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Describing the Nordic Forward Electric-Power Market: A Stochastic Model Approach

P. B. Solibakke
Associate Professor
Molde University College, Norwegian School of Logistics
Servicebokս 8, N-6405 Molde, Norway
per.b.solibakke@himolde.no

ABSTRACT

The paper calibrates stochastic differential equation (SDE) models for the mean and volatility of the Nordic forward electric power market. The main objective is to find appropriate descriptions of commodity markets emphasising schemes for derivative pricing purposes. Our estimation reveals that a two-factor stochastic specification is successful for moments matching. As for path dependent derivatives, simulation can be considered an appropriate numerical methodology for pricing purposes. Re-projections are used to evaluate model characteristics and extract the latent volatility process. Simulation based derivative pricing schemes are implemented.

JEL Classification: *C14*

Keywords: *Stochastic differential equations; Efficient method of moments; Re-projection; Monte Carlo simulation*

I. INTRODUCTION AND MOTIVATION

This paper estimates and analyses the partially observable Nordic one-year forward electric-power data series. The objective is to see whether versions of stochastic differential equations (SDEs) can appropriately describe the characteristics of the Nordic electric-power market. If the methodology is appropriate the use of simulation may contribute significantly to a higher understanding of market behaviour. Moreover, the SDEs output might enhance derivative pricing methodologies in the forward market. The electric-power market deregulation in 1992 introduced a period for the Nordic electric-power market showing growth in liquidity (lower trading costs and lower bid-asks spreads) and the evolution of rather sophisticated derivative instruments. The Nordic electric-power market has shown European integration inducing prices closer to other European power markets[1]. Considering that the Nordic electric market is the most liquid in Europe, a successful model implementation might contribute strongly to market understanding and enhanced risk management activities. SDE markets implementations will indicate non-predictive market features and weak-form market efficiency introducing these commodity markets to conventional funds and enhanced risk management activities. The foundation for a non-predictive and an efficient European electric-power market is important for political acceptance of market allocations and product pricings to both private and industrial end consumers. Moreover, for the market participants in general, a SDE implementation induces market price efficiency reflecting that all historical information is accounted for. Hence, the random walk induces no prediction power for the commodity market.

The theoretical motivation for the use of stochastic calculus and SDEs in estimation studies is that new (unpredictable) information is revealed continuously in an open market and decision-makers may face instantaneous changes in randomness. For example, the relevant "time interval"[2 3] may be different on different trading days, due to volatility changes. Changing volatility may require changing the basic observation period. Although seen in equity markets, these features may become more important in electric power markets due to both higher number and a higher sensitivity of systematic market factors. As numerical methods used in pricing securities are costly, the pace of activity may make the analyst choose coarser or finer time intervals depending on the level of volatility. Such approximations can best be accomplished using random variables defined over continuous time (stochastic calculus). A technical advantage of stochastic calculus is that a complicated random variable can have a very simple structure in continuous time, once the attention is focused on infinitesimal intervals. If the time interval is "infinitesimal" then asset prices may safely be assumed to have two likely movements; up-tick or downtick[4]. Under some conditionals, such a binomial structure may be a good approximation to reality during an infinitesimal interval, but not necessarily in a large discrete time interval.

Several procedures have been proposed for fitting models based on stochastic calculus[5]. This paper employs Efficient Method of Moments (EMM) proposed by Bansal, Gallant, Hussey and Tauchen (1993, 1995) and developed in Gallant and Tauchen (1996) and shown used in Gallant, Hsieh and Tauchen (1997) to estimate and test the stochastic differential equation (SDE) model. EMM is a (Monte-Carlo) simulation-based moment matching procedure with certain advantages. The moments that get matched are the scores of an auxiliary model called the score generator (SNP). Bayes Information Criterion[6] (BIC) values are used to approximate conditional densities from data series, applying serial correlation, volatility clustering, and hermite polynomial series expansions to model data features. The SNP model is fitted using conventional maximum likelihood together with a statistical objective model selection strategy (BIC). If this score generator approximates the distribution of the data well, estimates of the SDE parameters are as efficient as if maximum likelihood had been employed (Tauchen, 1996 and Gallant and Long, 1997)[7]. Importantly, failure to match the moments can be used to indicate features of series that the SDE model cannot accommodate (Tauchen, 1995). Hence, the EMM methodology may relatively easily obtain the preferred SDE specification suggesting improved model specifications relative to classical method of moments.

Due to recent theoretical achievements in finance, diffusion models have become more important for empirical time series applications. The EMM approach is only one of several alternative simulation strategies[8]. Despite recent progress, some of these alternatives are not for general purpose and are limited in their ability to deal with latent variables. State space methods for example, described in Durbin and Koopman (2001) covering non-gaussian observations, non-linearities and heavy-tailed densities, show adequate basis for many of the time series that are encountered in practice. The use of importance sampling and antithetic variables suggest strong efficiency improvements. However, unobservable and latent variables seem not to be adequately modelled in state space models. The EMM procedures have recently found several new applications [9] and accessibility and software for EMM implementations are available from several sources[10].

The rest of the paper is organised as follows. Section II defines the dynamics of stochastic differential equations. Section III defines the series, describes the adjustment procedures, and expands sequentially the score generator (SNP). Section IV reports the score characteristics that are important for the SDE specification details and reports the parameters and significance of the stochastic differential equations (SDE) and re-projects volatility. Section V reports the empirical findings and predicts the unobservable volatility process. Section VI summarises and concludes.

II. THE DYNAMICS OF STOCHASTIC DIFFERENTIAL EQUATIONS

A. Setup, Notation and Intuition

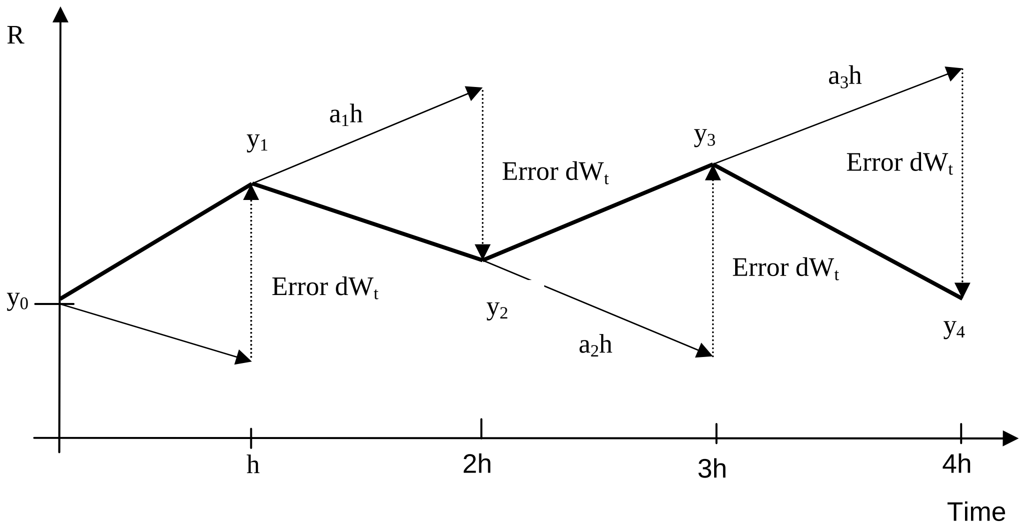
Let y_t denote the first difference (logarithmic) over a short time interval of the price of a financial asset traded on an active speculative market. The notation for dy_t in a stochastic differential equation is $dy_t = a(y_t, t)dt + \sigma(y_t, t)dW_t$, $t \in [0, \infty)$, justified as a symbolic way of writing

