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Calendar Corrected Chaotic Forecast of Financial Time Series

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ABSTRACT

Using daily returns from the NASDAQ Composite and TSE 300 Composite indices from 1984 to 2003, we specify a method that corrects the chaotic forecasting of financial time series taking into account the day-of-the-week, the turn-of-the-month and the holiday effects. When calendar effects are present in the series, the forecasting ability of the model leads to profitable opportunities compared to a buy-and-hold strategy.

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I. INTRODUCTION

Empirical studies on financial time series have revealed the presence of calendar effects in the behavior of stock returns. Calendar studies questioned whether irregularities exist in the rates of return during the calendar year. Knowing this, would better allow investors to predict returns on stocks. According to the Efficient Market Hypothesis (EMH) such seasonal patterns should not persist since their existence implies the possibility of obtaining abnormal returns applying market-timing strategies.

The day-of-the week effect, first documented by Osborne (1962); the weekend effect (significantly lower returns over the period between Friday's close and Monday's close), first documented by French (1980); the January effect (relatively higher returns in January), first reported by Wachtel (1942); the trading month effect studied by Ariel (1987); and the holiday effect documented by Lakonishok and Smidt (1988), are among the most important calendar effects. These calendar effects have been studied extensively in international level (e.g. Dubois and Louvet (1995), Hiraki and Maberly (1995), Aggarwal and Schatzberg (1997), Mookerjee and Yu (1999), Mills *et al.* (2000)) and the general conclusion is that there are particular periods of time where the investors' behavior changes significantly, affecting the distribution of returns.

Given the existence of the aforementioned market anomalies, that provide evidence of market inefficiencies, the fundamental question is how this information can be utilized by forecasting models, leading to better portfolio performance. Jensen (1978) highlights the importance of trading profitability when assessing market efficiency: "if a trading rule is not strong enough to outperform a buy and hold strategy on a risk-adjusted basis then it is not economically significant", while Roll (2000) argues that "if calendar time anomalies represent evidence of market inefficiencies, then they ought to represent an exploitable opportunity".

Extending previous work of Lisi and Medio (1997), Cao and Soofi (1999), among others, who applied non-linear techniques in financial applications providing better results compared to the random walk forecasts, this study utilizes a nonlinear chaotic forecasting method on the NASDAQ and Toronto Stock Exchange Composite indices, taking into account specific stylized irregularities of stock returns, reported in empirical finance literature, such as the calendar effects. The methodology applied in the present study overcomes the limitations of previous empirical work, in which either the calendar effects were not taken into account or the predictive ability of the forecasting models was not tested extensively.

The rest of the study is organized as follows: Section 2 describes the data set. Section 3 introduces the algorithm that takes into account the day-of-the-week, the turn of the month and the holiday effect, while Section 4 presents the results of the proposed method. Finally, Section 5 concludes proposing directions for future research.

II. THE DATA SETS AND PRELIMINARY DIAGNOSTICS

The dataset used is comprised of the NASDAQ Composite and the Toronto Stock Exchange 300 Composite (TSE 300), both indices belonging to mature markets having a high trading volume. It is noted that because the first one is a U.S. index while the second one a Canadian one they do not share the same holidays. Additionally, they belong to two extremes: NASDAQ is a technology-based index, therefore a speculative behavior is expected, while TSE 300 is an industry-based index, therefore a more stable behavior is expected. Indicatively, the standard deviation of NASDAQ is about 165% the standard deviation of TSE 300, which means that NASDAQ's behavior is more speculative. Also, a high kurtosis value for each index is observed.

Table 1
Descriptive statistics

	NASDAQ	TSE 300
Observations	4,796	4,775
Max	0.0576	0.0376
Min	-0.0523	-0.0521
Mean	0.0002	0.0001
Std. dev.	0.0063	0.0038
Skewness	-0.2499	-1.1800
Kurtosis	8.3209	17.7803

The period under study for both indices covers the period from February 1984 till December 2003. We transformed the daily closes to first logarithmic differences, so each time series has nearly 4,800 observations. On Table 1 the descriptive statistics for each index are reported.

