it. Once the product has made its desired market inroad the price is raised to the desired level. We argue that instead of introducing a new good to the public by setting a very low introductory price (which could have negative image implications), it might be appropriate to do just the opposite, i.e. to introduce the good at a very high inflated price. That would position the good in the eyes of the public as a high quality prestigious good and would tend to raise the reservation price of the customers. Once that is accomplished the price could be lowered.

This type of consumer affect is observed in a variety of situations. For example, often young people are enticed by the ban on selling alcohol to minors to become drinkers (and perhaps heavy drinkers) when they become adults (and often even before they become adults...). The ban on alcohol has apparently imbued its consumption with a certain degree of luster in the eyes of the consumer. Similarly, a monopolist can generate this kind of inter-temporal effect by adopting an initial very high (perhaps unaffordable) price in the first introductory period. This will widen the gap between the high initial reference price and the deal/sale price and enable more sales at higher prices in period 2 than would have otherwise been attainable. We wish to again point out that this is the exact opposite of the common practice of introducing a product through a low or even free introductory price such as free food and wine samplings.

In fact, the concept of 'conspicuous consumption' was first introduced by Veblen (1899), who argued that individuals often consume highly attention-getting goods and services in order to signal their wealth and thereby achieve greater social status. 'In order to hold the esteem of men it is not sufficient merely to possess wealth or power. The wealth and power must be put in evidence, for esteem is awarded only on evidence' (Veblen, 1899, p. 36). The extreme form of such behavior is known as the 'Veblen effect', witnessed whenever individuals are willing to pay higher prices for functionally equivalent goods (for further discussion, see also Leibenstein, 1950, and Frank, (1985)). The Veblen effect may indeed be empirically significant in some luxury good markets (see Creedy and Slottje, 1991, and Heffetz, 2004). We propose that the reverse is also true. Often people who can't afford to achieve status in the Veblen manner achieve a kind of status by bragging about the good deal they got and how they managed to purchase a really valuable product (based on a high price in period 1) at an unusually low price.

Our goal is to analyze what the optimal price range would be in terms of a monopolist's profit maximization goals. Our approach may add another dimension to the important issue of sales promotion. The standard consensus is that sales promotion can be generated either through advertising or by price reductions that tempt customers to try out new items or to use them more often, and thereby hopefully create a future brand loyalty (see Paroush and Spiegel, 1995).

We suggest that in many cases a reverse approach might be more effective. We argue that an initially high price could contribute toward fixing the product in the public's mind as a prestige item. This effect, which we term an **inter-temporal substitution effect**, could potentially challenge the standard substitution effect which always points to a reverse relationship between current and future quantity demanded and price.

The idea that a first period reference price has a positive effect on the second period reference price can be defended on the following grounds: (a) price is often an

indicator of quality in the eyes of customers who lack alternative methods of estimating quality, and (b) price adds a prestige factor—especially in the case of brand name products. Furthermore, a low initial price could make the product so widely used that a kind of "snob effect" might take place resulting in a reduced demand in the later period. Thus the **inter-temporal substitution effect** could well be different in sign from that of the traditional substitution effect. This will be demonstrated in our paper through a simple structured algebraic model.

If indeed the price in period 1 affects positively the price in period 2, then the profit maximizing monopolist will of course want to take that into account in setting the price for each period. This means that in the case of a two period model with inter-temporal effects, the issue of optimal pricing or the nature of the price trajectory is interesting and important.

II. THE MODEL

We assume that the reference price (suggested price or retail price) of the first period positively affects the reservation price in the second period.

Reservation Price (noted as P_{\max}^i) is a function of the Reference Price (RP) as follows:

$$P_{\max}^2 = P_{\max}^1 + \alpha \cdot RP, \quad (1)$$

where $0 < \alpha < 1$ ¹ and P_{\max}^1 is determined from the point of view of the seller arbitrarily. It can be any positive price (based on the consumer's general impression of the product and/or the image derived from brand loyalty, and his level of desire and need - either objective or subjective- for the product). In addition consumers have developed over time (during the first period) additional general knowledge, feeling, and instinct based on past experience as to what such a product should sell for. This kind of goodwill toward the product is reflected in equation (1) by the term, $\alpha \cdot RP$.

In the second period we assume a new linear demand curve where P_{\max}^2 of the second period depends on an initial value of reservation price P_{\max}^1 , of the first period, and the consumers' experience of $\alpha \cdot RP$ from the first period.

