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Long-Range Dependence in Daily Volatility on Tunisian Stock Market

Mondher Bellalah[a], Chaker Aloui[b], Ezzeddine Abaoub[c]

[a]*THEMA, University de Cergy, and ISC Group, Paris, 33 Boulevard du port, 9501 Cergy, France, France, Mondher.Bellalah@eco.u-cergy.fr*

[b]*Faculty of Economics and Management Sciences of Tunis, Boulevard du 7 Novembre, Tunis- El Manar, Tunisia, Chaker.Aloui@fsegt.rnu.tn.*

[c]*Faculty of Economics and Management Sciences of Tunis, Boulevard du 7 Novembre, Tunis-El Manar, Tunisia, Ezzeddine.Abaoub@fsegt.rnu.tn*

ABSTRACT

The aim of this paper is to surround the volatility dynamics on the Tunisian stock market via an approach founded on the detection of persistence phenomenon and long-term memory presence. More specifically, our object is to test whether long-term dependent processes are appropriated for modelling Tunisian stock market volatility. The empirical investigation has been driven on the two Tunisian stock market indexes IBVMT and TUNINDEX for the period (1998-2004) in daily frequency. Through the estimation of FIGARCH processes, we show that long-term component of volatility has an impact on stock market return series.

JEL: *C22, C52*

Keywords: *Volatility; Long-term memory; Fractional integration; FIGARCH process*

I. INTRODUCTION

Volatility persistence is a subject that has been thoroughly investigated since the introduction of ARCH models by Engle (1982). It is not only important in forecasting future market movements but also is central to a host of financial issues such as portfolio diversification, risk management, derivative pricing and market efficiency. Although, it is common to find a significant statistical relationship between current measures of volatility and lagged values, it has been very difficult to find models that adequately specify the time series dependencies in volatilities in speculative returns data. Ding, Granger and Engle (1993) show that stock market absolute returns exhibit a long-memory property in which the sample autocorrelation function decay very slowly and remain significant even at high order lags. Evidence in favour of long-range dependence in measure of volatility has been largely documented. Despite the fact that emerging markets in the last two decades had attracted the attention of international investors as means of higher returns such as with diversification of international portfolio risk. Few studies had investigated the issue of volatility persistence using nonlinear estimation models.

Emerging markets differ from developed markets. The former are in most, cases are characterized the by lack of institutional development, thinly traded markets, lack of corporate governance and market microstructure distortions. Theses factors hinder the flow of information to market participants. Moreover, in most of these markets, participants slowly react to information due to the lack of equity culture. This paper will focus on Tunisian Stock Exchange (henceforth, TSE) revisiting the issue of volatility persistence in stock market returns. We attempt to investigate empirically market returns, volatility persistence in a distinct approach from previous researches and this by testing for presence of fractional dynamics (i.e. long memory process in TSE volatility). Thus, this investigation proves to be a first essay in the Tunisian context. As we raised, the categorical absence of empirical studies founded on the fractional integrated behaviour in the conditional variance of Tunisian stock returns. Data used are the two Tunisian stock indexes (IBVMT index and TUNINDEX) daily returns during the period from December 31, 1997 till April 16, 2004. The empirical results provided evidence that the daily stock market volatility exhibits long-range dependency. The fractional integrated behaviour in the conditional variance of the daily Tunisian stock indexes have important implications on efficiency tests and on optimal portfolio allocations and consequently for optimal hedging decisions. The remaining sections are organized as follows. The next respected on the theoretical background of long memory and discusses its measurement. Section III presents some practical considerations of long memory processes. Section IV provides an overview on the Tunisian stock market while section V reviews the fractionally integrated GARCH model. Results are presented in section VI with conclusions in section VII.

II. THEORETICAL BACKGROUND

To this level, it seems to be worth to elucidate the conceptual issues of volatility, standard deviation and risk. In financial market theory, volatility is often used to refer to standard deviation, σ or variance σ^2, estimated from an historical return time as follows:

$$\sigma^2 = \frac{1}{N-1}\sum_{t=1}^{N}(r_t - \bar{r})^2$$

where $\bar{r}$ is the mean return. The sample standard deviation statistic $\hat{\sigma}$ is the distribution free parameter representing the second moment characteristic of the sample. Only when σ is attached to a standard distribution, such as normal or Student-t distributions, the required probability density and cumulative probability density can be derived analytically. In fact, σ can be estimated from an irregular shape distribution, in which case the probability density will have to be derived empirically. In a continuous time context, σ is a scale parameter that multiplies and reduces the size of the fluctuations generated by a standard Wiener process. Indeed, different shapes of financial assets returns are specified either through the dynamic of the underlying stochastic process or whether or not the parameter are time varying. Therefore, it would be disjointed to assimilate the standard deviation to a good measure of risk dice at the time of that it is neither attached to a distribution data to a dynamics of assessment. In the same way, the using of the standard deviation as measure of uncertainty often implicitly implies the presence of a normal distribution in the financial assets returns distribution. However, the junction between concepts of volatility and risk is ambiguous. In particular, the risk is often associated to a possible presence of weak or negative returns; whereas, most measures of distribution make no such distinction (e.g., Poon and Granger 2002, p. 5). According to Sharpe (1964), the measure of portfolio performance management is defined as being the return in excess of risk free rate divided by the standard deviation. The Sharpe measure incorrectly penalizes the occasionally high returns. To this consideration, Markowitz (1959) advances the notion of the "semi-variance". The underlying idea consists in taking in account only square returns below the mean return. However, this notion didn't know a big success among portfolio managers.

A. Absolute and Squared Returns As Volatility Proxies

As mentioned previously, volatility is often estimated through a sample standard deviation. Researchers have pointed out methods for volatility estimation that are designed to exploit or to attenuate the influence of extreme values. Ding, Granger and Engle suggest measuring volatility directly from absolute returns. Indeed, Davidian and Cornell (1987) show that absolute returns volatility is more robust against asymmetry and non-normality. Some empirical studies such as Taylor (1986), present evidence that absolute returns based models generate better volatility forecasts than models founded on squared returns. Given that volatility is a latent variable, the actual volatility is usually estimated from a sample using σ^2 expression that presents some inaccuracies when the sample size is small. Before high frequency data becomes widely available, many researchers have resorted to using daily squared returns, computed from closing prices as daily proxy of volatility.

B. Defining and Measuring Long Memory

According to Ding and Granger (1996), a series is said to have a long-memory if it displays a slowly declining autocorrelation function (ACF) and an infinite spectrum at zero frequency. Specifically, the series $\{y_t\}_{t=0}^{\infty}$ is said to be a stationary long-memory process if the ACF, $\rho(k)$ behaves as,

$$\rho(k) \approx c|k|^{2d-1} \text{ as } |k| \to \infty \tag{1}$$

where $0 < d < 0.5$ 0 and c is some positive constant. The left-hand side and the right-hand side in equation (1) tends to 1 as $k \to \infty$. The ACF in (1) displays a very slow rate of decay to zero as k goes to infinity and $\sum_{k=-\infty}^{\infty}|\rho(k)| = \infty$. This slow rate of decay can be contrasted with ARMA processes, which have an exponential rate of decay, and satisfy the following bound,

$$|\rho(k)| \le ba^k, \quad 0 < b < \infty, \ 0 < a < 1. \tag{2}$$

And consequently, $\sum_{k=-\infty}^{\infty}|\rho(k)| < \infty$. A process that satisfies (2) is termed short-memory. Equivalently, long-memory can be defined as a spectrum that goes to infinity at the origin. This is,

$$f(\omega) \approx c\omega^{-2d} \text{ as } w \to 0 \tag{3}$$

A simple example of long-memory is the fractionally integrated noise process, $I(d)$, with $0 < d < 1$. Which is,

$$(1-L)^d y_t = u_t \tag{4}$$

where L is the lag operator, and $u_t \sim iid(0, \sigma^2)$. This model includes the traditional extremes of a stationary process, $I(0)$ and a nonstationary process $I(1)$. The fractional difference operator $(1-L)^d$ is well defined for a fractional d and the ACF of this process displays a hyperbolic decay consistent with equation (1). A model that incorporates the fractional differencing operator is a natural starting point to capture long-memory. This is the underlying idea of the ARFIMA and FIGARCH class of processes. In practice we must resort to estimating the ACF with usual sample quantities

$$\hat{\rho}(k) = \frac{\frac{1}{T}\sum_{t=k+1}^{T}\left(y_t - \overline{y_t}\right)\left(y_{t-k} - \overline{y_t}\right)}{\frac{1}{T}\sum_{t=k+1}^{T}\left(y_t - \overline{y_t}\right)^2} \tag{5}$$