III. CHAOTIC FORECAST

Given the existence of calendar anomalies, the fundamental question is how this information can enhance portfolio performance and financial forecasting. This section describes the simple chaotic forecasting method, and then introduces an algorithm that embodies the calendar effects.

A. Simple Chaotic Forecast

The dynamics of a time series $\{x_t\}_{t=1}^{N}$ are reconstructed using the vectors $\mathbf{x}_t=(x_t,x_{t+1},...,x_{t+m-1})$, where m is the embedding dimension and the total number of the vectors is T=N-(m-1). The components of these vectors should be linearly uncorrelated otherwise the calculated dimension will approximate 1 regardless of its true value. If m is more than 2 times the dimension of the original (and unknown) dynamic system, then the reconstructed dynamic system preserves all the characteristics of the original one.

In experimental situations such as the reconstruction of the phase space of the financial dynamics we have to relax the previous condition and try different reconstructions (i.e. m may vary). For the empirical results we set m=3 since for that value of m the best results for each index are reported (see also Siriopoulos and Leontitsis 2002).

The prediction is obtained as follows: For the last point of the phase space ($\mathbf{x}_T$) we consider the K nearest neighbors of $\mathbf{x}_T$, denoted as $\mathbf{x}_{T(k)}$, k=1,2,… K. This is a hyper-sphere of radius r. The forecast of the next value of the time series (x_{N+1}) is the mean of $x_{T(k)+(m-1)+1}$. This model can be expanded to take into account linear relation present in the data of each neighborhood, but since the noise level is large (Siriopoulos and Leontitsis 2002) the estimated linear relations can be spurious.

B. Introducing the Calendar Effects

The calendar effects distort not only the distribution of a financial time series but also its dynamics. Therefore an accurate model should be able to restore the time series at least in mean and variance. We consider 3 kinds of calendar effects: the day-of-the-week, the first and the last trading month days, and the holiday effects. In a case that a day falls into 2 or more kinds, the latter holds.

The above algorithm is tested for different values of K. For each K, a minimization algorithm helps to restore the time series. Since we have 8 location parameters and 8 scale parameters we work on a minimization space of 16 dimensions. In this space an optimum combination of these parameters is able to enhance the performance of the algorithm, given that the calendar effects exist on the time series.

Since the calendar effects do not follow the normal distribution (see Table 1), we standardize each calendar effect in a robust way estimating the robust location and the robust scale parameters using the Least Median of Squares (LMS) method proposed by Rousseeuw and Leroy (1987). The LMS location parameter minimizes the median of the squared errors, and the scale parameter is a function of the median of the squared residuals.

Thus, we obtain the calendar corrected time series after subtracting each effect’s bias on location and dividing by each effect’s bias on scale, as in eq. (1).

$$\tilde{x}_i = \frac{x_i - loc_i}{sca_i} \qquad (1)$$

where x_i is an element of the vector of the fist logarithmic differences that belong to a particular calendar event i, loc_i and sca_i are the location and the scale parameters of the calendar effect i, with i=‘Monday’, ‘Tuesday’,..., ‘Last day of trading month’, ‘Holiday’, and $\tilde{x}_i$ is the calendar corrected observation.

Then we rerun the algorithm and measure its performance. Finally we do the inverse transformation to the predicted value that is multiplied by sca_i and add loc_i:

$$\hat{p}_i = \tilde{p}_i \cdot sca_i + loc_i \qquad (2)$$

where $\tilde{p}_i$ is a predicted value for a calendar event i based on the data of the right hand side of eq. (1), and $\hat{p}_i$ is the calendar-corrected predicted value.

IV. EMPIRICAL RESULTS AND COMPARISON OF METHODS

This section applies the method analyzed above to the NASDAQ Composite and TSE 300 Composite indices, and discusses the results. Initially, out-of-sample forecasts on the two indices for the year 2003 and for the 4-year period 2000-2003 are applied. Every forecasted value is the result of the nearest neighbor methodology applied in all the previous values of the forecasted observation.

The most common way to evaluate the forecasting accuracy of a model, how close are the predicted values to the actual ones, is by using the normalized mean squared error. We also focus on the correct forecast of next day’s sign (i.e. to forecast if the time series will go up or down). We adopt the following trading rule for evaluating the performance of our forecasts:

“Start with investing 1$ on the first trading day of the period under study. Do the next day's prediction. If it is positive buy the next day's return, otherwise keep capital unchanged”. Tables 2 and 3 summarize the results.

