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How to Calculate An Internal Rating? Synthesis and Proposition for A Contingent Approach Using A Monte Carlo Simulation and A Rate Stochastic Model Following Poisson's Law

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ABSTRACT

The decisions taken by the Basle Committee have encouraged the banks to develop more innovative methodologies in order to assess the financial risks. The Credit Risk can be measured with a homogeneous measuring unit called VaR. This measuring unit stands for various and complementary approaches: VaR Rating, VaR Spread, VaR contingent. This paper aims at presenting these various methodologies and suggests a contingent approach with an interest rate stochastic diffusion process (including jump functions distributed following a law of Poisson) and a Monte Carlo simulation. Empirical tests are also included in this paper.

JEL Classification: *G19*

Keywords: *Credit risk model; Basle II; Rating; Contingent approach; Interest rate stochastic process*

I. INTRODUCTION

We shall demonstrate in the first part that credit risk is still a current issue with the Basle II agreements and the slackening of the economic activity. Today, the credit risk is fully integrated in the precautionary surveillance either using regulatory methods (Cooke– Mac Donough weighting) or internal methods.

Presently the banks are turning toward internal methodologies when these latter require less risk covering equity funds than the regulatory methods. Internal approaches either rely on the ratings and default probabilities associated, or on the connexion between the credit spreads noticed on the markets and the ratings or else, on contingent approaches using option models. Whatever the approach used, the risk management banking managers turn the risk into a homogeneous measuring unit called value at Risk (VaR). In the second part, we shall present the VaR rating and the VaR spread.

VaR credit approaches are grounded on ratings. These ratings are issued by credit rating agencies. This rating only concerns a minority of companies. The purpose of the third part is to present a synthesis of the latest synthetic appraisal methodologies of credit risk.

We shall conclude the article by suggesting a particular modelling of credit risk using market variables for listed companies. Ideally, the risk can be analysed by calculating the company's call with the company's assets as underlying asset and the strike price with the debt value. We shall try, during this presentation, to answer the following question: Do market parameters or contingent approaches enable the credit risk to be assessed, and if so, to what extent? The article suggests a particular modelling of the option with an interest rate stochastic diffusion process following a Poisson's law. A Monte Carlo simulation will also be proposed to optimise the examination of the underlying asset's volatility, namely the economic assets. An illustration over ten companies will be presented after the model.

II. THE STANDARD APPROACH OF BASLE II

Adaptability and innovation are two of the many features of Finance. On the contrary, banking regulations quickly prove to be obsolete. The new Basle agreement is part of a will to define a better apportionment of equity funds that better corresponds to banking practices. Indeed, since 1988, (last Basle agreements), these banking practices have been greatly modified in particular in the credit risk domain. Let us mention, titrisation, with the development of A.B.S.[1] – C.L.O[2], titrised retail supports to spare owner's equity or the common use of credit derivatives such as the default swap credit or even better, the total return swap which allows to handle concomitantly market risks and credit risks. A rethink of the 1988 Basle agreements and of Cooke ratio turn out to be necessary to take into account these new practices, new risks such as the operational risk and, to reduce inconsistent balance sheet's arbitrages in terms of risk, whenever the banks make use of the regulatory capital method. Concerning this latter, the stocks issued by the OECD country members are considered as risk-free whatever the rating. Moreover, *corporate* issues have an identical weighting in regulatory capital whatever the default risk.

The Basle committee aims at making up for Cooke ratio's limitations and builds the **new standard approach** on three points.

The first one is to take the operational risk into account. There is a regulatory approach to that risk which rests on a weighting proportional to the bank's GNP. The bank may follow its own model. The Bayesienne approach of incidents and seriousness allow the modelling of this risk.

The second is the market risk management, which remains unmodified further to the 1996 amendments.

The last point we are more directly concerned with in relation with this article is to make the credit risk measures more accurate. The basic or standard approach is improved by creating a regulatory capital not only with regard to the issuer's category (enterprise – State...) but also with regard to the ratings provided by rating agencies such as S&P, Moody's or even FITCH. This basic approach clearly aims at eliminating regulatory arbitrages conflicting with the control of credit risk. The agreement also encourages the banks to develop their own models or internal models with a possible reduction of owner's equity requirement as a reward.

There are mainly two methodological trends for these **internal models**. The first models use the descriptive and historical statistical analysis among the ratings and default probabilities noticed. This is a universal approach and may also be built using rating transition matrix. Another approach directly models the credit spreads, comparing the spreads noticed on the market with the ratings. This latter, of course, only works with listed companies

III. CREDIT RISK APPROACHES BUILT ON INTERNAL MODELS

A. VaR Credit Approaches Using Default Probabilities

Credit rating agencies publish transition matrix from one rating to the other relying on historical observations (Moody's as well as Standard and Poors' data). Therefore, we have probabilities of relative credit risk when dealing with rating deterioration and probabilities of absolute credit risk when a default occurs for a considered rating.

a. Resorting to statistics[3]

- EL (*Expected loss*) is the default mathematical life span but the credit risk depends on the volatility of that risk measured thanks to the UL (*unexpected loss*) or the standard deviation of credit losses. EL and UL are actually determined by the default probability distribution.
- EAD (*Exposure at default*) deals with outstanding credits and confirmed lines not used when calculating off balance sheet or the par value for bond portfolios.
- DP (*Default probability*) is the probability that the counterpart be lacking over a determined time span. For products with a maturity lasting more than a year, the probability over the period may be annualised. To the contrary, a maturity shorter than a year requires an extrapolation to obtain the yearly probability (linear interpolation , polynomial or else)

- LGD (*Loss Given Defaults*) is the loss linked to the default once guaranties and subsequent collections have been taken into account.