$\int_{t}^{t+h} dy_u = \int_{t}^{t+h} a(y_u, u)du + \int_{t}^{t+h} \sigma(y_u, u)dW_u$, when *h* is infinitesimal. The dW_t is an innovation term representing unpredictable events that occur during the infinitesimal time interval dt. The $a(y_t,t)$ and the $\sigma(y_t,t)$ are the drift and the diffusion coefficients, respectively. Given that the decomposition is done using the information set available at time t, then to the extent different investors may have access to different sets of information, the SDE may also be different. An "insider" may learn all the random events that influence price changes in advance. This implies a zero diffusion term. Since the participant knows how dy_t is going to change, he or she can predict this variable perfectly, and $\sigma(y_t,t) = 0$ for all t. This result also implies that the exact form of the SDE, and hence the definition of the error term dW_t, always depends on the family of information sets $\{I_t, t \in [0, T]\}$. Formally summarised by saying that the Wiener process W_t is adapted to the family of information sets I_t. Hence, $a(y_t,t)$ and $\sigma(y_t,t)$ are both I_t-adapted. Moreover, the derivation of SDEs is compatible with the way dealers behave in financial markets. In fact, during a given trading day a trader is continuously trying to forecast the change in the price of an asset and record the realisation of the new events as time passes. These events always contain some parts that are unpredictable until one observes the dy_t. After that they become known and become part of the new information set (I_t) the trader possesses. The drift and diffusion parameters of the SDE are allowed to depend on y_t and t. These variables are themselves random variables. Given the information set at time t, they are observed by the market participants. Conditional on available information, they "become" constant. This is the consequence of the important assumption that these parameters are I_t - adapted. Moreover, we usually assume that these parameters be well behaved. These regularity conditions say that the $a(y_t,t)$ and $\sigma(y_t,t)$ parameters are assumed to satisfy the conditions:

$$P\left(\int_{0}^{t} | a(y_u, u) | du < \infty\right) = 1 \text{ and } P\left(\int_{0}^{t} | \sigma(y_u, u)^2 | du < \infty\right) = 1 \qquad (1)$$

which imply that the drift and the diffusion parameters are functions of bounded variation with probability one. Consider now a geometric behaviour of the SDE. Figure 1 shows an example path for y_t. We consider small but discrete intervals of length h. From Figure 1 we see that over time, the behaviour of y_t can be decomposed into two types of movements. First there is an expected path during the interval. Upward- or downward-sloping arrows indicate these changes. Then at each $t_k = k{*}h$, there is a second movement orthogonal (uncorrelated) to the predicted changes, which are unpredictable. Vertical arrows represent these changes. Sometimes they are positive; other times they are negative. The actual movement of y_t over t is determined by the sum of these two components and is indicated by the heavy line. The derivation emphasises that the trajectories of y_t are likely to be very erratic when h becomes infinitesimal.

Figure 1
A geometric description of paths implied by SDEs



where $y_0,\ldots,y_4$ is returns at period 1 to 4, h is time interval and dW_t is a wiener process.

B. Efficient Method of Moments [11]

We should like to estimate the parameter ρ that appears in the system of stochastic differential equations $dU_t = a(U_t;\rho)dt + b(U_t;\rho)dW_t$ where $t \in [0, \infty)$, and ρ defines the parameter space for the model. The parameter ρ has dimension p_ρ, the state vector U_t has dimension d, W_t is a k-dimensional vector of independent Wiener processes; $a(\cdot;\rho)$ maps $\Re^d$ into $\Re^d$, and $b(\cdot;\rho)$ is a d x k- matrix comprised of the column vectors $b_1(\cdot;\rho)$, ... $b_k(\cdot;\rho)$, each of which maps $\Re^d$ into $\Re^d$. U_t is interpreted as the solution of the equations $U_t = U_0 + \int_0^t a(U_u, u;\rho)du + \sum_{i=1}^{k}\int_0^t b_i(U_u, u;\rho)dW_{iu}$, where U_0 is the initial condition at time t=0, and $\int_0^t b_i(U_u, u;\rho)dW_{iu}$ denotes the Ito stochastic integral (Gihman and Skorohod, 1972, Øksendal, 1992). The system is observed at, equally spaced time intervals t = 0,1,… and selected characteristics $y_{t-L} = T(U_t)$ and t = 0, 1, ... of the state are recorded, where y_t is an M-dimensional vector and L > 0 is the number of lagged variables that enter the formulas which follow. The continuous time version of the stochastic volatility model that was proposed by Clark (1973), Tauchen and Pitts

(1983), and others, is a description of speculative markets. Hence, for daily price observations on two securities the model is

$$\left.\begin{aligned} dU_{1t} &= (\rho_1 - \rho_2 \cdot U_{1t})dt + \rho_3 \cdot U_{1t} dW_{1t} \\ dU_{2t} &= (\rho_4 - \rho_5 \cdot U_{2t})dt + \rho_6 \cdot \exp(U_{1t}) dW_{2t} \\ dU_{3t} &= (\rho_7 - \rho_8 \cdot U_{3t})dt + \rho_9 \cdot \exp(U_{1t}) dW_{3t} \end{aligned}\right\} \; t \in [0, \infty), \quad \left.\begin{aligned} y_{1t} &= U_{2t} \\ y_{2t} &= U_{3t} \end{aligned}\right\} \; t = 0, 1, ..., n \tag{2}$$

where U_{1t} represents an unobserved flow of new information to the market that influences the volatility of asset prices U_{2t} and U_{3t} by changing the instantaneous conditional variances of U_{2t} and U_{3t}. The observed data $\tilde{y}_{1t}$ and $\tilde{y}_{2t}$ are prices recorded at the end of each trading day. To achieve identification in estimation, normalisation rule such as $\rho_9 = 1$ should be imposed[12 13.] For each setting of parameter ρ and lag length L, there exists a time invariant density $p(y_{-L},\ldots,y_0|\rho)$ so that

$$E_\rho(g) = \lim_{N \to \infty} \frac{1}{N} \sum_{t=0}^{N} g(\hat{y}_{t-L},\ldots,\hat{y}_t) = \int \ldots \int g(y_{-L},\ldots,y_0).p(y_{-L},\ldots,y_0 \mid \rho) dy_{-L} \ldots dy_0 \tag{3}$$

where $\hat{y}_{t=-L}^{N}$ is realisation of length N + L + 1 from the system. This assumes that the function g is integretable and that either U_0 is a sample from the stationary distribution of U_t or that a longer realisation was observed and enough initial observations were discarded for transients to have dissipated[14].