The linear demand curve of each period i is:

$$P_i = P_{\max}^i - \beta Q_i \quad (1')$$

For simplicity, we further assume that at each period the cost function is proportional to output:

$$TC_i = \gamma \cdot Q_i, \text{ or} \quad (2)$$

$$MC_i = \gamma \quad (2')$$

The value of the consumer surplus $\equiv CS$ for each period i is determined as:

$$CS = \sum_{i=1}^2 \frac{(P_{\max}^i - P_i) Q_i}{2} \quad (3)$$

and the profit function at each period, i ($i=1, 2$) is

$$\Pi_i = (P_i - \gamma) Q_i \quad (4)$$

Q_1 units are sold in the first period in accordance with the RP. Q_2 units of the second period will be sold at price P_2 which is positively related to RP-- the reference price determined in the first period.

The non-myopic seller's objective function that recognizes the inter-temporal effects is to maximize profit of the two periods that is defined as follows:

$$\begin{aligned} \text{Max}_{Q_1, Q_2} \Pi &= (P_{\max}^1 - \beta Q_1) Q_1 + \\ &+ (P_{\max}^1 + \alpha RP - \beta Q_2) Q_2 - \gamma(Q_2 + Q_1) = \\ &= (P_{\max}^1 - \beta Q_1) Q_1 + \left\{ P_{\max}^1 + [\alpha(P_{\max}^1 - \beta Q_1)] - \beta Q_2 \right\} Q_2 - \\ &- \gamma(Q_2 + Q_1) = (P_{\max}^1 - \beta Q_1) Q_1 + \left\{ P_{\max}^1(1 + \alpha) - \alpha\beta Q_1 - \beta Q_2 \right\} Q_2 - \gamma(Q_2 + Q_1) \end{aligned} \quad (5)$$

where we assume a discount rate of zero and RP, the reference price (or retail price) is given by:

$$RP = P_{\max}^1 - \beta Q_1 \quad (6)$$

Q_1 determines RP and the latter will affect the actual price that will be charged in the second period (along with the value of Q_2 , the output that is sold in the second period). The F.O.C. are

$$\Pi_{Q_1} = P_{\max}^1 - 2\beta Q_1 - \alpha\beta Q_2 - \gamma = 0 \quad (7)$$

and

$$\Pi_{Q_2} = P_{\max}^1(1 + \alpha) - \alpha\beta Q_1 - 2\beta Q_2 - \gamma = 0 \quad (8)$$

The S.O.C. are:

$$\begin{aligned} \Pi_{Q_1 Q_1} &= -2\beta < 0 \\ \Pi_{Q_2 Q_2} &= -2\beta < 0 \\ \Pi_{Q_1 Q_1} \cdot \Pi_{Q_2 Q_2} - (\Pi_{Q_1 Q_2})^2 &= 4\beta^2 - \alpha^2 \beta^2 > 0 \end{aligned} \quad (9)$$

where $\Pi_{Q_1 Q_2} = -\alpha\beta$. S.O.C. hold if $\alpha < 2$.

From (7) and (8) we can derive the iso-marginal profit functions that guarantee F.O.C.:

$$Q_2 = \frac{P_{\max}^1 - \gamma}{\alpha\beta} - \frac{2}{\alpha} Q_1 \quad (7')$$

and

$$Q_2 = \frac{P_{\max}^1(1+\alpha) - \gamma}{2\beta} - \frac{\alpha}{2} Q_1 \quad (8')$$

By equating (7') and (8') we find:

$$\frac{P_{\max}^1}{\alpha\beta} - \frac{\gamma}{\alpha\beta} - \frac{2}{\alpha} Q_1 = \frac{P_{\max}^1(1+\alpha)}{2\beta} - \frac{\gamma}{2\beta} - \frac{\alpha}{2} Q_1 \quad (10)$$

Thus,

$$\left(\frac{2}{\alpha} - \frac{\alpha}{2}\right) Q_1 = \frac{2\beta(P_{\max}^1 - \gamma) - \alpha\beta \cdot [P_{\max}^1(1+\alpha) - \gamma]}{2\alpha\beta^2} \quad (10')$$

or

$$\left(\frac{4 - \alpha^2}{2\alpha}\right) Q_1 = \frac{2\beta(P_{\max}^1 - \gamma) - \alpha\beta \cdot [P_{\max}^1(1+\alpha) - \gamma]}{2\alpha\beta^2} \quad (10'')$$

Therefore we get the optimal value of Q_1^* as:

$$Q_1^* = \frac{(2-\alpha)(P_{\max}^1 - \gamma) - \alpha^2 P_{\max}^1}{(4-\alpha^2) \cdot \beta} = \frac{(2-\alpha-\alpha^2) \cdot P_{\max}^1 - (2-\alpha)\gamma}{(4-\alpha^2) \cdot \beta} = \frac{(2-\alpha-\alpha^2) \cdot P_{\max}^1}{(4-\alpha^2) \cdot \beta} - \frac{\gamma}{(2+\alpha) \cdot \beta} \quad (11)$$