The risk modelling can start from a binomial random variable. By taking the capital multiplier into account we obtain an endowment in owner's equity.

b. Advantages and drawbacks of the method

1. Advantages

This empirical methodology has the advantage of being universal provided you have ratings or credit notes at disposal.

2. Drawbacks

The notion of capital multiplier and the correlations of defaults generate weak points. These two notions are valued by the bank and maintain a subjective character, which does not guarantee the impartiality of the credit VaR.

B. The Credit VaR Approach Using Bond Spreads

a. Presentation of the methodology

The credit risk is altogether the risk of non-payment of the principal, coupons, interests or, further to a degradation of the issuer's financial quality. This credit risk is to be noticed and/or anticipated in the spread. That spread is to be accurately defined and is not a trivial approach by merely calculating the difference between two rate curves, the risk-free curve and the issuer's curve.

The first stage consists naturally in choosing a curve of reference rates without zero coupon risk. The reference curve is generally that of the French, American government bonds. Other approaches will prefer a SWAP curve. Although as liquid, this latter already integrates a mean spread credit linked to the participants to that market. Trading room managers estimate that spread between 20 and 50 bps. On the other hand, the use of STRIP curves is to be proscribed on account of liquidity problems. These curves without risk or with the smallest risk on the market must follow the zero coupon methodology in order to obtain a zero coupon curve for each currency.

- The method used is the gradual spreading naturally adapted to the payment conditions of the coupons (biannual in the USA and annual in France). The academic literature suggests some methods to obtain a more sophisticated zero coupon curve such as that of MacCulloch[4].
- McCulloch's methodology: this method consists in estimating the zero coupon curve using a polynomial function. Time is a polynomial function associated to each cash flow. It is actually a mere multiple linear regression estimated by the least ordinary square number. This function gives the bond its market price.

According to Anderson and Al.[5], this method is suitable when cash flows are hardly separated in terms of time.

- Nelson – Siegel's model[6]: This model proposes modelling of implicit forward rates rather than modelling the term structure of the interest rates. The modelling of the forward rates takes the shape of an exponential polynomial function.

To obtain the spread, the next stage is to compare both curves over the same term. The zero coupon risk-free curve is obtained at definite periods. In order to know the rates of the intermediary maturities, interpolations must be resorted to. There is an extensive literature on that issue. In practice, linear extrapolation according to Thalles principle is just resorted to. Unfortunately it is ill-adapted to curves.

- B-Spline model: this method proposed by Steely[7] and Fisher and Al.[8] improves the prior method by proposing a « cubic basis spline » interpolation.
- Vasicek-Fong model: Vasicek and Fong[9] observe that polynomial smoothing methods generate unsteady Forward rates. Rate curves are often rising and a lot closer, mathematically speaking, than an exponential function. The authors suggest the use of exponential functions to approximate the dots of the zero coupon curve. At first sight, this method has not proved its superiority in empirical tests.
- Elbantli [10] model : Elbantli proposes a cubic polynomial.

The knowledge of historical spreads per rating enables to calculate a spread VaR over a whole portfolio and implicitly, to calculate an associated loss. The great advantage of this method is the direct use of a market data more reactive than information stemming from a rating.

b. Advantages and drawbacks of the method

1. Advantages

Contrary to the previous method, the « migration risk » is included.

2. Drawbacks

This method only deals with big companies, which possess a rating as well as a quotation of their debt.

IV. RATINGS

The internal ratings based approach requires ratings – *Documentation of rating system design*[11]. We present in this section the various appraisal sources for the credit quality of a counterpart.

A. Rating Agencies

In order to present rating methods, we submit to the reader the article published in the *Banque Magazine*.[12]

B. Scoring on Non-listed Companies

Since the 70's, for lack of rating, banking circles have been using statistical tools initially designed by Altman and Beaver (1968) to forecast a failure. That tool or score provides us with the client's failure probability. This probability may be translated into a mark or a score. The statistical approach of the credit risk has proved its efficiency in case of credit extension to individuals and in the control of risks. The score is a scientific approach of a decision process. It requires a statistical pooling and treatment procedure to analyse the information of the process to be considered. Presently, the bankruptcy process and counterpart risk are described and synthesised in a linear combination of financial ratios. The risk level of a company depends on the value Z gathered by the linear combination.

a. The score: a forecasting tool for the credit risk?

For private banking services, scores are better at predicting the default risk. The variables explaining a low risk would bear on the stability of the individual (home – work) rather than on the income level variables only. On the other hand, scoring have proved acknowledged weaknesses to assess the risk of bankruptcy over companies. Moreover, coherence difficulties are easily noticed in the bankruptcy scores of companies. The contribution to the risk of a financial ratio in the score may differ from the financial theory. It is common to have company bankruptcy scores whose redemption capacity, added value rate or earning power rate increase the risk of bankruptcy since these ratios are associated to a negative coefficient.

b. The improvement sources proposed[13]

Of course, the above mentioned coherence problems damage the relevance and the efficiency of the scores used. There may be many improvement sources.

- The first improvement is technical. All scores are calculated using the financial data of the corporate income tax return form. The score's relevance will depend on how fast the company's balance sheet will be available. The interest of calculating the score using proper banking data is evident since banks have the customers' corporate income tax return form in their possession before the commercial inquiry agencies. Being more rapidly in possession of the financial data, the banks are likely to anticipate the other scores and the most suitable baking strategy for the customer considered.
- The second source of improvement is financial. Most of the trading systems are based on the financial data issued from the corporate income tax return form.