C. The Relative efficiency

A measure of relative efficiency for the three moment functions, classical method of moments (CMM), maximum likelihood (ML) and efficient method of moments (EMM), is reported in Gallant and Tauchen (1999) assuming independency and identical distribution. Their results induce that EMM dominates relative to CMM and that the EMM efficiency increases rapidly relative to ML, once $f_k(\cdot \mid \theta^0)$ begins to approximate $p(\cdot \mid \theta^0)$. Hence, applying standard statistical model selection criteria for $f_k(\cdot \mid \theta^0)$ determination, and include as many parameters as is required for well approximation between EMM and ML, suggest advantageous of EMM relative to other methodologies. Moreover, standard statistical methods, both classical and Bayesian, are usually not applicable for the SDE specification either because it is not practical to obtain the likelihood for the entire state vector or because the integration required to eliminate unobservable series from the likelihood is infeasible.

Consequently, as shown by Gallant and Tauchen (1996), if an auxiliary model (i.e. ARMA-GARCH or SNP) encompasses the true data generating process, then quasi-maximum likelihood estimates make EMM fully efficient. Moreover, if this auxiliary model is a close approximation to the data generating process, then one can expect the efficiency of EMM to be close to that of maximum likelihood (Gallant and Tauchen, 2001) [15].

Table 1

Characteristics of the forward price changes in the electric power market

	Mean / Std.dev.	Maximum / Minimum	Kurtosis / Skew	Q(6) / Q2(6)	K-S Z-test	ARCH (12)	RESET (12;6)	BDS m=2;ε=1	m=3;ε=1
Raw	0.041	12.365	6.543	18.068	2.003	48.189	28.837	5.823	7.161
Series	1.435	-14.123	0.744	69.404	{0.001}	{0.000}	{0.004}	{0.000}	{0.000}
Adjusted	0.041	9.962	4.227	22.153	1.421	156.278	37.363	4.814	6.048
Series	1.435	-8.225	0.279	296.920	{0.035}	{0.000}	{0.000}	{0.000}	{0.000}

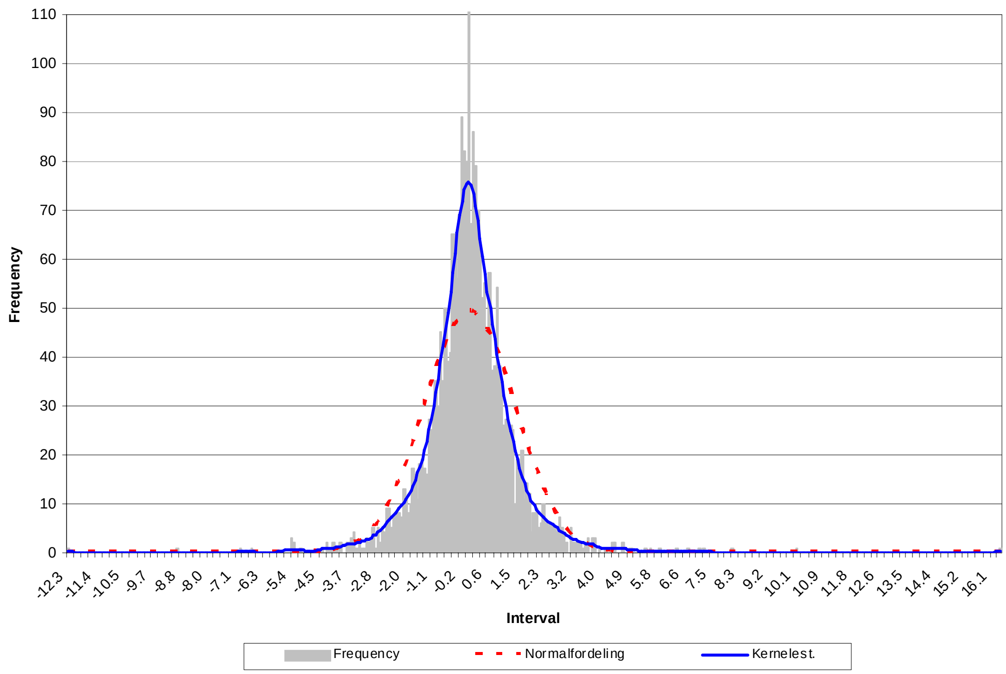
Skew is a measure of heavy tails and asymmetry of a distribution (normal) and kurtosis is measure of too many observations around the mean for a distribution (normal). Q/Q^2: Ljung and Box (1978) autocorrelation test for adjusted raw returns and squared returns. K-S Z-test: Used to test the hypothesis that a sample comes from a normal distribution. The value of the Kolmogorov-Smirnov Z-test is based on the largest absolute difference between the observed and the theoretical cumulative distributions. ARCH (12) is a test for conditional heteroscedasticity in returns. Low {·} indicates significant values. We employ the OLS-regression $y^2 = a_0 + a_1 \cdot y^2_{t-1} + \ldots + a_{12} \cdot y^2_{t-12}$. $T \cdot R^2$ is χ^2 distributed with 12 degrees of freedom. T is the number of observations, *y* is returns and R^2 is the explained over total variation. a_0, a_1 … a_6 are parameters. RESET (12,6) : A sensitivity test for mainly linearity in the mean equation. 12 is number of lags and 6 is the number of moments that is chosen in our implementation of the test statistic.$T \cdot R^2$ is χ^2 distributed with 12 degrees of freedom. BDS (m=2,ε=1): A test statistic for general non-linearity in a time series. The test statistic BDS $= T^{1/2}.[C_m(\sigma \cdot \varepsilon) - C_1(\sigma \cdot \varepsilon)^m]$, where *C* is based on the correlation-integral, m is the dimension and ε is the number of standard deviations. Under the null hypothesis of identically and independently distributed (i.i.d.) series, the BDS-test statistic is asymptotic normally distributed with a zero mean and with a known but complicated variance.