From (11) and (8') we find Q_2^*

$$\begin{aligned}
Q_2^* &= \frac{P_{\max}^1(1+\alpha) - \gamma}{2\beta} - \frac{\alpha}{2} \left[\frac{(2-\alpha-\alpha^2)P_{\max}^1 - (2-\alpha)\gamma}{(4-\alpha^2)\beta} \right] \\
&= \frac{(4-\alpha^2)P_{\max}^1(1+\alpha) - (4-\alpha^2)\gamma - (2\alpha-\alpha^2-\alpha^3)P_{\max}^1 + (2\alpha-\alpha^2)\gamma}{2(4-\alpha^2)\beta} \\
&= \frac{P_{\max}^1(4+2\alpha) - \gamma(4-2\alpha)}{2(4-\alpha^2)\beta} \\
&= \frac{P_{\max}^1(2+\alpha) - \gamma(2-\alpha)}{(4-\alpha^2)\beta} \\
&= \frac{P_{\max}^1}{(2-\alpha)\beta} - \frac{\gamma}{(2+\alpha)\beta}
\end{aligned} \tag{11'}$$

From (11) and (11') we can conclude that for an internal solution of $Q_1^* > 0$ and $Q_2^* > 0$, while $P_{\max}^1 > \gamma$, the following is required:

$$(2-\alpha-\alpha^2) > 0 \quad \text{or} \quad \alpha(1+\alpha) < 2$$

Therefore $\alpha < 1$ is a necessary condition for the S.O.C. to exist, which is a more restricting constraint over and above the previous condition that $\alpha < 2$.

From (6) and (11) we can derive the RP as follows:

$$\begin{aligned}
RP &= P_{\max}^1 - \beta Q_1 = P_{\max}^1 - \frac{[(2-\alpha-\alpha^2)P_{\max}^1 - (2-\alpha)\gamma]}{(4-\alpha^2)} = \\
&= \frac{4P_{\max}^1 - \alpha^2 P_{\max}^1 - 2P_{\max}^1 + \alpha P_{\max}^1 + \alpha^2 P_{\max}^1 + (2-\alpha)\gamma}{(4-\alpha^2)} = \\
&= \frac{(2+\alpha)P_{\max}^1 + (2-\alpha)\gamma}{(4-\alpha^2)} = \frac{P_{\max}^1}{2-\alpha} + \frac{\gamma}{2+\alpha}
\end{aligned} \tag{12}$$

From (12) we can make a comparative static analysis by taking the derivatives as follows:

$$\frac{\partial RP}{\partial \alpha} = \frac{P_{\max}^1}{(2-\alpha)^2} - \frac{\gamma}{(2+\alpha)^2} = \frac{(2+\alpha)^2 P_{\max}^1 - (2-\alpha)^2 \gamma}{(2-\alpha)^2 (2+\alpha)^2} > 0, \tag{12'}$$

since $0 < \alpha < 1$ and $P_{\max}^1 > \gamma$, $\frac{\partial RP}{\partial P_{\max}^1} = \frac{1}{(2-\alpha)}$. As $0 < \alpha < 1$ we conclude that

$$\frac{1}{2} < \frac{\partial RP}{\partial P_{\max}^1} < 1. \quad (12'')$$

In the extreme case where $\alpha = 0$, i.e., no inter-temporal effect exists then we get as expected:

$$\begin{aligned} \frac{\partial RP}{\partial P_{\max}^1} &= \frac{1}{2} \\ \frac{\partial RP}{\partial \gamma} &= \frac{1}{(2+\alpha)}. \end{aligned} \quad (12''')$$

Thus, as $0 < \alpha < 1$ we get $\frac{1}{3} < \frac{\partial RP}{\partial \gamma} < \frac{1}{2}$, and again for $\alpha = 0$ we get (as expected),

$$\frac{\partial RP}{\partial \gamma} = \frac{1}{2}.$$

The same discussion can be applied to see how changes in the variables α , P_{\max}^1 and γ will affect the values of the dependent variables Q_1^* and Q_2^* :

From (11) we find that

$$\begin{aligned} \frac{\partial Q_1^*}{\partial \alpha} &= \frac{\left[-(1+2\alpha)P_{\max}^1 + \gamma \right] (4-\alpha^2)\beta + 2\alpha\beta \left[(2-\alpha-\alpha^2)P_{\max}^1 - (2-\alpha)\gamma \right]}{(4-\alpha^2)^2\beta^2} = \\ &= \frac{(4+8\alpha-\alpha^2+2\alpha^3)P_{\max}^1 + (4-\alpha^2)\gamma + \left[4\alpha-2\alpha^2-2\alpha^3 \right] - (4\alpha-2\alpha^2)\gamma}{(4-\alpha^2)^2\beta} \\ &= \frac{-\left[4+4\alpha+\alpha^2 \right] \cdot P_{\max}^1 + \left[4-4\alpha+\alpha^2 \right] \gamma}{(4-\alpha^2)^2\beta} = \\ &= \frac{(4+\alpha^2)(\gamma-P_{\max}^1) - \alpha^2(P_{\max}^1-\gamma)}{(4-\alpha^2)^2\beta} < 0 \end{aligned} \quad (13)$$