The whole qualitative information on the company (problem of strategy, financial management, quality, market positioning…) is synthesized in the quantitative evolution of the financial variables. The accountancy is a representation of economic facts and deeds. Nevertheless, the scope allowed by this latter may warp the accuracy of this representation. The improvement consists in setting up a financial filter to assess the accuracy of the financial information. That filter is actually an expert system for detecting window dressing. This system refers to sensitive financial variables likely to warp the honesty of the company's financial situation. For example, a change in the settlement date of an account for a more favourable period in the company's activity cycle, removals of provisions to display a more positive result. This expert system is also useful when using the score to invalidate a note or an assessment of the credit risk further to a suspicion of financial tampering.

- The scoring construction methods may be sharpened by using a step by step scoring construction method under both statistical and financial constraint and using the latest financial restatement techniques.[14] Thus, classical methodologies for multi-function discriminating analyses are abandoned.

C. Contingent Approaches or Structural Models for Listed Companies – KMV Rating

a. Theoretical models

Contingent approach modelling on the Credit Risk relies on Merton and Vasicek – Kealhofer's works, who consider that an action made by a company is a perpetual call on the assets' value with the debt value as exercise price. According to this contingent assets theory, the company has gone bankrupt when the assets' value is inferior to the debt value. The call of the contingent approach is therefore a barrier option. This approach enables to determine the default probability, which is the difference between the assets' value and that of the debt or « *distance to default* ». The difficulty of this approach is to calculate the debt value using realistic hypotheses on the distribution mode of the rate curve. Moreover, the concept of debt is not homogeneous since there are subordinate debts (« *senior debt* ») non pre-emptive in case of default, market debts and « *retail* » *debts.* The debt as a whole also differs according to the term.

It is even more difficult to assess the value of the assets as well as their volatility. As a whole, the exercise is solved by calculating a two-equation system while considering that the assets' value is equal to the total of share and debt values.

The first equation is the Call value in function of the assets' value and volatility, of the capital structure and the interest rates: Call = $f(V, \sigma_V)$. The second equation is the one that links the assets value volatility with the volatility of the of S share value: $\sigma_{VA} = f(S, \sigma_s)$. Solving the system allows to define the distribution principles of the assets value and to transform the « *distance to default* » into default probability. A great number of scientific works have proposed several distribution models for interest rates and assets' value.

Table 1[15]

Summary of the various structural models

References	Asset Value	Default Risk Free Rate
Black and Scholes (1973) ; Merton (1974)	$dV_A = \mu V_A dt + \sigma V_A dz$	dr = rdt (the rate is a constant)
Black and Cox (1976)	$dV_A = (\mu-\delta)V_A dt + \sigma V_A dz$	dr = rdt
KMV Model or Vasicek – Kealhofer (1993)	$dV_A = V_A dt + \left(\mu - \left(\frac{\sigma_1^2}{2}\right)dt\right) + \sigma_1 \sqrt{t}\, dz_1$	dr = rdt
Leland (1994) ; Leland and Toft (1996)	$dV_A = (\mu(V_A, t)-\delta)dt + \sigma V_A dz$	dr = rdt
Shimko, Tejima, and Van Deventer (1993)	$dV_A = \mu V_A dt + \sigma_1 V_A dz_1$	$dr = k(\gamma - r)dt + \sigma_2 dz_2$ [16]
Kim, Ramaswany, and Sundaresan (1993)	$dV_A = V_A dt + \sigma_1 V_A dz_1$ [A]	$dr = k(\gamma - r)dt + \sigma_2 \sqrt{r}\, dz_2$
Longstaff and Schwartz (1995)	$dV_A = \mu V_A dt + \sigma_1 V_A dz_1$	$dr = (\gamma - kr)dt + \sigma_2 dz_2$
Briys and de Varenne (1997)	$dV_A = rV_A dt + \sigma_1(\rho dz_2 + \sqrt{1-\rho^2}\, V_A dz_1)$ [17]	$dr = k(t)\,\gamma(t) - r)dt + \sigma_2 \sqrt{r}\, dz_2$
Zhou (1997)	$dV_A = (\mu - \lambda\delta)V_A dt + \sigma_1 V_A dz_1 + (\prod -1)Dj$ [18]	$dr = (\gamma - kr)dt + \sigma_2 dz_2$

μ : is the instantaneous profitability rate anticipated on the company's assets and σ the variance in a Wiener Standard process

V_A : asset's value

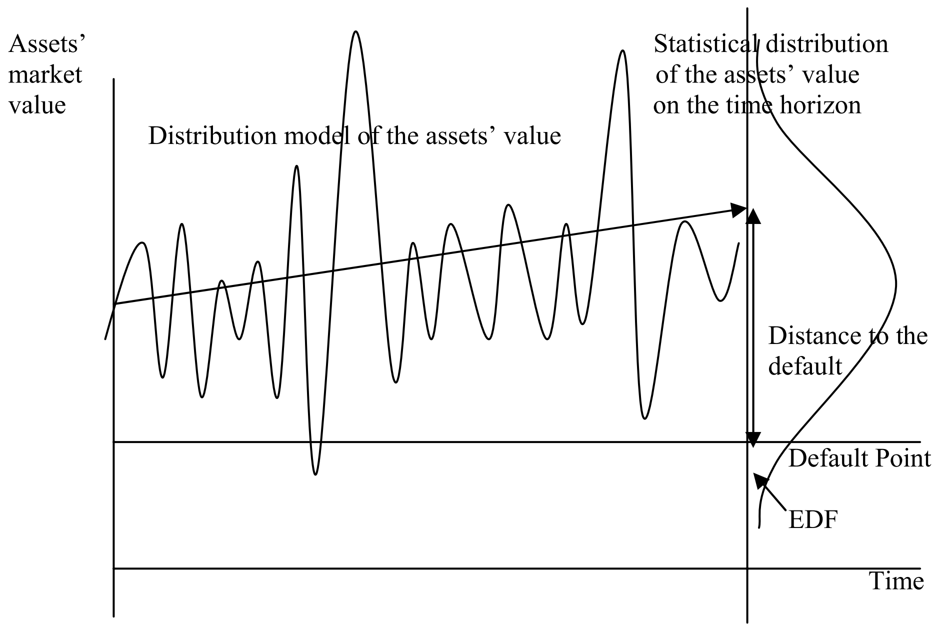
b. Calculation of the distance to the default

There are six variables that allow to determine the default probability for a defined time horizon.