A second approach to measure the degree of long-memory has been to use semiparametric methods. This allows one to review the specific parametric form, which is misspecified and could lead to an inconsistent estimate of the long memory parameter. In this paper, we consider the most two frequently used estimators of long memory parameter d. The first is the Geweke and Porter-Hudak (1983) (henceforth GPH) estimator, based on a log-periodogram regression. Suppose $y_0, y_1, \ldots y_{T-1}$ is the dataset and define the periodogram for the first m ordinates as,

$$I_j = \frac{1}{2\pi T}\left|\sum_{t=0}^{T-1} y_t \exp\left(i\omega_j t\right)\right|^2 \tag{6}$$

where $\omega_j = 2\pi j / T, j = 1,2,\ldots m,$ and m is chosen positive integer. The estimate of $\left(\hat{d}\right)$can then be derived from linear regression of $\log I_j$on a constant and the variable $X_j = \log\left|2\sin\left(\omega_j / 2\right)\right|$, which gives,

$$\hat{d} = -\frac{\sum_{j=1}^{m}\left(X_j - \overline{X}\right)\log I_j}{2\sum_{j=1}^{m}\left(X_j - \overline{X}\right)} \tag{7}$$

Robinson (1995a) provides formal proofs of consistency and asymptotic normality for the Gauss case with $-0.5 < d < 0.5$. The asymptotic standard error is $\pi / \sqrt{24m}$. The bandwidth parameter m must converge infinitely with the sample size, but at a slower rate than $\sqrt{T}$. Clearly, a larger choice of m reduces the asymptotic standard error, but the bias may increase. The bandwidth parameter was set to (T) in Geweke and Porter-Hudack (1983). While Hurvich, Deo and Brodsky (1998) show the optimal rate to be $O\left(T^{4/5}\right)$. Recently, Hurvich and Deo (1999) have shown that the GPH estimator is also valid for some non Gaussian time-series. Velasco (1999) has shown that consistency extends to $0.5 < d < 1$and asymptotic normality to $0.5 < d < 0.75$. The other popular semiparametric estimator is due to Robinson (1995b). Essentially, this estimator is based on the log-periodogram and solves:

$$\hat{d} = \arg\min R(d) \tag{8}$$

$$R(d) = \log\left(\frac{1}{m}\sum_{j=1}^{m}\omega_j^{2d} I_j\right) - \frac{2d}{m}\sum_{j=1}^{m}\omega_j \tag{9}$$

The estimator is asymptotically more efficient that the GPH estimator and consistency and asymptotic normality of $\hat{d}$ are available under weaker assumptions than for the Gaussian case. The asymptotic standard error for $\hat{d}$ is $1/\left(2\sqrt{m}\right)$. Robinson and Henry (1996) have shown that this estimator is valid in the presence of some forms of conditional heteroskedasticity

III. THE PRACTICAL CONSIDERATIONS

Previous studies of long-memory and fractional integration in time series are numerous. Barkoulas, Baum, and Oguz (1999a, b), studied the long run dynamics of long-term interest rates and currencies. Recent studies of stock prices include Cheung and Lai (1995), Lee and Robinson (1996), Andersson and Nydahl (1998). Batten, Ellis, and Hogan (1999) dealt with credit spreads of bonds. Wilson and Okunev (1999) searched for long-term co-dependence between stock and property markets. While the results on the level of returns are mixed, there is general consensus that the unconditional distribution is non-normal and that there is long-memory process in squared and absolute returns. The following are some issues. Though not mutually exclusive, they are separated by headings for easier discussions:

A. Risk and Volatility

Standard deviation is a statistical measure of variability and it has been called the measure of investment risk in the finance literature. Balzer (1995) argues that standard deviation is a measure of uncertainty and it is only a candidate, among many others, for a risk measure. Markowitz (1959) and Murtagh (1995) found that portfolio selection based on semi-variance tend to produce better performance than those based on variance. A normal distribution is completely characterised by its first two statistical moments, namely, the mean and standard deviation. However, once nonlinearity is introduced, investment returns distribution is likely to become markedly skewed away from a normal distribution. In such cases, higher order moments such as skewness and kurtosis are required to specify the distribution. Standard deviation, in such a context, is far less meaningful measure of investment risk and does not seem to be a good proxy for risk. While recent developments are interested in the conditional volatility and long memory in squared and absolute returns, most practitioners continue to think in terms of unconditional variance and continue thus to work with unconditional Gaussian distribution in financial applications. Recent publications are drawing attention to the issue of distribution characteristics of market returns, especially in emerging markets, which cannot be summarized by a normal distribution (Bekaert et al., 1998).