This is expected due to the fact that $0 < \alpha < 1$ and $P_{\max}^1 > \gamma$. Similarly, we find from (11) that as expected:

$$\frac{\partial Q_1^*}{\partial P_{\max}^1} = \frac{(2 - \alpha - \alpha^2)}{(4 - \alpha^2)\beta} > 0 \quad (13')$$

And also as expected:

$$\frac{\partial Q_1^*}{\partial \gamma} = \frac{(2 - \alpha)}{(4 - \alpha^2)\beta} < 0 \quad (13'')$$

In order to analyze the effect of the above variables on Q_2^* . We derive Q_2^* of (11') with respect to α , P_{\max}^1 and γ as follows:

$$\frac{\partial Q_2^*}{\partial \alpha} = \frac{P_{\max}^1}{(2 - \alpha)^2\beta} + \frac{\gamma}{(2 + \alpha)^2\beta} > 0 \quad (14)$$

$$\frac{\partial Q_2^*}{\partial P_{\max}^1} = \frac{1}{(2 - \alpha)\beta} > 0 \quad (14')$$

$$\frac{\partial Q_2^*}{\partial \gamma} = -\frac{1}{(2 + \alpha)\beta} < 0 \quad (14'')$$

At this stage, we investigate the price of equilibrium for the second period. Thus, we get from (1) and (1') and (11) and (12) the following:

$$\begin{aligned} P_2 &= P_{\max}^2 - \beta Q_2^* = P_{\max}^1 + \frac{\alpha P_{\max}^1}{2 - \alpha} + \frac{\alpha \gamma}{2 + \alpha} - \beta \left[\frac{P_{\max}^1}{\beta(2 - \alpha)} - \frac{\gamma}{\beta(2 + \alpha)} \right] \\ &= \frac{2P_{\max}^1}{(2 - \alpha)} + \frac{\alpha \gamma}{(2 + \alpha)} - \frac{P_{\max}^1}{(2 - \alpha)} + \frac{\gamma}{(2 + \alpha)} \\ &= \frac{P_{\max}^1}{(2 - \alpha)} + \frac{\gamma(1 + \alpha)}{(2 + \alpha)} \end{aligned} \quad (15)$$

Comparison of RP and P_2 , based on (12) and (15) leads to the conclusion that $P_2 > RP$ and the gap increases as the coefficient variable α increases.

Based on (11) and (11') we can compare the optimal quantities at each period by subtracting Q_1^* from Q_2^* , i.e.,

$$Q_2^* - Q_1^* = \left[\frac{P_{\max}^1}{(2-\alpha)\beta} - \frac{\gamma}{(2+\alpha)\beta} \right] - \left[\frac{(2-\alpha-\alpha^2)P_{\max}^1}{(4-\alpha^2)\beta} - \frac{\gamma}{(2+\alpha)\beta} \right] \quad (16)$$

or

$$\begin{aligned} Q_2^* - Q_1^* &= \frac{P_{\max}^1}{\beta} \left[\frac{1}{(2-\alpha)} - \frac{(2-\alpha-\alpha^2)}{(4-\alpha^2)} \right] = \\ &= \frac{P_{\max}^1}{\beta} \left[\frac{(2+\alpha) - (2-\alpha-\alpha^2)}{(4-\alpha^2)} \right] = \\ &= \frac{P_{\max}^1}{\beta} \left[\frac{\alpha(2+\alpha)}{(4-\alpha^2)} \right] = \frac{P_{\max}^1}{\beta} \left[\frac{\alpha}{(2-\alpha)} \right] > 0 \end{aligned} \quad (16')$$

since $0 < \alpha < 1$. Thus, we can conclude that both prices and quantity purchased in the second period are larger than the quantity in the first period.

These results in some sense are interesting and "surprising". In the regular promotion policy used and discussed in the literature, the seller who desires to promote future sales and profits introduces the product at a low price to get consumers to try the product and get used to it. Several methods are used by sellers to promote purchases such as discount prices, coupon distributions, free samples, and larger quantities for a given price. This leads to further sales and increases the demand of the future (second) period leading to more profits, while the future price is higher than the reduced discount price of the first period. However, the future quantity purchased by consumers can be smaller or larger than that of the first period, as a result of the price increase on the one hand, and the addiction process based on the previous price on the other hand.