Table 2

Performance of the simple chaotic forecast algorithm with and without calendar correction of the time series of NASDAQ Composite versus the buy-and-hold strategy

NASDAQ Composite					
Test period 2000-2003			**Test period 2003**		
Performance	Buy-and hold	0.49	Performance	Buy-and hold	1.50
	Without cal. correction	1.29 (K=3)		Without cal. correction	1.52 (K=3)
	With cal. correction	1.89 (K=3)		With cal. correction	1.81 (K=49)
Effect	**Location**	**Scale**	**Effect**	**Location**	**Scale**
Mon	0	1	Mon	0.002198	1.004
Tue	0	1	Tue	-0.0022	1
Wed	0	0.996	Wed	0	1
Thu	0	1	Thu	-0.0022	1
Fri	0	1	Fri	0	1
First	0.002198	0.996	First	0	1
Last	0	0.996	Last	0	1.004
Hol	0	1.012	Hol	0	1

Table 3

Performance of simple chaotic forecast algorithm with and without calendar correction on the time series of TSE 300 Composite versus the buy-and-hold strategy

TSE 300 Cmp.					
Test period 2000-2003			**Test period 2003**		
Performance	Buy-and hold	1.00	Performance	Buy-and hold	1.22
	Without cal. correction	1.35 (K=4)		Without cal. correction	1.18 (K=2)
	With cal. correction	1.97 (K=4)		With cal. correction	1.32 (K=6)
Effect	**Location**	**scale**	**Effect**	**location**	**scale**
Mon	0	0.996	Mon	0	1
Tue	0	1.008	Tue	-0.00179	1
Wed	0	1	Wed	0	1
Thu	0	1	Thu	0	1
Fri	0	0.988	Fri	0	1
First	0	1.004	First	0	1
Last	0	1	Last	0	1
Hol	0.001794	0.996	Hol	0.001794	1.004

The buy-and-hold strategy serves as a benchmark to the performance of the chaotic forecast. The number of neighbors is optimized to result the maximum possible profit on the training period according to the above-mentioned trading rule. One would expect NASDAQ Composite to be a more calendar sensitive index than TSE 300 Composite, due to the sensitive nature of the stocks it represents. Nevertheless, the results indicate that both indices are subject to calendar effects, although these are less evident in the TSE 300 index. This means that the dynamical estimation and correction of the calendar effects may add a new insight to this issue.

V. DISCUSSION AND CONCLUSIONS

Over the past twenty years financial economists have documented numerous stock return patterns related to calendar time. The list includes patterns related to the month-of-the-year, day of-the-week, day-of-the-month, and market closures due to exchange holidays to name a few. Calendar anomalies are not in accordance with the concept of the Efficient Market Hypothesis. However, the simple observation of these anomalies are far from convincing, and do not contradict the EMH unless they can be used to provide exploitable profit opportunities.

This paper presents a nonlinear forecasting method for financial time series taking into account calendar effects, improving the quality of the forecasts and leading to the development of profitable trading strategies (excluding taxes, transaction and other costs).

The study shows that there is an improvement on out-of-sample forecasting results, for calendar-corrected time series. More precisely, both indices were found to be calendar-sensitive. This means that many systematic biases affect their structure. If one subtracts all these pieces of bias from the time series, makes forecasts, and adds them back, the results are improved compared to the buy-and hold strategy or to non-corrected results. It turns out that the calendar effects distort the deterministic structure of a time series. However, for the TSE 300 Composite index, where the presence of calendar effects is weaker compared to the NASDAQ index, the forecasting ability of the model is limited.

Future research may focus on the fact the nonlinear forecasting model is a parameter sensitive technique which results are highly dependent on the selection of the parameters, in contrast to other stochastic techniques such as ARIMA and ARFIMA. Also, focus may be paid to a possible improvement of the proposed neighbor selection method. Finally, another route for future research concerns the application of the proposed methodology to other mature and emerging markets, and the inclusion of more calendar anomalies.

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