B. Estimating and Forecasting Asset Prices

Earlier perception was that deseasonalised time series could be viewed as consisting of two components, namely, a stationary component and a non-stationary component. It is perhaps more appropriate to think of the series consisting of both a long and a short memory components. A semiparametric estimate d can be the first step in building a parametric time series model as there is no restriction on the spectral density away from the origin. Fractional ARIMA or ARFIMA can be used in forecasting although of the debates on the relative merits of using this class of models is still inconclusive (Hauser, Pötscher, and Reschenhofer, 1999, and Andersson, 1998). Lower risk bounds and properties of confidence sets of so called ill-posed problems associated with long-memory parameters are also discussed in Pötscher (1999). The paper casts doubts on the used statistical tests in some semiparametric models on the ground that a priori assumptions regarding the set of feasible data generating processes have to be imposed to achieve uniform convergence of the estimator.

C. Portfolio Allocation Strategy

The results of Porterba and Summers (1988) and Fama and French (1988) provided the evidence that stock prices are not truly random walk. Based on these observations, Samuelson (1992) has deduced on a rational basis that it is more appropriate to have more equity exposure with long investment horizon than with short horizon. Optimal portfolio choice under processes other than white noise can also suggest lightening up on stocks when they have risen above trend and loading up when they have fallen below trend. This coincides with the conventional wisdom that long-horizon investors can tolerate more risk and therefore gain higher mean returns. As one grows older, one should have less holding of equities and more assets with lower variance than equities. This argues for “market timing” asset allocation policy and against the use of “strategic” policy by buying and holding as implied by the random walk model. Then, there is the secondary issue of short-term versus long-horizon tactical asset allocation. Persistence or a more stable market calls for buying and holding after market dips. This would likely to be a mid to long-horizon strategy in a market trending upwards. Whereas, in a market that exhibits antipersistence, asset prices tend to reverse their trend in the short term creating thus short-term trading opportunities. It is unclear, taking transaction costs into account, whether trading the assets would yield higher risk adjusted returns. This is an area of research that may be of interest to practitioners.

D. Diversification and Fractional Cointegration

If assets are not close substitutes for each other, one can reduce risk by holding such substitutable assets in the portfolio. However, if the assets exhibit long-term relationship (e.g., to be co-integrated over the long-term), then there may be little gain in terms of risk reduction by holding such assets jointly in the portfolio. The finding of fractional cointegration implies the existence of long-term co-dependencies, thus reducing the attractiveness of diversification strategy as a risk reduction technique.

Furthermore, portfolio diversification decisions in the case of strategic asset allocation may become extremely sensitive to the investment horizon if long-memory is present. As Cheung and Lai (1995) and Wilson and Okunev (1999) have noted, there may be diversification benefits in the short and medium term, but not if the assets are held together over the long term naturally if long-memory is present.

E. Multifractal Model of Asset Returns and FIGARCH

The recently developed multifractal model of asset returns (henceforth MMAR) of Mandelbrot, Fisher and Calvet (1997) and FIGARCH process of Baillie, Bollerslev, and Mikkelsen (1996) incorporate long-memory and thick-tailed unconditional distribution. These models account for most observed empirical characteristics of financial time series, which show up as long tails relative to the Gaussian distribution and long-memory in the volatility (absolute return). The MMAR also incorporates scale-consistency, in the sense that a well-defined scaling rule relates return over different sampling intervals.

F. Stock Market Weak Form Efficiency

A time series that exhibits long memory process violates the weak form of efficient market hypothesis developed by Fama (1970); it states that the information in historical prices or returns is not useful or relevant in achieving excess returns. Consequently the hypothesis that prices or returns move randomly (random walk hypothesis) is rejected.

IV. TUNISIAN STOCK MARKET OVERVIEW

A. The Main Reform Measures Concerning the TSE

1. Fiscal regime for holdings

Any company listed on the stock exchange and holding, directly or indirectly, at least 95% of capital in other companies can, as the parent company, opt for tax assessment on the basis of the combined earnings which. They should priorly be subject to corporate tax law, they must have the same accounting year opening and closing dates and be both established in Tunisia [see Note 1]. Tax incentives to companies which open their capital to the public were initially granted for a period of three years starting from January 1999, in the form of a reduced tax rate from 35 to 20%. This was extended for an additional period of three years starting from February 2002, with a view to encourage companies to be more transparent and also mobilising public savings by increasing the range of offerings and this by posting new stocks on the stock market.

