

A General Formula for the WACC: A Comment

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ABSTRACT

This note builds on the paper of Farber, Gillet and Szafarz (2006). The WACC is a discount rate widely used in corporate finance. However, the correct calculation of the WACC rests on a correct valuation of the tax shields. The value of tax shields depends on the debt policy of the company. Many authors, (e.g. Inselbag and Kaufold (1997), Booth (2002), Cooper and Nyborg (2006), Farber, Gillet and Szafarz (2006)) consider that debt policy may only be framed in terms of maintaining a fixed market value debt ratio (Miles-Ezzell assumption) or a fixed dollar amount of debt (Modigliani-Miller assumption).

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I. INTRODUCTION

The value of tax shields (VTS) defines the increase in the company's value as a result of the tax saving obtained by the payment of interest. However, there is no consensus in the existing literature regarding the correct way to compute the VTS. Modigliani and Miller (1963), Myers (1974), Brealey and Myers (2000) and Damodaran (2006) propose to discount the tax savings due to interest payments on debt at the cost of debt (r_D), whereas Harris and Pringle (1985) and Ruback (2002) propose discounting these tax savings at the cost of capital for the unlevered firm (r_A). Miles and Ezzell (1985) propose discounting these tax savings the first year at the cost of debt and the following years at r_A .

Equation (18) of Farber, Gillet and Szafarz (2006) is a general formulation of the WACC:

$$\text{WACC} = r_A - \frac{\text{VTS}}{D + E} (r_A - r_{\text{TS}}) - \frac{D}{D + E} r_D \cdot T \quad (1)$$

where E is the value of the equity, D is the value of the debt, V_u is the value of the unlevered equity, VTS is the value of tax shields, and r_E , r_D , r_A and r_{TS} are the required returns to the expected cash flows of equity, debt, assets (free cash flow) and tax shields.

This equation is correct if the required returns are always constant over time. Farber, Gillet and Szafarz (2006) consider that r_{TS} can be r_D (as Modigliani-Miller) or r_A (as Harris-Pringle). These two scenarios correspond to two different financing strategies: the first one is valid for a company that has a preset amount of debt and the second one should be valid for a company that has constant debt ratio in market value terms. However, as Miles and Ezzell (1985) and Arzac and Glosten (2005) prove, the required return for the tax shield (r_{TS}) of a company with a constant debt ratio in market value terms is r_D for the tax shields of the first period and r_A thereafter. It is not possible to derive a debt policy such that the appropriate discount rate for the tax shields is r_A in all periods. $D_t = L \cdot (D_t + E_t)$ implies that D_t is also proportional to FCF_t . The Miles and Ezzell (1985) correct formula for the VTS of a perpetuity growing at a rate g is:

$$\text{VTS}^{\text{ME}} = \frac{D r_D T (1 + r_A)}{(r_A - g) (1 + r_D)} \quad (2)$$

Formula (2) is identical to formulae (21) of Miles and Ezzell (1985), (13) of Arzac and Glosten (2005) and (7) of Lewellen and Emery (1986). Formula (2) arises from considering that $r_{\text{TS}} = r_D$ in the first period ($t = 1$) and $r_{\text{TS}} = r_A$ for the following periods ($t > 1$). However, Farber, Gillet and Szafarz (2006) assume, as Harris and Pringle (1985), that $r_{\text{TS}} = r_A$ in all periods. The formula for the VTS offered by Harris and Pringle (1985) and implied by Farber, Gillet and Szafarz (2006) in their equations (28) and (29) is:

$$VTS^{HP} = \frac{Dr_D \cdot T}{(r_A - g)} \quad (3)$$

If debt is adjusted continuously, not only at the end of the period, then the ME formula (2) changes to

$$VTS = D \cdot \rho \cdot T / (\kappa - \gamma) \quad (4)$$

where $\rho = \ln(1 + r_D)$, $\gamma = \ln(1 + g)$, and $\kappa = \ln(1 + r_A)$. Equation (4) is quite similar to equation (3) (but then r_D , g and r_A should also be expressed in continuous time). However, (3) is incorrect for discrete time: (2) is the correct formula.

Therefore, equations (14) and (28) of Farber *et al* (2006) for discrete time should be:

$$r_E = r_A + \frac{D}{E}(r_A - r_D) \frac{1 + R_F(1 - T)}{1 + R_F}, \text{ instead of } r_E = r_A + \frac{D}{E}(r_A - r_D) \quad (5)$$

Equations (25) and (29) should be:

$$WACC = r_A - L \frac{r_D T(1 + r_A)}{(1 + r_D)}, \text{ instead of } WACC = r_A - L r_D T \quad (6)$$

II. REQUIRED RETURN TO EQUITY AND WACC FOR PERPETUITIES WITH A CONSTANT GROWTH RATE

For perpetuities with a constant growth rate (g), the relationship between expected values in $t=1$ of the free cash flow (FCF) and the equity cash flow (ECF) is:

$$ECF_0(1+g) = FCF_0(1+g) - D_0 r_D (1-T) + g D_0 \quad (7)$$

The value of the equity today (E) is equal to the present value of the expected equity cash flows. If r_E is the average appropriate discount rate for the expected equity cash flows, then $E = ECF_0(1+g) / (r_E - g)$, and equation (7) is equivalent to:

$$E r_E = V u r_A - D r_D + VTS g + D r_D T \quad (8)$$

And the general equation for the r_E is:

$$r_E = r_A + \frac{D}{E} [r_A - r_D (1 - T)] - \frac{VTS}{E} (r_A - g) \quad (9)$$

Equation (9) is equivalent to equation (10) of Farber *et al* (2006) because $VTS = D r_D T / (r_{TS} - g)$.

The WACC is the appropriate discount rate for the expected free cash flows, such that $D_0 + E_0 = FCF_0(1+g) / (WACC-g)$. The equation that relates the WACC and the VTS is (10):

$$WACC = r_A \left(1 - \frac{VTS}{D+E} \right) + \frac{gVTS}{D+E} \quad (10)$$

Equation (10) is equivalent to equation (18) of Farber et al (2006) because $VTS = D r_D T / (r_{TS} - g)$.

III. CONCLUSIONS

The WACC is a discount rate widely used in corporate finance. However, the correct calculation of the WACC rests on a correct valuation of the tax shields. The value of tax shields depends on the debt policy of the company. When the debt level is fixed, Modigliani-Miller applies, and the tax shields should be discounted at the required return to debt. If the leverage ratio is fixed at market value, then Miles-Ezzell applies. Other debt policies should be explored. For example, Fernandez (2006) develops valuation formulae for the situation in which the leverage ratio is fixed at book values and argues that it is more realistic to assume that a company maintains a fixed book-value leverage ratio than to assume, as Miles-Ezzell do, that the company maintains a fixed market-value leverage ratio because the company is more valuable, and because it is easier to follow for non quoted companies.

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