- Current asset's value
- Distribution of the asset's value
- Volatility of the asset's value
- Default point or debt's value
- Expected growth rate of the asset's value
- Time horizon

Once the model is parameterised, we can define the EDF (*expected Default Frequency*) which is the probability that the assets' value be inferior to the debt's value. The distribution of the distance to the default is not standard. Moreover, there are substantial correlations between the financial leverage level and the assets' value. For that reason, Vasicek- Kealhofer KMV model measures the distance to the default using the number of standard deviations from the assets' value to reach the default point . In order to improve the relevance of this approach, KMV statistically binds the distance to the default with empirical data on the defaults which allows frequency tables of defaults noticed to be set up for each level of distance to default calculated (option Vasicek – Kealhofer).

Figure 1



c. Empirical tests

We cannot but acknowledge the increasing sophistication of credit risk modelling. On the other hand, empirical tests are loosing ground. In the literature, the first empirical researches date back to 1984, in particular those carried out by Jones, Mason, and Rosenfeld. The results show that the contingent approach reduce the assessment of the credit risk compared to the credit spread noticed on the market. Later, Franks and Torous (1989) demonstrate that the contingent approach is a coherent approach to assess credit spreads. More recently, Wei and Guo (1997) have proposed the models of both Merton and Longstaff – Schwartz. These tests favourable, in principle, to Merton have actually a limited scope due to the small size of the sample.

d. Possible sources of improvement

We notice that the variation in the assets' value depends on the volatility of the debt value. A possible source of improvement is to upgrade the interest rate distribution models.

V. OUR CONCEPTUAL CONTRIBUTION: A CONTINGENT APPROACH OF THE CREDIT RISK WITH A DISTRIBUTION MODEL OF POISSONIAN RATES APPLIED TO AN F.R.N.[19] PORTFOLIO AND AN ASSET SWAP

A. The Suggested Model

In compliance with the contingent approach, the share is considered as a perpetual call on the assets' value with the debt value as exercise price.

a. Models of rate distribution

A calculation of the debt value requires a rate distribution model. There are two types of rate distribution models[20].

1. Deformation models. These models examine the evolution of the price whenever the interest rate structure is subjected to deformations.
2. Arbitrage models. These models rest on arbitrage reasoning and on hypotheses on rate distribution stochastic processes.

We suggest a deformation model. Unlike the discreet binomial model of Ho and Lee, we use a discreet model of the poissonian type. We shall justify this methodological choice. To finish with, we shall present an illustration of the model in order to assert the coherence of contingent approaches resting on rate deformation models.

b. Poissonian model of deformation

Over a 4-year history report on the EUR and 5 years on the USD, the usual tests[21] show that the statistical distribution of each point of the curve (3 months, 1 year…) does not follow a regular law. This finding invalidates the models that are in fact, gaussian stochastic processes with a return to average. More confident with these findings, we can assert that the distribution of each point on the curve undergoes Poisson jump functions with a historical intensity $\prod$ based on a random selection of a reduced centred regular law N(0,1) [22]. This random process uses the average and standard deviation of the point on the curve as descriptive data over the observation period. Therefore, by using the random walk model terminology, we can claim that the rate model is a sub-martingale: $E(\tilde{R}_t / \varphi_t) \geq r$ or $\leq r$, with r as interest rate and φ_t as the whole information available on t time.

This approach allows us gauge the random process for each point of the rate curve (ideally a swap curve) but also for each currency. The debt's current value will provide us with an exercise price in the call a lot more realistic than the models built on constant rates or stochastic processes with return to average.

c. Definition of the debt

The definition of the debt is also a critical point. The call strike depends on this definition. On that issue, an arbitrage between a thorough financial analysis and an operationality research is seriously called for. On second thoughts, we take in Bloomberg all the information related to the debt of the parent company concerned to calculate the current stochastic value (standard issuances, convertible bonds evaluated as a standard bond and a call valued according to B&S' model). The complementary debt stemming from the consolidated balance-sheet data owns, out of hypothesis, the same average characteristic. The current value of the consolidated short-term debt is equal to its book value. Some other points remain unanswered, such as the treatment of some information off balance-sheet, the leasing, the stock options often difficult to exploit on a large scale because of missing information.

The current stochastic value of the debt is calculated from the stochastic rate model presented previously with 35 iterations per point on the curve and 750 Poisson functions that is to say, on the whole, 700 iterations and 525 000 jump functions for the USD and the EUR. Bellow, the mathematical formula of our rate distribution model of stochastic rates with jump functions following a Poisson law:

$$d_r = [(d_{z1} + \Pi) \times \sigma_r^{\theta} + \bar{r}^{\theta}]dt \times P(\lambda)$$

or

$$d_r = \tilde{r}^{\theta} dt \times P(\lambda)$$

with $d_{z1} = \frac{1}{\sqrt{2\Pi}} e^{-t^2/2}$ a Wiener process $P(\lambda) = e^{-\lambda} \times \frac{\lambda^k}{k!}$. where λ=1 in this Poisson law; k is an intensity parameter of the jump function; Π repeats the range of the jump function noticed on the history reports previously mentioned; and θ represents the points of the rate curve [1,30] years in EUR and USD. The advantage of this model is to take the cyclical effect of the interest rate into account by gauging σ_r, r of each point on the curve and by taking Π.