2. Amendment of financial market council

This amendment supports greater transparency in public calls for savings by requiring that companies, seeking this kind of funding, to provide a more complete and reliable information. Thus, companies will have to provide to the Financial Market Council (henceforth, CMF) and to shareholders the required information. To encourage new issues and transactions on the financial market, commissions to the CMF and the TSE were reduced. Previously calculated on the basis of the amount of the issue, commissions to he CMF are henceforth set at 0.2% of the nominal value of the issue.

B. Tunisian Stock Exchange Trends

TSE sent a higher level of public securities and a greater volume of transactions for the second straight year. But no new companies were posted on the stock exchange in 2000; despite larger fiscal incentives [see Note 2] that encourage new companies, already posted, to open their capital to the public. The CMF published regulations concerning public call for savings, which specify conditions, procedures and responsibilities of stockbrokers and companies issuing securities through public calls for savings. Concerning the official quotation, stock market activity was characterised by two distinct phases. Over the first nine months of the year, there is sustained demand for securities, especially for active stocks. Volume on stock market picked up in the light of figures of 1999 and the first half of 2000 concerning posted companies, dividend distribution and 13 capital increases operations. Total profits posted by listed companies on the basis of 1999 activity were up in 2000 by 14%, while dividends per share increased by an average 16%. But despite the overall improvement in distributed profit, the average market price earning ratio (PER) indicating the time required to recover investment was up from 13 in 1999 to 16 in 2000, tied to the higher cost of stock exchange quotations. The same forces that marked trading also accounted for an improvement in the securities ration rate, which reached 23.6% vs. 16.7% in 1999 and 9.7% in 1998. Likewise, the average market liquidity rate was up slightly from 46% in 1999 to 51% in 2000. But trade remained insufficiently diversified, concentrating on a limited number of stocks: almost two thirds of total transactions involved just 10 stocks. In 2001, stock market quotations were marked by a process to adjust stock prices which had increased significantly during the last two years; and by weak demand for securities which sought mainly new issues made that year. Lack of confidence on the part of investors was the reason behind low demand, despite the favourable financial results published by listed companies; this became even more complicated, during the last quarter after the events of September 11th. Companies listed on the TSE increased from 42 at the end of 2000 to 45 at the end of 2001. The new members were included by public sale and by public subscription to capital increase transactions. During 2003, financial market activity showed timid improvement, with a slight increase in the TUNINDEX and BVMT indexes and a drop in the volume of issues by public call for savings and transactions on quotations. Concerning the stock market activity, it was characterized by gradual recovery starting in the third quarter, as seen in higher prices for key stocks or for strong market capitalisation ones. This upward trend was influenced in particular by improved national economic conditions, the 87.5 base point drop in the Central Bank of Tunisia's key rate, and heightened confidence on the part of

Table 1
Main Tunisian stock market indicators (1997-2004) (in MTND unless otherwise indicated)

Description	1997	1998	1999	2000	2001	2002	2003
BVMT index in points (base 100 on 30 September, 1990, adjusted on 31 march 1998 to 465.77	455.64	464.56	810.24	1,424.91	996.09	782.93	939.78
TUNINDEX in points (base 1000 on 31 December 1997	1,000	917.08	1,192.57	1,442.61	1,266.89	1,119.15	1,250.18
Stock market capitalisation [(a)]	2,632	2,452	3,326	3,889	3,275	2,842	2,976
Stock market capitalisation/GDP (in %)	12.6	10.9	13.5	14.6	11.4	9.5	9.2
Number of listed companies	34	38	44	42*	45	46	45
Overall volume of transaction	590	927	881	1 814	1 204	1 006	948
of witch: official quotation [(b)]	287	237	554	919	508	343	238
Rotation rate (in %) [(a/b)]	10.9	9.7	16.7	23.6	15.5	12.1	8.0
Liquidity rate (in %)	36	37	46	51	49	42	33
PER	12	10	13	16	10	12	13

operators, particularly the return of foreign investors. With no new entries on the market, the number of companies quoted on the stock exchange dropped from 46 in 2002 to 45 in 2003 [see Note 3]. The volume of transactions on the market fell by 225 105 MTD (31%) in 2003 to 238 MTD, an average daily volume under a million dinars, compared to 1.4 MTD in 2002. Some 12.9 million securities were transacted in 2003, down from 17 million in 2002, denoting a drop of 24.2%. Exchange of securities and transacted capital did not show much diversity, focusing on a limited number of stocks. Six stocks accounted for more than 60% of total capital transacted in 2003. Sector-related breakdown of traded stocks showed a 34% share for the banking sector in 2003, down from 38% in 2002. The share of the industrial sector also decreased, from 38% in 2002 to 29% in 2003. But the share of the services sector increased in 2003 to 27%, up from 16% in 2002.