III. DATA AND PROJECTIONS

The data to which we fit the univariate stochastic volatility model is a Nordic time series comprised of 2134 daily observations, $\{\tilde{y}_t\}_{t=1}^{2134}$ on adjusted movements on the Nordpool Exchange forward price series, 1995-2004. The series, which consists of all available quotes from the financial market, shows seasonal effects in the whole period and we therefore adjust for systematic location and scale effects. The log first difference of the price index is adjusted but the unit of measurement of the adjusted series is the same as that of the original series and is reported in Table 1. The adjusted series shows a mean of –0.0561 (negative drift), standard deviation of 0.7516, strong ARCH effects (Engle, 1982) signalling volatility clustering, highly significant RESET test statistic (Ramsey, 1969) signalling a non-linear mean, and the BDS test statistics (Scheinkman, 1991) signal general non-linear dependencies in the data series. The K-S Z-test together with the kurtosis and skew numbers, report that the series have non-normal density characteristics. The three test statistics ARCH, RESET and BDS, suggest data dependence. A classical sequence plot shows no apparent trend and regime switches in the adjusted time series. In Figure 2 shows the frequency distribution of the whole adjusted series together with a Gaussian kernel, and the normal distribution. From both the frequency and the kernel plots characteristics, the series deviate from a normal distribution showing too many observations around the mean, too few observations at one standard deviation (both positive and negative) and heavy tails.

Figure 2

Frequency distribution, Kernel & Normal distribution



The EMM estimator is implemented using the SNP score generator *f(y | x, θ)* (Gallant and Tauchen (1989)), which is verified by Gallant and Tauchen (1996) and Gallant and Long (1997), and shown to be the best choice of a moment function to implement simulated method of moments for dynamic systems. Hence, the best choice has the form $\tilde{\Psi}_f(x,y) = \frac{\partial}{\partial\theta}\log f^*(y \mid x, \tilde{\theta}_n)$, where $\tilde{\theta}_n = \arg\min\ s_n(\theta_n)$ and $s_n(\theta) = -\frac{1}{n}\sum_{t=1}^{n}\log f(\tilde{y}_t \mid \tilde{x}_{t-1}, \tilde{y}_t)$. Moreover, the covariance terms can be neglected when the auxiliary model closely approximates the transition density of the system (Gallant and Long, 1997). The tuning parameters L_μ, L_g, L_r, L_p, K_z, and K_x follow the protocol that is described in detail in Bansal, Gallant, Hussey, and Tauchen (1995). The BIC (Schwarz, 1978) model selection criterion expands the model[16]. The protocol for the forward series reports computed BIC, AIC and HQC values in Table 2[17]. The first block of Table 2 (cases 1-3) increases L_μ to determine the preferred VAR fit. The second block of the table (cases 4-7) increases L_r to determine the Schwarz preferred ARCH fit. Introducing GARCH by increasing L_g and adjusting L_r accordingly, determine the Schwarz preferred GARCH fit (case 10-12). Hence, this Gaussian GARCH Score model specifies two lags in the mean equation and one moving average (ARCH) and one autoregressive (GARCH) lag for the variance equation. The Schwarz preferred Semi-parametric GARCH score (K_z>0) is shown in Table 2 case 10-12. Finally, a fully non-linear model specification is evaluated. The results in Table 2 suggest that the preferred linear model is $(L_\mu, L_g, L_r, L_p, K_z, I_z, K_x, I_x)$[18] = (1, 1, 2, 1, 4, 0, 0, 0). Specification tests are shown in Table 3 for the preferred linear SNP model. The test statistics induce no data dependence suggesting no need for a non-linear extension, as the non-linear BIC values show non-preferable values (case 13 Table 2).

Table 2
Optimized likelihood and model selection criteria

Case	L_m	L_g	L_r	L_p	K_z	I_z	K_x	I_x	p_q	s_n	BIC	HQ	AIC
1	0	0	0	1	0	0	0	0	3	1.4231	1.4267	1.4250	1.4241
2	1	0	0	1	0	0	0	0	4	1.4150	1.4204	1.4179	1.4164
3	2	0	0	1	0	0	0	0	5	1.4135	1.4208	1.4174	1.4154
4	1	0	1	1	0	0	0	0	5	1.3699	1.3771	1.3737	1.3718
5	1	0	7	1	0	0	0	0	11	1.3007	1.3189	1.3104	1.3055
6	1	0	8	1	0	0	0	0	12	1.2937	1.3137	1.3044	1.2989
7	1	0	9	1	0	0	0	0	13	1.2923	1.3142	1.3039	1.2980
8	1	1	1	1	0	0	0	0	6	1.2868	1.2959	1.2916	1.2892
9	1	1	2	1	0	0	0	0	7	1.2846	1.2955	1.2904	1.2874
10	1	1	3	1	0	0	0	0	8	1.2828	1.2956	1.2896	1.2862
11	1	2	1	1	0	0	0	0	7	1.2857	1.2966	1.2915	1.2885
12	1	2	2	1	0	0	0	0	8	1.2842	1.2969	1.2909	1.2875
13	1	1	2	1	4	0	0	0	11	1.2683	1.2865	1.2780	1.2730
14	1	1	2	1	5	0	0	0	12	1.2651	1.2852	1.2758	1.2704 *
15	1	1	2	1	6	0	0	0	13	1.2646	1.2865	1.2762	1.2703
16	1	1	2	1	5	0	1	0	17	1.2605	1.2914	1.2769	1.2686

L_μ is the number of lags in the linear part of the SNP model; L_g is the number of lags in the GARCH part; L_r is the number of lags in the ARCH part; L_p is the number of lags in the polynomial part, P(z,x). The polynomial

P(z,x) is the degree K_z in z and K_x in x; by convention, $L_p = 1$ if $K_x = 0$. *p* is the number of parameters. The values of I_z and I_x are irrelevant because the series is univariate.

Table 3

Test statistics for the BIC preferred semiparametric GARCH SNP Model*

Panel A.	Mean	Standard deviation	Max. Min.	Kurtosis Skew	Q(12)	Q2(12)	ARCH (12)	RESET (12;6)
Residual	-0.0025	1.0004	5.4249	1.1776	9.3020	9.3290	9.2311	13.8047
			-3.3997	0.1445	{0.6770}	{0.6750}	{0.6831}	{0.3134}
Panel B.	BDS-statistic (ε=1)						K-S	Joint
	m=2	m=3	m=4	m=5	m=6		Z-test	Bias
Residual	-1.5988	-1.3171	-1.4998	-0.3636	-0.8526		-1.1890	7.2323
	{0.1111}	{0.1676}	{0.1296}	{0.3734}	{0.2774}		-{0.1183}	{0.0649}

* See Table 1 for description of the specification test statistics.