In our deal-prone cases, in order to promote future sales, the seller **increases** prices in the first period, thus reducing quantities in the first period generating an image of high quality good and a greater future demand. However, as a result he can sometimes charge even higher prices in the future and sell larger quantities, or reduce prices and sell those units that are only purchased by deal-prone customers. For example, one way that might convince a deal-prone customer to buy a Ralph Lorene (RL) shirt would be for the seller to charge an initial reference price that would be very high, thus discouraging the customer from buying the good in the first period, which would in turn develop expectations and inter-temporal snob effect. The seller would then prepare a change in behavior that would encourage the customer to look for the price of the shirt in the second period. When the second period arrives and the new reservation price is already high enough, the seller can then charge the customer either a higher price for the shirt, much higher than what he would have charged otherwise, or he can offer him a lower price, thereby encouraging him to buy more of the shirts, so that ultimately the same kind of people would buy more shirts of the RL brand and end up paying either more or less per shirt.

This model is in a sense an extension to the Veblen Snob Effect where we implement the idea of inter-temporal effect, where price reflects on quality (or at least image of quality) over time.

In the discussion below we demonstrate the three different inter-temporal effects. In the first case, the actual price in the first period represents the reference price for the future (second) period, with a positive effect (represented by the coefficient α) on the demand of the second period. In the second case the demand in the second period is affected positively by the gap between the reference (actual) price of the first period and the current price of the second period. The larger the gap, the greater the perception on the part of the consumer that he is getting an excellent deal and this will be reflected in greater quantities being purchased in the second period. In case three the effect of the reference price (RP) is more significant, namely RP affects positively, *although not symmetrically*, the demands of both periods. This means that in this case RP does not represent the actual price charged by the seller either in the first or second period. Each unit of the good is actually sold at P_1 and P_2 during period 1 and period 2 respectively, and both prices are different from the RP. However, the RP generates some expectation of quality and/or some expectation of an "appropriate" price which may affect positively, but not necessarily symmetrically, the demands of both periods.

Case 1:

In this case we assume demand functions of the two periods where the Reference Price, RP, is the actual price of the first period that affects positively the demand of the second period.

$$Q_1 = A_1 - \beta RP \quad (17)$$

$$Q_2 = A_2 + \alpha RP - \gamma P_2 \quad (18)$$

The objective function of the seller is:

$$\text{Max}_{RP, P_2} \Pi = A_1 RP - \beta RP^2 + A_2 P_2 + \alpha P_2 RP - \gamma P_2^2 \quad (19)$$

The F.O.C. for maximization are:

$$\Pi_{RP} = A_1 - 2\beta RP + \alpha P_2 = 0 \quad (20)$$

$$\Pi_{P_2} = A_2 + \alpha RP - 2\gamma P_2 = 0 \quad (21)$$

Thus, the ISO marginal profit curves, RC_i are:

$$RC_1 : P_2 = \frac{-A_1}{\alpha} + \left(\frac{2\beta}{\alpha} \right) RP \quad (20')$$

$$RC_2 : P_2 = \frac{A_2}{2\gamma} + \left(\frac{\alpha}{2\gamma}\right)RP \quad (21')$$

where the S.O.C. are:

$$\Pi_{RP,RP} = -2\beta < 0, \Pi_{P_2,P_2} = -2\gamma < 0,$$

$$\Pi_{RP,RP}\Pi_{P_2,P_2} - (\Pi_{RP,P_2})^2 = 4\beta\gamma - \alpha^2 > 0$$

Rewriting (20') and (21') as (20'') and (21'') below yield the solutions of RP^* and P_2^*

$$2\beta RP - \alpha P_2 = A_1 \quad (20'')$$

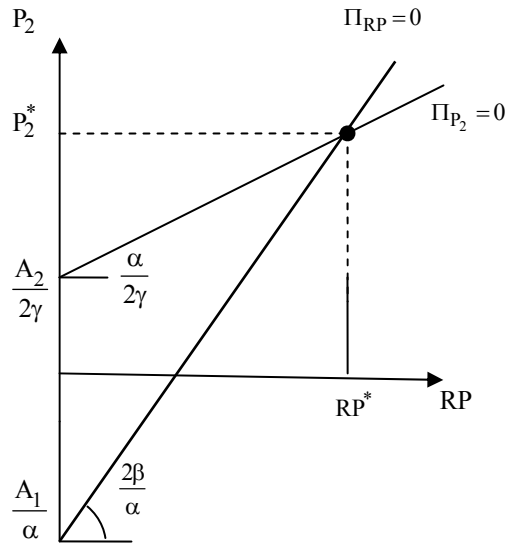
$$-\alpha RP + 2\gamma P_2 = A_2 \quad (21'')$$

$$RP^* = \frac{2\gamma A_1 + \alpha A_2}{\Delta} > 0 \quad (22)$$

$$P_2^* = \frac{2\beta A_2 + \alpha A_1}{\Delta} > 0 \quad (23)$$

where $\Delta = 4\beta\gamma - \alpha^2 > 0$. The solution is demonstrated by the graphical illustration in Figure 1.