V. MODELING THE LONG MEMORY OF THE VOLATILITY

Traditionally, the time series econometrics centred itself around an alternative: the presence of a unit root, indicating a nonstationarity of the set, on the one hand, and the absence of such a unit root indicating that the set is stationary. On the other hand, these two cases correspond to cases of processes of short memory of ARIMA (p,d,q) and ARMA(p,q). These classic modeling doesn't take in account the intermediate cases to know the existence of a fractional integration parameter. However, the presence of such a coefficient no whole is especially interesting since it permits to characterize processes of long memory. These processes, called ARFIMA, have been introduced by Granger and Joyeux (1980) and Hosking (1981). They present the interest to take account at a time of the short-term behaviour of the set through autoregressive and moving average and the behaviour of long term by means of the fractional integration parameter. The ARFIMA (p, d, q) process can be defined as follows:

$$\Phi(L)(1-L)^d y_t = \Theta(L)\varepsilon_t \tag{10}$$

where, $\Phi(L)$ and $\Theta(L)$ are lag polynomials of p and q respectively. ε_t is a White noise process, and :

$$(1-L)^d = 1 - dL - \frac{d(1-d)}{2!}L^2 - \frac{d(1-d)(2-d)}{3!}L^3 - ....$$

ARFIMA (p,d,q) processes are stationary and inversible when $d \in \left]-1/2, 1/2\right[$ and $d \neq 0$.

A. Short and Long Term Memory and FIGARCH Processes

Considering a possible fractional integration of the conditional variance has been evoked initially by Ding and Granger (1996) and Ding, Granger and Engle (1993). Positively, FIGARCH processes have been introduced by Baillie, Bollerslev and

Mikkelsen (1996). The starting point is a GARCH (p,q) process. It can be written as follows:

$$\sigma_t^2 = \alpha_0 \sum_{i=1}^{q} \alpha_i \varepsilon_{t-1}^2 + \sum_{j=1}^{p} \beta_i \sigma_{t-j}^2 = \alpha_0 + \alpha(L)\varepsilon_L^2 + \beta(L)\sigma_t^2 \tag{11}$$

where α^2 is the conditional variance; $\alpha_0 > 0; \alpha_i \geq 0; \beta_j \geq 0, i = 1,...,q.$ GARCH(p,q) process are short memory processes since the effect of a shock on the conditional variance decreases at an exponential rate. GARCH(p,q) can be also written as follows:

$$[1 - \alpha(L) - \beta(L)]\varepsilon_t^2 = \alpha_0 + [1 - \beta(L)]\mu_t \tag{12}$$

Consequently, when the lag polynomial $[1 - \alpha(L) - \beta(L)]$ contains a unit root, the GARCH process becomes an integrated GARCH process, denoted as IGARCH(p,q)). IGARCH (p,q) process can be written as:

$$\Phi(L)(1 - L)\varepsilon_t^2 = \alpha_0 + [1 - \beta(L)]\mu_t \tag{13}$$

with $$\Phi(L) = [1 - \alpha(L) - \beta(L)](1 - L)^{-1}$$

FIGARCH processes constitute an alternative between GARCH processes and IGARCH processes and result with the equation (4) by replacing the operator $(1 - L)$ by the operator $(1 - L)^d$, where d is the fractional integration parameter. A FIGARCH process can be written as follows:

$$\Phi(L)(1 - L)^d \varepsilon_t^2 = \alpha_0 + [1 - \beta(L)]\mu_t \tag{14}$$

Roots of $\Phi(L)$ and $[1 - \beta(L)]$ polynomials being outside the unit circle. Thus, if $d = 0$, FIGARCH(p,d,q) process will be reduced to a GARCH(p,q). if $d = 1$, FIGARCH process will be an IGARCH. By replacing μ_t by its value according to σ_t^2, one can write equation (5) as follows:

$$[1 - \beta(L)]\sigma_t^2 = \alpha_0 + \left[1 - \beta(L) - \Phi(L)(1 - L)^d\right]\varepsilon_t^2 \tag{15}$$