Naturally, there are other rate distribution models whose main default lies in their overall modelling of the interest rate curves whereas a point by point or section modelling (monetary part and bondholding part or curve swap) seem to be preferable. This aspect reinforces the attractiveness of the model proposed. Our proposition to model the curve point by point (3 months, 6 months, 1 year, 2 years...) is totally compatible with a Monte Carlo simulation. Indeed, there is no particular coherence for each type of curve generated but there is coherence among all the curves produced through the rate distribution model.

d. The call evaluation model

Let VA= the asset's value; $B_t = d_{z1}$ a standard brownian movement or a Wiener process:

$$E(VA_t) = E\int_0^{\infty} VA_t e^{-rt} dt$$

Just like B&S, we consider that the initial formulation of the distribution process of the company assets' value takes the shape of a stochastic differential equation of the type:

$$dVA_t = VA_t(rdt + \sigma' dB_t) \quad (1)$$

with a specificity in this case since σ' is a function of the volatility of the company's shares. Function to be defined

$$\sigma'_{VA} = f(S, \sigma_s) \quad (2)$$

As a rule, the equation (1) is integrated using the lemma of Itô. This lemma can be used when the SDE (Stochastic Differential Equation) has a Wiener's process as a component whose essential mathematical property is a continuous quadratic variation. In annexes (A1), the unique solution of the integral can easily be demonstrated by considering the function g(t,B)

$$g(t, B) = VA_0 e^{((r - \frac{\sigma^2}{2})t + \sigma B)}$$

on the interval [0,T] with T as the expiration date of the option, the equation (1) is explicitly solved

$$VA_t = VA_0 e^{((r - \frac{\sigma^2}{2})t + \sigma B)} = VA_0 e^{(\sigma dVA - \frac{\sigma^2}{2} t)}$$

The random variable, like the KMV model, follows a lognormal law.

The call price takes this algebraic form, considering that the distribution process of the underlying asset is a Wiener process and, following the stochastic model previously presented [23]. At last, we get the valuation of the option:

$$\left(\begin{array}{l} PCALL = VA_0\, N(d1(VA_0, PE_D, T, r, \sigma')) - PE_D e^{T\left[\left(d_{z1} + \Pi\right)\times\sigma_r^{\theta} + \bar{r}^{\theta}\right]\times P(\lambda)}\, N(d2(VA_0, PE, T, r, \sigma')) \\ d1 = \dfrac{\ln\dfrac{VA_0}{PE_D} + \left(\left[\left(d_{z1} + \Pi\right)\times\sigma_r^{\theta} + \bar{r}^{\theta}\right]\times P(\lambda) - \dfrac{\sigma'^2}{2}\right)T}{\sigma'\sqrt{T}} \\ d2 = d1 - \sigma'\sqrt{T} \end{array}\right)$$

On average, this formulation is mathematically correct.

e. A Monte Carlo simulation in order to know the volatility of the company's asset or the underlying asset

In our formal approach, the only missing parameter is the volatility of the company assets' value. To assess this parameter, we suggest a Monte Carlo simulation approach. Indeed, at this stage, we have a detailed valuation o f the debt with an interest rate generator. The current stochastic value may be calculated at each stage or day by day. We also have a day-by-day stock market valuation of the company and through summation according to M.M. well-grounded principles; we shall infer the asset's volatility. We regard the asset as following a Brownian movement. The asset's volatility will be the volatility left unexplained by this Brownian process.

B. Illustration of the Model

a. Presentation of the results

The empirical tests bear on a sample of 10 enterprises from various activity sectors. The data on the debt come from Bloomberg and from the Internet sites displaying the enterprises' consolidated accounts. Thus, we calculated the debt average duration, in order to pick the point in the rate curve to be accounted with in the calculation of the call. The stock market capitalisation and the shares' value also come from Bloomberg. You will find hereafter, a full presentation of the test results with this optional model, in particular the distance to the default, the underlying assets' volatility (economic asset) and the call valuation over several periods (0.25 year, 1 year, 3 years, 10 years, 30 years). In fine, we have the ratings of the period studied (S&P, Moody's, Fitch) as well as the beta of the share.

Table 2

Enterprises Study date 10/2002	Duration of the debt on EUR zone market	Duration of the debt On USD zone market
Swedish Match AB	2.7037	-
Metro	5.27	-
Goldman Sachs	3.32	5.28
Pinault Printemps La Redoute	2.6275	-
Siemens	3.05	6.40
Ahold	3.80	12.91
Lafarge	3.29	9.60
France Telecom	3.90	7.26
Hutchinson	-	7.3
Renault	2.036	2.9732

			Distance to default in percentage of the economic asset			
Enterprises Study date 10/2002	Daily Volatility of the economic asset	Beta of the share	Average	Standard deviation	Kurtosis	Skewness
Swedish Match AB	1.98%	0.32	74.26	0.83	-0.66	0.20
Metro	1.21%	1	26.52	2.35	-1.13	0.03
Goldman Sachs	2.98%	1.07	98	0.07	-1.22	-0.25
Pinault Printemps La Redoute	1.29%	1.17	28.19	1.69	-0.35	0.17
Siemens	1.61%	1.19	39.65	2.6	-1.41	-0.13
Ahold	1.82%	0.99	36.07	1.91	-0.007	-0.25
Lafarge	1.23%	0.99	39.79	1.6	-0.4413	0.74
France Telecom	0.72%	1.15	14.54	1.5	-0.57	-0.58
Hutchinson	2.39%	1.12	65.37	2.51	-1.63	0.27
Renault	0.80%	0.99	22.54	1.4	-0.60	-0.137