Figure 1



From equations (22) and (23) we can reach several conclusions regarding the relationship between P_2^* and RP^* :

- when $\alpha = \gamma < \beta$ and $A_2 > A_1$ then $P_2^* > RP^*$
- when $\alpha = \gamma > \beta$ and $A_2 < A_1$ then $P_2^* < RP^*$
- However, when $\gamma > \beta$ $\gamma > \alpha$ and $A_2 < A_1$ then $P_2^* < RP^*$ still holds.

Case 2:

In this case RP is again the actual price in the first period; however, the demand in the second period is affected positively by the gap between the reference price and the actual price in the second period. The demand functions are represented as follows:

$$Q_1 = A_1 - \beta RP \quad (24)$$

$$Q_2 = A_2 + \alpha(RP - P_2) - \delta P_2 \quad (25)$$

The objective function

$$\text{Max}_{RP, P_2} \Pi = A_1 RP - \beta RP^2 + A_2 P_2 + \alpha RP P_2 - (\alpha + \delta) P_2^2 \quad (26)$$

The F.O.C. in this case are as follows:

$$\Pi_{RP} = A_1 - 2\beta RP + \alpha P_2 = 0 \quad (27)$$

$$\Pi_{P_2} = A_2 + \alpha RP - 2(\alpha + \delta) P_2 = 0 \quad (28)$$

Thus,

$$RC_1 : P_2 = \frac{-A_1}{\alpha} + \left(\frac{2\beta}{\alpha} \right) RP \quad (27')$$

$$RC_2 : P_2 = \frac{A_2}{2(\alpha + \delta)} + \left(\frac{\alpha}{2(\alpha + \delta)} \right) RP \quad (28')$$

where the S.O.C. are:

$$\Pi_{RP, RP} = -2\beta \quad \text{and} \quad \Pi_{P_2, P_2} = -2(\alpha + \delta)$$

and

$$\Pi_{RP, RP} \Pi_{P_2, P_2} - (\Pi_{RP, P_2})^2 = 4\beta(\alpha + \delta) - \alpha^2 > 0$$

Rewriting (27') and (28') as (27'') and (28'') below yield the solutions of RP^* and P_2^*

$$\alpha P_2 - 2\beta RP = A_1 \quad (27'')$$

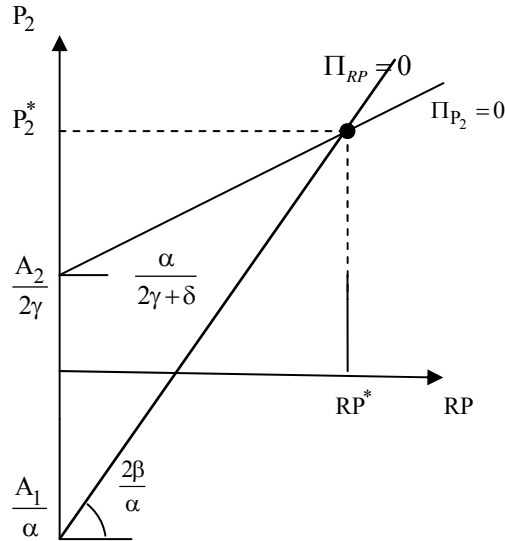
$$2(\gamma + \delta)P_2 - \alpha RP = A_2 \quad (28'')$$

$$RP^* = \frac{2(\alpha + \delta)A_1 + \alpha A_2}{\Delta} \quad (29)$$

$$P_2^* = \frac{2\beta A_2 - \alpha A_1}{\Delta} \quad (30)$$

where $\Delta = 4\beta(\alpha + \delta) - \alpha^2 > 0$. The solution is demonstrated by the graphical illustration in Figure 2.

Figure 2



From equations (29) and (30) we can derive a straightforward conclusion regarding the relationship between P_2^* and RP^* as follows:

For $(\alpha - 2\beta)A_2 \geq (\alpha - \delta)A_1$ then $RP^* \geq P_2^*$, respectively.