The variance equation is then given by:

$$\sigma_t^2 = \alpha_0[1 - \beta(1)]^{-1} + \lambda(L)\varepsilon_t^2 \tag{16}$$

with $$\lambda(L) = \left[1 - [1 - \beta(L)]^{-1}\Phi(L)(1 - L)^d\right]$$

$= \lambda_1 L + \lambda_2 L + \ldots$ and $\lambda_k \geq 0$ et $k = 1,2,\ldots,n$

Baillie, Bollerslev et Mikkelsen (1996) note that the effects of a shock on the conditional variance of FIGARCH(p,d,q) decreases at an hyperbolic rate when $0 \leq d < 1$.

B. Data and Statistical Distribution

Our empirical investigation is conducted using daily returns of two Tunisian stock indexes (IBVMT [see Note 4] and TUNINDEX [see Note 5]). The data cover the period (1997/12/31- 2004/4/16) and totalling 1593 observations. Daily returns are calculated for the two indexes as continuously returns at time t ; $r_{i,t}$. In other words, as the natural log difference in the closing market index P_t between two days as shown below: $r_{i,t} = 100\text{Log}(P_t / P_{t-1})$. Results reported in Table 2 call the following commentaries:

Table 2
Descriptive statistics

	Daily frequency		Weekly frequency	
	IBVMT returns	TUNINDEX returns	IBVMT returns	TUNINDEX returns
Mean (%)	4.89967	1.58515	23.8233	7.9747
t-statistic	2.3365	1.3428	2.0231	1.9781
S. deviation (%)	83.6171	47.0108	2.385634	1.81103
Kurtosis	5.00454	7.15012	13.88606	53.62137
Excess Kurtosis	2.00454	4.015012	10.88606	50.62137
Skewness	0.254762	0.639271	0.697379	0.2266645
Jarque-Béra normality test	244.34^{***}	435.43^{***}	345.32^{***}	354.22^{***}
Augmented Dickey-Fuller test [see Note 7]	-19.63^{***}	-21.43^{***}	-10.589^{***}	-12.613^{***}
Phillips-Perron unit root test [see Note 8]	-26.39^{***}	-27.01^{***}	-16.758^{***}	-20.804^{***}
KPSS test	0.66016 (3)	0.27769 (3)	0.66431 (1)	0.136 (1)
ARCH- test	231.358 Prob. (0.000)	306.345 Prob. (0.000)	73.067 Prob. (0.000)	66.005 Prob. (0.000)
Maximum	4.000052	3.040505		
Minimum	-3.06502	-2.04465		
Sample period	31/12/1997	16/04/2004	31/12/1997	16/04/2004
Observation	1590	1590	328	328

Note: The Jarque-Béra test for normality distributed as Chi-square with 2 degrees of freedom. The critical value for the null hypothesis of normal distribution is 5.99 at the 5% significance level. Higher test values reject the null hypothesis. *** denotes significance at 1% level.

1. Mean returns of the IBVMT are the highest compared to the TUNINDEX. According to the t-statistics, only IBVMT mean returns are significantly different from zero at 5% significant level. Medians' returns are positive and confirm the same ranking of the indices, implying skewed series with departure from normality.
2. It is evident that the two indices returns are volatile. This has been confirmed by ARCH test where the null hypothesis of returns that are homoscedastistic is rejected at 1% significance level. There is evidence of heteroscedasticity in the daily and weekly two indices and for the frequencies. In other words, the BVMT and TUNINDEX returns exhibit clustering volatility and that there is a tendency for large (small) asset price changes to be followed by other large (small) price changes of either sign and tend to be time dependent.
3. Indices' returns display significant positive skewness where the null hypothesis of skewness coefficients conforming to the normal distribution value of zero is rejected. This result is in compliance with means greater than the medians in (1).
4. The null hypothesis of kurtosis coefficients conforming to the normal distribution value of three is rejected at five percent significance level for the BVMT and TUNINDEX weekly and daily returns. Thus, the returns of both indices are leptokurtic and their distributions have thicker (fatter) tails than that of a normal distribution.
5. Results of both (3) and (4) have been confirmed by rejecting the null hypothesis of the bivariate Jarque-Bera test for unconditional normal distribution of the two stock market weekly and daily index returns.
6. With respect to Dickey-Fuller and Phillips-Perron unit root statistics, the null hypothesis for both tests whether indices returns, using t-statistics, have unit root is rejected in favour of the alternative that the four series are trend stationary process with a degree of predictability.
7. In sum, the BVMT and TUNINDEX weekly and daily returns tend to be characterized by positive skewness, excess kurtosis and departure from normality. The two indexes, also, display a degree of heteroscedasticity. The findings are in conformation with other market indexes and consistent with several other empirical studies [see Note 6] in which emerging markets returns depart from normality and the null hypothesis for a random walk is rejected.