IV. SDE SPECIFICATIONS FOR THE FORWARD ELECTRIC-POWER MARKET

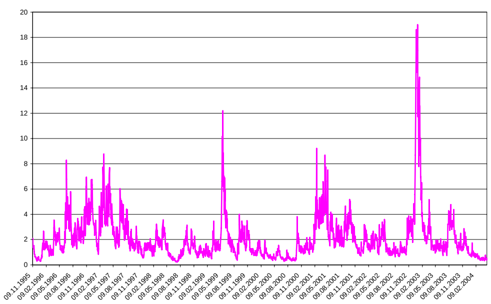
A. Characteristic details

Some characteristics of the time series, supporting the stochastic differential equation (SDE), are reported in Figure 3. The daily conditional volatility is plotted in panel A. The one-step-ahead density $f_K(\tilde{y}_t \mid x_{t-1}.\hat{\theta})$, conditional on the values for $x_{t-1} = (\tilde{y}'_{t-L}, \cdots, \tilde{y}'_{t-2}, \tilde{y}'_{t-1})'$, is plotted in panel B. All lags are set at the unconditional mean of the data. The conditional variance function is plotted in panel C where we show the average over all $x_{t-1} = (y_{t-L}, \cdots, y_{t-2}, y_{t-1})$ in the data of the conditional variance $VAR(y_t \mid y_{t-L}, \cdots, y_{t-1} + \delta)$ plotted against δ, the percentage growth. The variance profile is plotted in panel D. The profiles $\hat{\Psi}_j(x)$ are the forecast of the one-step variance j steps ahead, conditional on x_t = x, where $x = (y'_{-L+1}, y'_{-L+2}, \cdots, y'_0)'$. Typically one plots $\hat{\Psi}_j(x)$ against j at three points: x^0, which defines a baseline initial condition, x^+ that corresponds to a positive impulse δy^+, and x^-, which corresponds to a negative impulse δy^-. For each sub-vector of x a baseline is chosen to the unconditional mean of the data y. A conditional volatility profile may be computed by Monte Carlo integration as follows. Let $\{y_j^r\}_{j=1}^{\infty}$, r = 1, 2, ⋯, R, denote R simulated realisations of the process starting from x_0 = x. In other words, y_1^r is a random drawing from f(y | x) with $x = (y'_{-L+1}, y'_{-L+2}, \cdots, y'_0)'$; y_2^r is a random drawing from f(y|x) with

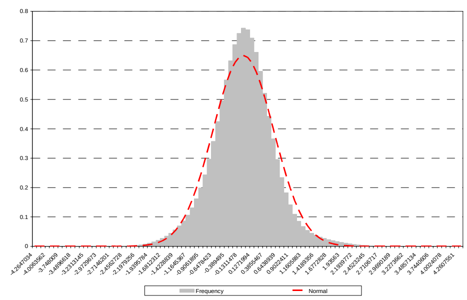
$x = (y'_{-L+2}, y'_{-L+3}, \cdots, y'_0, y'^{r}_1)'$, and so forth. Then $\hat{\Psi}_j(x) = \frac{1}{R} \cdot \sum_{r=1}^{R} \mathrm{Var}(y^r_j \mid y^r_{j-1-L}, \cdots, y^r_{j-1})$ for j = 1,2, … where $x = (y'_{-L+1}, y'_{-L+2}, \cdots, y'_0)'$ with the approximation error tending to zero almost surely as $R \to \infty$. The profile bundles for the mean and volatility is reported in panel E and F, respectively. The profile bundle for the volatility suggests considerable persistence in the series and suggests a need for modelling serial correlation for the latent volatility process. All these mean and volatility densities are very useful for the SDE implementation to follow.

Figure 3

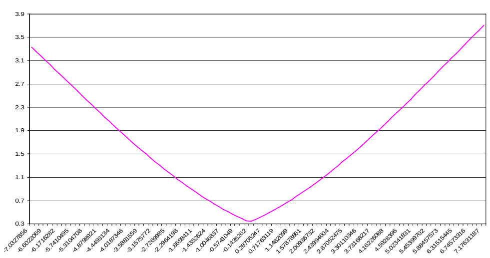
Characteristics for the semi-parametric GARCH score model



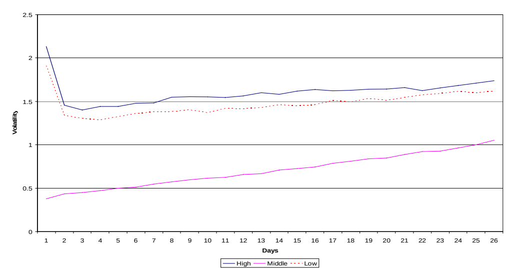
Panel A. Daily Latent Volatility



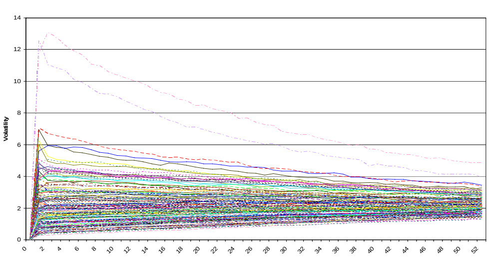
Panel B. Conditional Density Function



Panel C. Conditional Variance Function.



Panel D. Conditional Variance Profiles.



Panel F. Profile Bundle for the Volatility.

B. EMM implementation

Table 4 reports the score, diagnostics and parameter estimates for the various versions of the SDEs for the preferred Score generator. The first model may be thought of as a variant of a model with first order drift and diffusion allowing for heterogeneity in both the drift and the diffusion

$$\text{F}-\text{CHD}-\text{CHD} \qquad \begin{aligned} dy_t &= (a_y + a_{yy}y_t)dt + (b_y + b_{yy}y_t)\cdot e^{\omega_t}\cdot dW_{1t} \\ d\omega_t &= 0 \end{aligned}, \tag{4}$$

with ω_0 =1.0. Taking $\omega_0 = 1$ normalises $\omega_t = 1$ for all $t > 0$. The model's fit is reported in the first row of Table 4. The SDE model is rejected based on the χ^2(df) omnibus test and inspection of the polynomial (A's), the Var (psi's) and ARCH (tau's) parameters, indicate a mismatch to both the polynomial and the ARCH moments (t ratios are above 2.0 in magnitude). The specification therefore accommodates the linear aspects of the data but fails to account for the leptokurtic character of stock return movements and the ARCH-like behaviour of stock return movements. More elaborate SDE models appropriate for these features are therefore introduced. The second model introduces second order drift and diffusions and allow for a changing drift in the mean and a changing drift and diffusion in the volatility processes

$$\text{S}-\text{CHD}-\text{SVD} \qquad \begin{aligned} dy_t &= (a_y + a_{yy}y_t)dt + b_y\cdot e^{\omega_t}\cdot dW_{1t} \\ d\omega_t &= (a_\omega + a_{\omega\omega}\omega_t)dt + b_\omega\cdot dW_{2t} \end{aligned}, \tag{5}$$

with $a_\omega = a_{\omega\omega}$ for identification. Stochastic volatility is introduced applying the parameter b_ω and the Wiener process dW_{2t}. The model also introduces volatility clustering by means of the $a_{\omega\omega}$ parameter in the drift term of the volatility process. The model's fit and model parameters are reported in Table 4, line 5. The omnibus test statistic χ^2(df) cannot reject the model. However, inspection of the moment parameter t-ratios reveals that the model produces moment-ratios that match the features defined by the VAR and the ARCH parts of the Score Generator, but fails to match the features of the POLYNOMIALS. Hence, again, the specification fails to account for the leptokurtic character of price movements.