Case 3:

In this case the "declared" reference price, RP, differs and is greater than the actual current price determined by the seller in the first period. The "declared" RP positively affects the demand of each period, but not necessarily symmetrically. The demand for each period is given by:

$$Q_1 = A_1 + \beta RP^\varepsilon - \gamma P_1 \quad (31)$$

$$Q_2 = A_2 + \alpha(RP - P_2) - \delta P_2 \quad (32)$$

In this case three decision variables should be determined: RP, P₁, P₂, where the objective function is:

$$\text{Max}_{P_1, P_2, RP} \Pi = A_1 P_1 + \beta P_1 RP^\varepsilon - \gamma P_1^2 + A_2 P_2 + \alpha R P P_2 - (\alpha + \delta) P_2^2 \quad (33)$$

The F.O.C. with respect to the three decision variables are:

$$\Pi_{P_1} = A_1 + \beta RP^\varepsilon - 2\gamma P_1 = 0 \quad (34)$$

$$\Pi_{P_2} = A_2 + \alpha RP - 2(\alpha + \delta) P_2 = 0 \quad (35)$$

$$\Pi_{RP} = \varepsilon \beta RP^{\varepsilon-1} P_1 + \alpha P_2 = 0^2 \quad (36)$$

From (34) we can derive the optimal first period price, P₁ :

$$P_1 = \frac{A_1 + \beta RP^\varepsilon}{2\gamma} \quad (34')$$

and from (35) we can derive the optimal second period price, P₂ :

$$P_2 = \frac{A_2 + \alpha RP}{2(\alpha + \delta)} \quad (35')$$

Using the above last two equations we can conclude that assuming a high level of RP even if the actual price at each period is below RP, the relationship between the prices of each period is ambiguous. It may occur that the starting actual price in the first period will be high and is reduced in the second period such that $RP > P_1 > P_2$.

However, it is also possible that we may face a promotional strategy that uses the tool of prestige creation by use of a high RP in conjunction with a low actual price for

the first period, P_1 - in order to promote an addiction process, followed by an increase in price in the second period, P_2 . In such a case we can face an optimal pricing policy under which: $RP > P_2 > P_1$.

III. IMPLICATIONS AND CONCLUSIONS

In general sales promotion activities are undertaken for the purpose of attracting new customers to try the product. Deal prone customers are typically attracted by such devices as give-aways of small samples, larger packages for the same price, discount coupons, etc. All of this is designed to win over customers to the seller's product, thus enabling him to sell more and/or at higher prices in the future. Similar methods are undertaken when introducing a new product. We, however, analyze the possibility of adopting a reverse approach by introducing the product at a high price and thereby creating a type of prestige effect. Only when that aura of prestige has been firmly established will deal prone customers be allowed to purchase the product at a reduced price.

A policy of setting an initial high price which only attracts the hard-core prestige seeking Loyals who wish to show the world that they are able to afford this prestige item creates a situation in which those Loyals would prefer that the price continue to remain high in the future. But although the deal-prone customers can't afford this high price, they have become thoroughly impressed by the aura of status and prestige that emanates from this product. Thus when in the next period they are offered the chance to purchase at discount they will have a higher reference price and purchase more of the good at the discount price than they would had the aura of status and prestige been lacking.

Of course the seller must carefully weigh his pricing policy: should he continue to maintain a high price policy in the future thereby winning the approval of his prestige-seeking Loyals (such as Rolls Royce does for example), or perhaps should he start with a high price and then in the future drop it so as to attract the deal-prones. This approach, while appealing has its associated risks. *The New York Times* reported (Sept. 7, 2007) the iPhone customer rebellion:

"In June, they were calling it the God Phone. Yesterday, it was the Chump Phone. People who had rushed to buy the Apple iPhone over the last two months suddenly and embarrassingly found that they had overpaid by \$200 for the year's most coveted gadget. Apple, based in Cupertino, Calif., has made few missteps over the last decade, but it angered many of its most loyal customers by dropping the price of its iPhone to \$400 from \$600 only two months after it first went on sale. They let the company know on blogs, through e-mail messages and with phone calls.

Yesterday, in a remarkable concession, Steven P. Jobs acknowledged that the company had abused its core customers' trust and extended a \$100 store credit to the early iPhone buyers. "Our early customers trusted us, and we must live up to that trust with our actions in moments like these," Mr. Jobs wrote in a letter posted to Apple's Web site."

Another interesting story on pricing policy taken from that same article reflects the issues dealt with in this paper:

Mobile phones tend to be more prone to price declines because the pace of product introductions is faster than for televisions or DVD players. Motorola, for instance, introduced the ultra-thin Razr phone for \$499 with a two-year service contract in early 2005. Six months later, Motorola realized it had a hit on its hands and dropped the price to \$199 in an effort to aim at more mainstream buyers. By the end of 2005, the price was \$99. Ken Dulaney, a vice president at Gartner Research, said that in general starting high and dropping the price slowly was a smart strategy. By starting the price high, manufacturers can gauge early demand and reap greater profit from early adopters who are willing to pay any amount to be the first with a particular device. "It's probably a formula taught in business school," Mr. Dulaney said.