Enterprises	Call premium in million EUR					
	Intrinsic value	Time value T=0.25 year	Time value T=year	Time value T=3 years	Time value T=10 years	Time value T=30 years
Swedish Match AB	24902	71	280	814	2426	5409
Metro	7703	156	616	1785	5281	11591
Goldman Sachs	31462	1	6	17	54	136
Pinault Printemps La Redoute	9639	167	663	1925	5735	12787
Siemens	40192	428	1693	4916	14643	32646
Ahold	11431	181	717	2079	6166	13607
Lafarge	9392	120	474	1378	4104	9150
France Telecom	21043	921	3639	10549	31288	69043
Hutchinson	554	2.03	8	2.3	69	153
Renault	13448	356	1408	4086	12154	27006

Table 2 (continued)

Enterprises	Call premium proportional to the value of the underlying asset					
	Intrinsic value	Time value T=0.25 year	Time value T=1 year	Time value T=3 years	Time value T=10 years	Time value T=30 years
Swedish Match AB	74.43	0.21	0.83	2.43	7.25	16.16
Metro	30.30	0.6	2.42	7	20.77	45.59
Goldman Sachs	99.02	0	0.02	0.05	0.1	0.4
Pinault Printemps La Redoute	32.28	0.56	2.22	6.44	19.20	42.82
Siemens	43.77	0.46	1.84	5.35	15.94	35.55
Ahold	35.20	0.56	2.20	6.40	18.99	41.90
Lafarge	39.36	0.50	1.98	5.77	17.20	38.34
France Telecom	16.46	0.72	2.84	8.25	24.48	54
Hutchinson	69.64	0.25	1.01	2.92	8.70	19.34
Renault	24.08	0.64	2.52	7.32	21.77	48.37

Enterprises	RATING							
	Standard and Poor's	Moody's	Fitch	Risk classification in S&P rank	Tested model rank classification (average distance to the default)	Tested model rank classification (time value T=0.25 year)	Tested model rank classification (time value T=3 years)	Tested model rank classification (time value T=30 years)
Swedish Match AB	A-	Baa1	/	4	2	2	2	2
Metro	BBB	Baa1	BBB	6	8	8	10	8
Goldman Sachs	A+	Aa3	AA-	2	1	1	1	1
Pinault Printemps La Redoute	BBB-	/	/	8	7	6	6	7
Siemens	AA-	Aa3	A+	1	5	4	4	4
Ahold	BB+	Bb3	BB-	10	6	7	7	6
Lafarge	BBB	Baa1	/	5	4	5	5	5
France Telecom	BBB-	Baa3	BBB-	9	10	10	9	10
Hutchinson	A	A2	/	3	3	3	3	3
Renault	BBB	Baa2	BBB	7	9	9	8	9

b. Analysis of the results

The illustration provides congruent results for this exploratory research. The time value increasingly depends on the exercise price (debt value), the underlying asset's volatility (economic asset), the time and the interest rate. The rank correlation between S&P rating and the average distance to the default is 0.709 and the rank correlation between S&P rating and the time value of the option is 0.78[24] (over one quarter), 0.73[25] (over 3 years), 0.75[26] (over 30 years). We must acknowledge that this approach sheds a different and coherent light compared to the ratings. This approach can be more reactive than the rating-based models. Indeed, many articles have demonstrated that the market values better anticipate the risk modifications than the ratings. This approach allows to follow various approaches since it is based on a Monte Carlo simulation. This research work also shows that the credit risk analysis must be approached using several types of model, each modelling bringing about its own complementarity (rating model/default probability – spread credit model/ rating – structural model or optional contingent approach). This complementarity of modelling seems to be the current trend of the research departments in the banking system, be it either for the determination of internal models or for the pricing of complex products (of the structured E.M.T.N.[27] type)

On the other hand, the drawbacks are actually one drawback specific to all Monte Carlo simulation approaches, namely heavy calculations (700 iterations and 525 000 Poisson jump functions for each case exposed in this paper). This heaviness makes the application of these approaches difficult for big samples. The other hindrance is the difficult search for information, in particular on the consolidated debt and the debt structure. Naturally, this approach only applies to companies listed on the financial market.

VI. CONCLUSION

The credit risk management is becoming fully operational as its impressing development of credit derivatives proves it. Since it is under strict regulation, the credit risk is closely observed. At the same time, the development of the financial organisation creates more and more complex products for which the market risk from by-products through mutual agreement is evacuated. This coverage evacuates the market risk but creates credit risk on the counterparts.

The credit risk management is becoming fully operational as the existence of the universal analysis method, such as the VaR credit, proves it. This VaR credit is grounded on ratings and notes. This paper has exposed a great number of research leads to build-up credit notes for listed as well as non-listed companies.

At last, we have proposed a contingent approach with a deformation model and poisonnnian jump functions. This exploratory approach enables to show that the improvement tracks of these contingent models require the progress of the stochastic rate distribution models.

The illustration that we have presented also puts forward the operational character of this research and comes in addition to the tests of contingent models by Franks and Torous (1989) and those of Wei and Guo (1997) after Merton and Longstaff-Schwartz.model.

ENDNOTES

1. Asset back security
2. A.B.S.: Asset Back Securities: alternative assets for the private investor as well as for the institution. Beginning of the activity in the USA (1985)

M.B.S.: Mortgage-Backed securities: titrised assets with the guarantee of a mortgage. Beginning of the activity in the USA (1980)
C.B.O.: collateralised bond obligations
C.L.O.: collateralised loan obligations
C.D.O.: collateralised debt obligations (titrisation of the corporate debts of the commercial banks.