This SDE specification must therefore be rejected. The third SDE model therefore focuses leptokurtosis. The model maintains second order drift and diffusion but extends the SDE by introducing the instantaneous volatility in the diffusion process of the latent volatility. The model therefore becomes

$$\text{S}-\text{CHD}-\text{SVLD} \qquad \begin{aligned} dy_t &= (a_y + a_{yy}y_t)dt + b_y\cdot e^{\omega_t}\cdot dW_{1t} \\ d\omega_t &= (a_\omega + a_{\omega\omega}\omega_t)dt + (b_\omega + b_{\omega\omega}\omega_t)\cdot dW_{2t} \end{aligned}, \tag{6}$$

This two factor stochastic volatility model's fit is reported in Table 4 line 9. The $\chi^2(df)$ omnibus test statistic cannot reject the model. Moreover, applying a large number of simulations, the inspection of the A's, the psi's and the tau's parameters, suggest success (t ratios are well below 2 in magnitude). Finally, the fourth model includes an in-Mean specification allowing for volatility in the drift of the structural equation, modelling any form of risk compensation from volatility movements in the market. The model is

$$\begin{pmatrix} S-CHD-SVLD \\ in-Mean \end{pmatrix} \quad \begin{matrix} dy_t = (a_y + a_{yy}y_t + a_{y\omega}\omega_t)dt + b_{1y} \cdot e^{\omega_t} \cdot dW_{1t} \\ d\omega_t = (a_\omega + a_{\omega\omega}\omega_t)dt + (b_{2\omega} + b_{2y}y_t) \cdot dW_{2t} \end{matrix}, \qquad (7)$$

This two-factor stochastic volatility model is reported in Table 4 line 13. The $a_{y\omega}$ parameter for the in-Mean measurement is significant at 1%. Hence, risk compensation is present in the forward market.

Table 4
Stochastic differential equations:
Parameters, t-ratios and Wald confidence intervals

SDE Model	*Score*	*ay*	*ayy*	*ayw*	*aww*	*by*	*byy*	*bw*	*bww*
F-CHD-CHD	51.168	3.61384	-496.84206	---	---	10.6152	-0.0778	---	---
	{11.804}	{0.2925}	-{11.1843}	---	---	{14.187}	-{0.345}	---	---
	95% Lower	-22.4596	-591.9720	---	---	9.0545	-0.5849	---	---
	95% Upper	30.8615	-415.2238	---	---	12.3545	0.4459	---	---
S-CHD-SVD	8.901	3.3553	-601.1014	---	-10.2695	13.4410	---	2.0108	---
	{0.8380}	{0.3285}	-{10.6931}	---	-{4.4938}	{14.258}	---	{10.310}	---
	95% Lower	3.3450	-601.4467	---	-10.2955	13.4351	---	2.0101	---
	95% Upper	3.8192	-600.6337	---	-10.2599	13.4421	---	2.0314	---
S-CHD-SVLD	8.4690	17.2191	-579.9288	---	-10.1522	14.4871	---	2.3235	-0.4844
	{1.097}	{1.0889}	-{15.9648}	---	-{9.0734}	{9.729}	---	{8.025}	-{4.066}
	95% Lower	17.1807	-579.9799	---	-10.2433	14.4817	---	2.3223	-0.4845
	95% Upper	17.2343	-579.8232	---	-10.1459	14.6368	---	2.3240	-0.4844
S-CHD-SVLD	6.6290	1.5155	550.0952	30.3331	-9.0841	2.5032	---	6.1116	-3.4610
in-Mean	{0.930}	{0.0695}	-{8.7252}	{2.6434}	-{0.8342}	{12.150}	---	{6.530}	-{5.987}
	95% Lower	1.2418	-550.7183	29.7616	-9.1381	12.5019	---	5.9920	-3.4635
	95% Upper	4.5547	-541.1356	30.35227	-8.7354	12.5093	---	6.1146	-3.3787

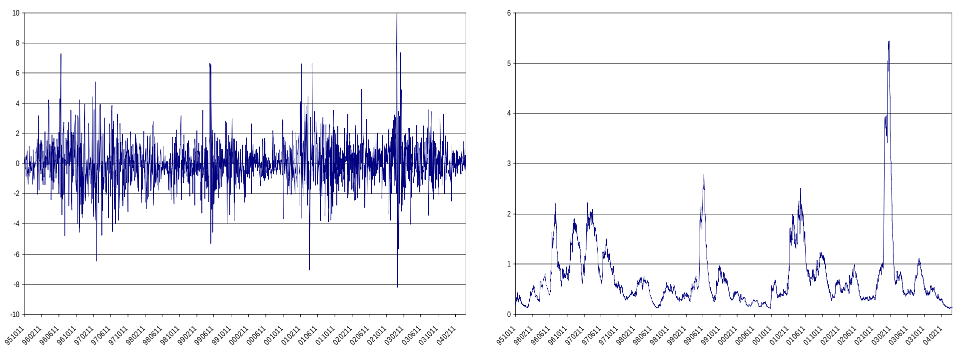
F-CHD-CHD = First order Conditional Heteroscedasticity drift and Conditional Heteroscedasticity Constant Volatility Diffusion; S-CHD-SVD = Second order Conditional Heteroscedasticity in the Drift and Stochastic Volatility Diffusion; S-CHD-SVLD = Second order Conditional Heteroscedasticity in the Drift and Stochastic Level Adjusted Volatility Diffusion; and S-CHD-SVLD-in-Mean = Second order Conditional Heteroscedasticity in Drift and Stochastic Level Adjusted Volatility Diffusion with in-Mean par. The *a*'s and *b*'s are parameters for the Stochastic differential equations (SDE), *y* is the mean process and ω is the volatility processes. The parameters $a_{\omega s}$ and $b_{\omega s}$ (volatility) are insignificant in all SDE models (not reported). dt = 1/6048; parameters are yearly numbers.

C. Re-projections[19]

Scatterplots (y_t-y_{t-1} against y_{t-1}) for raw- as well as projected values, being aware of the different series lengths the preferred projected model, seems to capture the features of the raw data. The trend lines and the R^2 statistics in these plots indicate a good match between the model-generated data and the raw data. Re-projected one-step ahead conditional volatilities seem to mimic the raw data characteristics. Shape characteristics of the re-projected transaction density are seen leptokurtic relative to the normal density mimicking the general characteristics from the projection phase. One-step-ahead volatilities with unforeseen and foreseen shocks show similarities and interestingly, show that the effects from unforeseen shocks seem twice as high as foreseen shock effects. The projected and re-projected volatility profiles (future volatility) mimic each other closely. Multi step dynamics are considered by examining re-projected profile bundles. The similarities are strong, showing mean reversion in the mean and strong persistence in the conditional volatility (relatively long persistence).