As an aside we should point out that in light of our previous discussion, we would suggest that the following also be taught in business schools: An additional reason to start with a high price and reduce it later is that the high introductory price encourages the deal prone customers to buy more of the good when offered a price discount. But the later price reduction not only encourages customers to enter the market due to the lower price itself, but also because of the gap between the initial reference price and the actual new price. This customer response is further increased in the case where the initial high price generates a prestige effect.

That must have been what Apple was counting on. But the size and speed of the price cut alienated some of Apple's most loyal supporters. *"I feel totally screwed," wrote one iPhone owner on the Unofficial Apple Weblog site. "My love affair with Apple is officially over."*

Thus, it is clear that when undertaking a future price cut strategy to attract the deal-prones, great care must be taken. Of course a third strategy is also available to the seller primarily in the case of non-durable goods. That strategy consists of selling initially at very low come-on prices (perhaps by the use of free samples, discount coupons, two for the price of one packaging, etc.) in order to get the public attached to the product and then to raise prices in the future (perhaps by canceling the special offers or allowing them to expire).

All the above possibilities are actually observed empirically in the daily economic life of free market countries. Since we do actually observe many cases of initially high prices and their gradual reduction over time (independent of end of season sales—which is a different topic altogether), we have presented in a simple model what the optimum inter-temporal pricing policy for the seller would be. There are different trajectories of prices along time which we have investigated. According to our understanding, a policy of creating a prestige image with an initially high price tends in some cases to dominate the other pricing possibilities. This policy can also be relevant in cases where the reference price of the good is not actually imposed on the customers but just "declared" in order to generate an image of prestige, glamour, and high quality. The actual price in the first period might be lower than the reference price, RP, and the trajectory of the actual prices along the two periods (current and future) is therefore ambiguous.

ENDNOTES

1. We prove below that a necessary condition for a non-corner solution is that $0 < \alpha < 1$.

2. The S.O.C. are presented in Appendix A.

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APPENDIX A

Investigation of the second order conditions (S.O.C) for maximum are:

$$(A.1) \quad \Delta \equiv \begin{vmatrix} -2\gamma & 0 & \varepsilon\beta RP^{\varepsilon-1} \\ 0 & -2(\alpha + \delta) & \alpha \\ \varepsilon\beta RP^{\varepsilon-1} & \alpha & (\varepsilon - 1)\varepsilon\beta RP^{(\varepsilon-1)} \end{vmatrix}$$

where: (A.2) $-2\gamma < 0$.

$$(A.3) \quad 4\gamma(\alpha + \delta) > 0 \text{ and}$$

$$(A.4) \quad 2 \left\{ \left[2\gamma(\alpha + \delta)(\varepsilon - 1)\varepsilon\beta RP^{(\varepsilon-1)} + \gamma\alpha^2 \right] + (\alpha + \delta)\varepsilon^2\beta^2 RP^{2(\varepsilon-1)} \right\} < 0$$

Thus for S.O.C. to exist if (A.4) holds the required (but not sufficient) condition for (A.4) exists, $\varepsilon < 1$.

However, equation (A.4) holds if the following exists:

$$(1 - \varepsilon) 4\gamma(\alpha + \delta)\varepsilon\beta RP^{\varepsilon-1} > 2 \left[\gamma\alpha^2 + \varepsilon^2\beta^2 RP^{2(\varepsilon-1)}(\alpha + \delta) \right]$$

or

$$(1 - \varepsilon) 4\gamma(\alpha + \delta)\varepsilon\beta RP^{\varepsilon-1} - 2\varepsilon^2\beta^2 RP^{2(\varepsilon-1)}(\alpha + \delta) > 2\gamma\alpha^2$$

$$2\varepsilon\beta(\alpha + \delta)RP^{(\varepsilon-1)} \left[2\gamma(1 - \varepsilon) - \varepsilon\beta RP^{(\varepsilon-1)} \right] > 2\gamma\alpha^2 > 0$$

The left-hand side of the inequality is positive only if:

$$(A.5) \quad 2\gamma(1 - \varepsilon) > \varepsilon\beta RP^{(\varepsilon-1)}$$

$$(A.6) \quad RP^{(1-\varepsilon)} > \frac{\varepsilon\beta}{2\gamma(1 - \varepsilon)}$$

$$(A.7) \quad RP > \left(\frac{\varepsilon\beta}{2\gamma(1 - \varepsilon)} \right)^{\frac{1}{1-\varepsilon}}$$