3. This approach does not take the "migration risk" or the rating depreciation risk into account. This inclusion of the « migration risk » is essential within the management of a portfolio. On the other hand, the banks adopt a default probability approach to better standardise the credit risk between listed assets and « retail » assets.
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8. Fisher, M., Nychka D., and Zervos D., "Fitting the Term Structure of Interest Rates with Smoothing Splines," Federal Reserve Board, Finance and Economics Discussion Series, 95-1 Working Paper.
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11. Quantitative Impact Study 3 Technical Guidance – Basel Committee on Banking Supervision October 2002 – BANK FOR INTERNATIONAL SETTLEMENTS.
12. Jeanne Françoise de Polignac, Directeur Général S&P; "du bon usage de la notation", Dossier le financement des entreprises, Banque Magazine May 2004.
13. Improvement submitted, in particular, in Pascal Damel's Thesis, "Détection avancée du risque de défaut : impact de l'information comptable sur le risque de faillite et le risque boursier", January 1996.
14. For example, by calculating the net cash flow in DORNIER balance sheet, thesis "de l'analyse financière à l'expertise financière", 1990.
15. J. R. BOHN, "A Survey of Contingent-claims Approaches to Risky Debt Valuation", *The Journal of Risk Finance*, Spring 2000.
16. The interest rate follows a stochastic distribution process from Vasicek (1977).
17. It is a stochastic formulation with neutral risk probabilities by taking the correlations between the rate distribution process and the assets' value distribution process into account.
18. DJ is a jump function which follows a Poisson law with an intensity parameter λ and a jump range equal to $\Pi > 0$.
19. F.R.N.: floating rate note.

20. CF "Asset and Risk Management : Finance orientée Risque", Louis Esch, Robert Kieffer, Thierry Lopez with the collaboration of Christian Berbé, Pascal Damel, Miche Debay and J-F Hannosset. De Boeck and soon in English at Wiley's.
21. Khi two test, kurtosis, skewness, SR test , wilks shapiro….
22. Rothschild and Stiglitz were the first ones to highlight jump functions on the yield distributions and shed a theoretical light on the interest of the poissonian process. Rothschild, M.E., and Stiglitz, J., 1970, "Increasing Risk: A Definition", *Journal of Economic Theory* 2, P. 225-243.
23. CF demonstration Annex (A2)
24. The rank correlation is validated by a correlation test of Spearman ranks.
25. The rank correlation is validated by a correlation test of Spearman ranks.
26. The rank correlation is validated by a correlation test of Spearman ranks.
27. E.M.T.N. : Euro medium term note

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ANNEXES (A1)
Interest rate stochastic models

Cf "Asset and Risk Management : Finance orientée Risque", Louis Esch, Robert Kieffer, Thierry Lopez with the collaboration of Christian Berbé, Pascal Damel, Miche Debay and J-F Hannosset. De Boeck and soon in English at Wiley's.

ANNEXES (A2)

$dVA_t = VA_t(rdt + \sigma' dB_t)$

We search suitable process so that the integrals $\int_0^t VA_t dt$ and $\int_0^t VA_t dB_t$ have a meaning

$$VA_t = VA_0 + \int_{t=0}^{T} rVA_t dt + \int_{t=0}^{T} \sigma VA_t dB_t$$

Itô's formula as the function of g with two variables (t,B) twice continuously differentiable over $\Re$

$$g(t, B_t) = g(0,0) + \int_{s=t}^{T} \frac{\delta g}{\delta B}(s, B_s)dB_s + \int_{s=t}^{T} (\frac{\delta g}{\delta t} + \frac{1}{2}\frac{\delta^2 g}{\delta B^2})(s, B_s)ds$$

where $\frac{\delta g}{\delta t}, \frac{\delta g}{\delta B}, \frac{\delta^2 g}{\delta B^2}$ designate the partial derivatives of g . This formula can be written in the shape of a differential increment.

$$dg(t, B_t) = \frac{\delta g}{\delta B}(t, B_t)dB_t + (\frac{\delta g}{\delta t} + \frac{1}{2}\frac{\delta^2 g}{\delta B^2})(t, B_t)dt$$

Considering the function g(t,B), a calculation with partial derivatives, combined with Itô formula demonstrate

$$g(t, B) = VA_0 e^{((r - \frac{\sigma^2}{2})t + \sigma B)}$$

$$\frac{\delta g}{\delta t} = r - \frac{\sigma^2}{2}, \frac{\delta g}{\delta B} = \sigma, \frac{\delta^2 g}{\delta B^2} = \sigma^2$$

$$g(t, B_t) = g(0,0) + \int_{s=t}^{T} ((r - \frac{\sigma^2}{2}) + \frac{1}{2}\sigma^2)(s, B_s)ds + \sigma\int_{s=t}^{T} g(s, B_s)dB_s \quad \Leftrightarrow$$

$$g(t, B_t) = g(0,0) + \int_{s=t}^{T} rg(s, B_s)ds + \sigma\int_{s=t}^{T} g(s, B_s)dB_s \quad \Leftrightarrow$$

$$g(t, B_t) = VA_0 + \int_{s=t}^{T} g(s, B_s)dVA_s$$

ANNEXES (A3)

Calculation of the Call value
PE_D : Strike current debt value
P_{CALL}= CALL Price
$P_{CALL}=e^{-rT}\ E(VA_T - PE_{DT})_+$
$P_{CALL}=e^{-rT}\ E(VA_{T\{VAT \geq PEDT\}}) - e^{-rT}\ PE_{DT}\ \text{Proba}\{VA_T \geq PE_{DT}\}$

with: $VA_t = VA_0 e^{((r-\frac{\sigma^2}{2})T+\sigma B_t)}$ and $B_T = N(0;\sqrt{T})$ with N a regular density law

$$N(x) = \int_{-\infty}^{x} e^{-\frac{\mu^2}{2}} \frac{d\mu}{\sqrt{2\Pi}} \quad \text{and} \quad d_r = [(d_{z1} + \Pi)\times\sigma_r^\theta + \bar{r}^\theta]dt \times P(\lambda)$$