Now we apply a backward process intending to infer the unobserved state vector from the observed process implied by the SDE model. The simulation results can be evaluated on the observed data series, but the unobservable volatility process cannot be evaluated. However, working with the simulation and the functional form of the conditional volatility distribution, functions of the volatility given the observed price change series $\{y_\tau\}_{\tau=1}^{t}$ can be constructed and evaluated on the observed data. Due to filtering and a large data set, very general functions of $\{y_\tau\}_{\tau=1}^{t}$ may be used. The identification procedure starts with estimation of a classic GARCH model on the simulated mean process ($\hat{y}_t$). The GARCH model is convenient as it provides a representation for the one-step ahead conditional variance σ_t^2 of $\hat{y}_{t-1}$ given the simulated series. Running regressions of $\hat{U}_t$ (latent volatility) on σ_t^2, $\hat{y}_t$ and $|\hat{y}_t|$ and lags of these series with relative long lag lengths determined by OLS t-ratios, calibrates functions that give predicted values of $\hat{U}_t$ given $\{y_\tau\}_{\tau=1}^{t}$. The estimated parameter values can be evaluated for the observed data series, giving re-projected values for the volatility at the data points. Figure 4 reports the raw returns (panel A) and the re-projected volatility (panel B) at the data points. Interestingly, Figure 4 shows that the turbulence in the spring of 1999 for example, is clearly manifested in the re-projected volatility process suggesting a temporary volatility increase. Moreover, the mean and the volatility plots together seem to suggest that the mean and the volatility are consistent over the whole time period.

Figure 4
Raw data series and the re-projected volatility process



Panel A. Raw Price Change Process   Panel B. Re-Projected Volatility Process

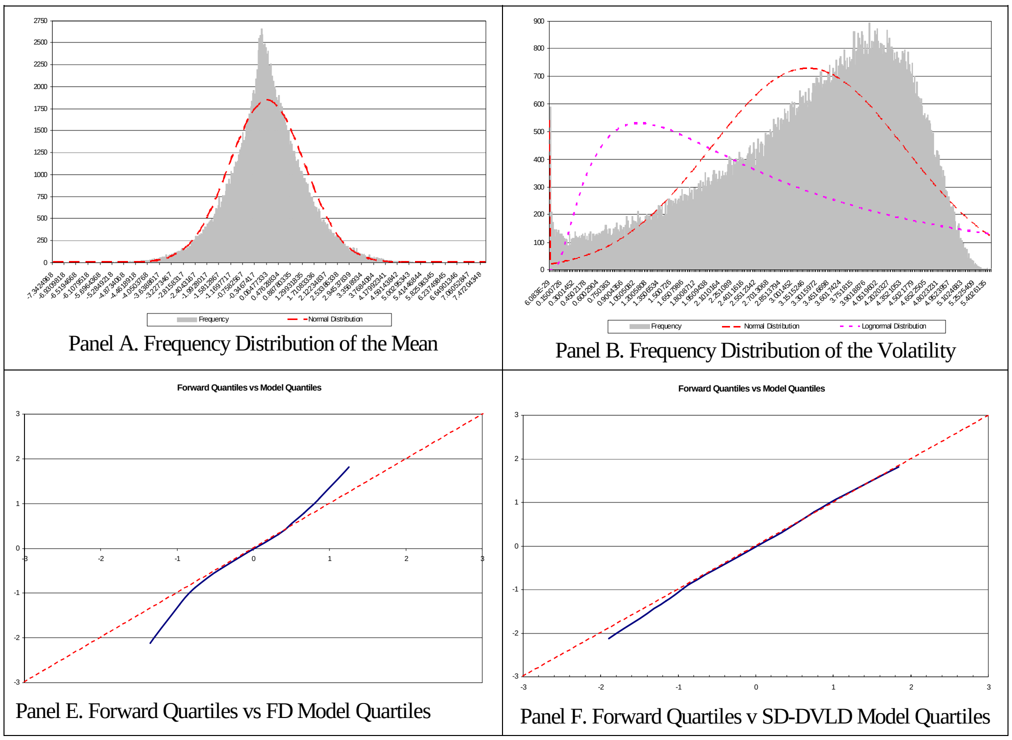
V. EMPIRICAL FINDINGS AND VOLATILITY PREDICTIONS FOR DERIVATIVE PRICING

Figure 3 has reported projection characteristics of the preferred semi-parametric Score Generator, which is a detailed and effective descriptor of the commodity market's one-year forward time series. The Scores show that the market exhibits characteristics that are in line with other equity, currency and commodity markets[20]. The series shows serial correlation, volatility clustering (possible GARCH) and a need for Hermite-Polynoms, controlling for leptokurtosis. The projections show symmetric volatility and positive skewness for the series suggesting higher positive than negative shock effects. In contrast to the conditional mean, which show mean reversion, persistence is found for the conditional volatility process.

The preferred SDE model is a two-factor model with a changing drift in the mean equation, stochastic volatility and in-Mean volatility adjustments. Figure 5 reports characteristics for a simulation of the simultaneously derived mean and volatility equations[21], derived from the preferred stochastic model. For this model, the simulation series shows leptokurtosis in the mean but not in volatility, illustrated in panel A and B. Serial correlation seems present for both the mean and volatility. Serial correlation in the mean may stem partly form non-synchronous trading and in the volatility from volatility clustering. Panel C and D show the forward market series percentiles versus model percentiles (1%-99%) for the One-factor model and the preferred Two-factor stochastic volatility model, respectively. As expected the plots of the unconditional percentiles suggest that the One-factor model fail to capture the tail properties of the data series. In contrast, the two-factor model seems to capture several more aspects of the raw data series.

Figure 5

Characteristics of the SDE simulations based on the preferred score model moments



Panel A. Frequency Distribution of the Mean

Panel B. Frequency Distribution of the Volatility

Panel E. Forward Quartiles vs FD Model Quartiles

Panel F. Forward Quartiles v SD-DVLD Model Quartiles

The information in the mean equation can be outlined using the following SDE interpretations. The two-factor diffusion model suggests that the mean drift equation change with the mean level and not only with the time increment. The mean equation shows a positive drift and a negative mean reversion per unit time. Moreover, the drift seems to incorporate a risk component from the latent volatility process. The mean diffusion function as an exponential stochastic volatility process seems appropriate. Importantly, the b_{yy} parameter is insignificant and the level of y_t does not influence the diffusion part of the equation. The mean therefore consists of (1) a constant and mean changing (predictable) factor, (2) an in-Mean effect compensating for risk and (3) a stochastic (unpredictable) instantaneous factor. The latent volatility process is a simple differential equation with a negative mean reversion and a loading factor associated with the drift for appropriate calibration. Both the drift and diffusions parts of the volatility equation are influenced by the level of the instantaneous volatility. The higher the instantaneous volatility the lower is the drift and the lower is the diffusion. The volatility equation includes no feedback from the mean series. Hence, the volatility consists of (1) a constant and changing (predictable) level factor and (2) a stochastic and a changing (unpredictable) instantaneous level factor.

Consequently, the price of a derivative of a forward/future may not be possible to determine, due to stochastic volatility. The reason is that there may not exist a self-financing portfolio strategy involving forwards/futures and risk-less bonds applying a tracking portfolio approach perfectly replication the derivatives pay-off (the arbitrage argument). As this simple two-factor SDE model approach matches the full complexity of the forward electric-power market, the long simulation may give derivative prices for the forward market. In absence of hedging possibilities (perfect tracking) the only available methodology for perfect derivative pricing is simulation based methodologies that closely replicate market characteristics and dynamic equilibrium models. Moreover, the results of Broadie and Glasserman (1996) give the direct path wise estimates for the hedge parameters within a single simulation run (greeks). Consequently, the stochastic volatility results suggest that as for valuation of path dependent options, simulation may be considered as the best numerical method for derivative valuation[22]. Hence, any derivative can be priced using the preferred SDE-specifications and parameters together with simulation techniques in Mathematica® or any other programming tools[23].

VI. SUMMARIES AND CONCLUSIONS

This investigation uses Efficient Methods of Moments estimation to estimate SDE parameters for the observed price changes and the unobserved and latent volatility, in continuous time for the forward Nordic electric power market. The results suggest success for a general and a simple two-equation and a two-factor stochastic differential equation (SDE) model that incorporates a changing drift, in-Mean effects and stochastic volatility for the structural equation and the incorporation of the instantaneous volatility level for the volatility equation. The preferred SDE model is a two-factor model with mean and volatility parameters reported in Table 4. As shown by Tauchen (1997) and Andersen and Lund (1997) a two-factor model is needed, as a single factor model cannot quite capture all the dynamics in this complex commodity market. Our methodology applies re-projection to report market characteristics and to extract a general formula for the latent volatility process from the original raw price change series applicable for volatility forecasting. As pricing formulas for derivatives incorporate functions of the parameters from the process driving the volatility of stock returns, the result may have practical relevance. For example, the above results suggest a need to replace σ with $\int_t^T \sigma(s) \cdot ds$ over the derivative's life in the Black and Scholes option pricing formula. Even more importantly, it may not be possible to determine a tracking portfolio and therefore a price of the option by arbitrage arguments.

Heuristically, because we now have two sources of uncertainty, the option is no longer "spanned" by a dynamic portfolio of stocks and bonds applying the tracking portfolio approach. Since Goldman, Sosin and Gatto (1979) show that the option to sell at the maximum is indeed spanned, we may apply the Cox-Ross method. However, it may be more difficult to verify spanning for more complex path-dependent derivatives. Hence, the results suggest that a simulation based stochastic volatility specification is a valid specification for the Nordic electric power market. The approach can be used for derivative valuation. Moreover, as the valuation of derivatives including path dependent derivatives, has become very important in the forward market, simulations incorporating stochastic volatility may be the most appropriate approach for derivative pricing.

ENDNOTES

1. The leading energy exchange in Central Europe is Germany's energy exchange located in Leipzig.
2. “time interval” is a reference to for example dynamic synthetic option adjustments. Volatility changes may change the need for portfolio adjustments in for example applying portfolio insurance techniques.
3. Can thinly traded markets be well approximated by continuous time diffusion? In fact, as shown by Solibakke (2000) asset volatility seems to exist without relation to the mean process. That is, in an open market the diffusion seems to a continuous process only adjusting for the time increment. Sudden changes in volatility may require changes in asset and portfolio evaluation, changing the traders relevant “time interval”.
4. With reference to the Binomial Distribution.
5. For method of moments see Duffie and Singleton, 1993 and Andersen and Sørensen, 1996; for bayesian methods see Jacquier, Polson and Rossi, 1994 and Geweke, 1994; for simulated likelihood see Danielson, 1994; for Kalman filtering methods see Harvey, Ruiz and Shephard, 1994 and Kim and Shephard, 1994.
6. Schwartz (1978)
7. If the auxiliary model encompasses the true data generating process, then quasi-maximum likelihood estimates become sufficient statistics and EMM if fully efficient (Gallant and Tauchen, 1996).
8. See e.g. Brandt and Santa-Clara (1999), Durham and Gallant (2002), Elerian, Chib and Shephard (2001).
9. See e.g. Andersen and Lund (1997), Dai and Singleton (2001) for interest rate applications. See e.g. Lui (2000), Andersen, Benzoni and Lund (2001), Cherov, Gallant, Ghysels, and Tauchen (2001) for long memory and jumps. See Chung and Tauchen (2001) for target zone models of exchange rates. See Valderama (2001) for a macroeconomic analysis and see Nagypal (2001) for labour economic applications.

10. Fortran version: ftp.econ.duke.edu; OX-version: see van der Sluis (1997); Jeffrey Wang is directing a project for the inclusion of EMM in S-PLUS; Donald Erdman is directing a project for the inclusion of EMM in SAS.
11. Our work is strongly influenced by lectures at Norwegian School of Economics and Business Administration in June 1998 of Prof. Kenneth Singleton (1998), Stanford University and papers of Gallant and Tauchen (1997, 1998). All ideas behind Moment Estimation are discussed fully from both sources.
12. We should remark that one does not have to restrict attention to models where latent variables affect only volatility. They can affect the drift as well; see, for example Andersen and Lund (1997).
13. We assume that U_t and hence y_t is stationary and ergodic. We further assume that the stationary distribution of y_t is absolutely continuous. However, note that the SDE specifications from y_t will indicate ergodicity and stationary series.
14. Several methods for generating $\hat{y}_{t=-L}^{N}$ are described in Gallant and Tauchen, (1997).
15. For a thorough description of efficiency of EMM relative to classical method of moments and maximum likelihood, see Gallant and Tauchen (2001).
16. BIC is the Bayes Information Criterion from Schwarz (1978).
17. AIC is the Akaike Information criterion (Akaike, 1969) and HQC is the Hannan Quality Criterion (Hannan, 1987).
18. The values of I_z and I_x are irrelevant as our data series is univariate.
19. Plots for projected versus re-projected characteristics are all available from author upon request.
20. Findings are serial correlation, volatility clustering and leptokurtosis.
21. Simultaneous derived equations mean data from a general-purpose simulation SDE subroutine. The applied routine produces a weak-2 approximate solution to an exact SDE solution (Kloeden and Platen, 1992) (one-term Ito-Taylor improvement on the elementary Euler Scheme).
22. Simulations of varying lengths are available from author upon request.
23. The t-ratios, serial correlation plots and the Mathematica® implementation for derivative pricing are all available from author upon request.

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