The following quantities must be calculated

$$\text{Proba}\{VA_T \geq PE_{DT}\} = \int_R \{VA_0 e^{((r-\frac{\sigma^2}{2})T+\sigma\mu)} \geq PE_D\} e^{-\frac{\mu^2}{2}} \frac{dv}{\sqrt{2\Pi}} \Leftrightarrow$$

$$\text{Proba}\{VA_T \geq PE_{DT}\} = \int_R \{VA_0 e^{((r-\frac{\sigma^2}{2})T+\sigma\sqrt{T}v)} \geq PE_D\} e^{-\frac{v^2}{2}} \frac{dv}{\sqrt{2\Pi}} \Leftrightarrow$$

Stating $u=\sqrt{T}v$

$$\text{Proba}\{VA_T \geq PE_{DT}\} = \int_R \{\ln VA_0 e^{\left(\left(r-\sigma'^2/2\right)T+\sigma'\sqrt{T}v\right)} \geq \ln PE_D\} e^{-\frac{v^2}{2}} \frac{dv}{\sqrt{2\Pi}} \Leftrightarrow$$

Using the properties of the nepierian logarithm

$$\text{Proba}\{VA_T \geq PE_{DT}\} = \int_R \{\frac{\ln\frac{VA_0}{PE_D} + (r-\frac{\sigma^2}{2})T}{\sigma\sqrt{T}} \geq -v\} e^{-\frac{v^2}{2}} \frac{dv}{\sqrt{2\Pi}} \Leftrightarrow$$

Specifying accurately the rate distribution model and stating v=-w

$$\text{Proba}\{VA_T \geq PE_{DT}\} = \int_R \{\frac{\ln\frac{VA_0}{PE_D} + [(d_{z1}+\Pi)\times\sigma_r^\theta + \bar{r}^\theta]\times P(\lambda) - \frac{\sigma^2}{2}]T}{\sigma\sqrt{T}} \geq w\} e^{-\frac{w^2}{2}} \frac{dw}{\sqrt{2\Pi}} \Leftrightarrow$$

$$\text{Proba}\{VA_T \geq PE_{DT}\} = N\left[\frac{\ln\frac{VA_0}{PE_D} + [(d_{z1}+\Pi)\times\sigma_r^\theta + \bar{r}^\theta]\times P(\lambda) - \frac{\sigma^2}{2}]T}{\sigma\sqrt{T}}\right]$$

The following quantity must also be calculated : $E(VA_{T\{VAT\geq PEDT\}})$

$$E(VA_{T\{VAT\geq PEDT\}}) = \int_R \{VA_0 e^{((r-\frac{\sigma^2}{2})T+\sigma\mu)} \geq PE_D\} VA_0 e^{((r-\frac{\sigma^2}{2})T+\sigma\mu} e^{-\frac{\mu^2}{2T}} \frac{d\mu}{\sqrt{2\Pi T}} \Leftrightarrow$$

$$E(VA_{T\{VAT\geq PEDT\}}) = VA_0 + e^{rT} \int_R \{\frac{\ln\frac{VA_0}{PE_D}+(r-\frac{\sigma^2}{2})T}{\sigma\sqrt{T}} \geq -v\} e^{-\frac{\sigma^2}{2}T+v\sigma\sqrt{T}-\frac{v^2}{2}} \frac{dv}{\sqrt{2\Pi}} \Leftrightarrow$$

$$E(VA_{T\{VAT\geq PEDT\}}) = VA_0 + e^{rT} \int_R \{\frac{\ln\frac{VA_0}{PE_D}+(r-\frac{\sigma^2}{2})T}{\sigma\sqrt{T}} \geq -v\} e^{-\frac{(v-\sigma\sqrt{T})^2}{2}} \frac{dv}{\sqrt{2\Pi}} \Leftrightarrow$$

Stating $w = v - \sigma\sqrt{T}$ and specifying accurately the rate distribution model

$$E(VA_{T\{VAT\geq PEDT\}}) =$$

$$VA_0 e^{rT} \int_R \{\frac{\ln\frac{VA_0}{PE_D}+[(d_{z1}+\Pi)\times\sigma_r^\theta+\bar{r}^\theta]\times P(\lambda)-\frac{\sigma^2}{2}]T}{\sigma\sqrt{T}} \geq -(w+\sigma\sqrt{T})\} e^{-\frac{w^2}{2}} \frac{dw}{\sqrt{2\Pi}} \Leftrightarrow$$

$$\text{Proba}\{VA_T \geq PE_{DT}\} = VA_0 e^{rT} N\left[\frac{\ln\frac{VA_0}{PE_D}+[(d_{z1}+\Pi)\times\sigma_r^\theta+\bar{r}^\theta]\times P(\lambda)-\frac{\sigma^2}{2}]T}{\sigma\sqrt{T}}\right]$$

At last, we obtain, the valuation of the option

$$\left(\begin{array}{l} PCALL = VA_0\ N(d1(VA_0, PE_D, T, r, \sigma')) - PE_D e^{T\left[(d_{z1}+\Pi\ )\times\sigma_r^\theta+\bar{r}^\theta\right]\times P(\lambda)}\ N(d2(VA_0, PE, T, r, \sigma')) \\ d1 = \dfrac{\ln\dfrac{VA_0}{PE_D}+\left(\left[(d_{z1}+\Pi\ )\times\sigma_r^\theta+\bar{r}^\theta\right]\times P(\lambda)-\dfrac{\sigma'^2}{2}\right)T}{\sigma'\sqrt{T}} \\ d2 = d1 - \sigma'\sqrt{T} \end{array}\right)$$