VI. FIGARCH MODELING

Before estimating FIGARCH processes, we proceed to the application of the modified R/S test (Lo (1991) in order to detect the presence, if any, of long-range memory in Tunisian stock market volatility series. Let us simply recall that the limiting distribution of the modified R/S statistic is known and it is thus possible to test the null hypothesis of short-term memory against the alternative of long-term memory. The critical values of this statistic have been tabulated by Lo (1991). The author demonstrated that this statistic was not robust to short-range dependence, and proposed the following one:

$$Q_T = \frac{1}{\hat{\sigma}_T(q)}\left[\max_{1\le k\le T}\sum_{j=1}^{k}\left(X_j - \overline{X}_T\right) - \min_{1\le k\le T}\sum_{j=1}^{k}\left(X_j - \overline{X}_T\right)\right]$$

which consists of replacing the variance by the HAC variance estimator in the denominator of the statistic. If $q = 0$, Lo's statistic R/S reduces to Hurst's statistic. Unlike spectral analysis which detects periodic cycles in a series, the R/S analysis has been advocated by Mandelbrot for detecting non periodic cycles. Under the null hypothesis of no long-memory, the statistic $T^{-1/2}Q_n$ converges to a distribution equal to the range of a Brownian bridge on the unit interval:

$$\max_{0\le t\le 1} W^0(t) - \min_{0\le t\le 1} W^0(t)$$

where $W^0(t)$ is a Brownian bridge defined as $W^0(t) = W(t) - tW(1)$, $W(t)$ being the standard Brownian motion. The distribution function is given in Siddiqui (1976), and is tabulated in Lo (1991). This statistic is extremely sensitive to the order of truncation q but there are no statistical criteria for choosing q in the framework of this statistic. Andrews (1991) rule gives mixed results. If q is too small, this estimator does not account for the autocorrelation of the process, while if q is too large, it accounts for any form of autocorrelation and the power of this test tends to its size. Given that the power of a useful test should be greater than its size; this statistic is not very helpful. For that reason, Teverovsky, Taqqu and Willinger (1999) suggest to use this statistic with other tests. Since there is no data driven guidance for the choice of this parameter, we consider the default values for $q = 5,\ 10,\ 25,\ 50$. Results reported in Table 3 indicate that the two volatility series display a strong dependent structure. To verify this result and to take into account long-term property, we estimate FIGARCH process.

A. Geweke Porter-Hudack (1983) Tests

In this respect, two procedures have been retained: the GPH method and the maximum likelihood technique. The GPH method is founded on the behaviour of the spectral density around low frequencies. It is a two-step technique since one estimate in the first stage the fractional integration parameter d and, in the second stage the parameter of the GARCH model. Concerning the maximum likelihood procedure (Sowel (1992)), it is a one-step procedure: all the parameters of the FIGARCH(p,d,q) specification are estimated simultaneously. The GPH estimation of FIGARCH processes are reported in table below. Let us recall that the function $g(T)$ used in the spectral technique, corresponds to the number of periodogram ordinates. T is the number of observations. In order to examine the stability of the estimation when the number of periodogram ordinates vary, we have chosen different values: $T^{0.45}$, $T^{0.5}$, $T^{0.55}$ and $T^{0.8}$.

Table 3
Lo R/S modified test

BVMT				TUNINDEX			
Daily returns		Weekly returns		Daily returns		Weekly returns	
Order	$\tilde{Q}_T$ statistic	Order	$\tilde{Q}_T$ statistic	Order	$\tilde{Q}_T$ statistic	Order	$\tilde{Q}_T$ statistic
5	4.2912*	5	1.6179	5	2.5101*	5	1.1036
10	3.6252*	10	1.5630	10	2.3412*	10	1.0553
25	2.8189*	25	1.4012	25	2.0516*	25	1.0523
50	2.3489*	50	1.2843	50	1.9224*	50	1.1748

Note: string vector containing the estimated statistic with its corresponding order. If the estimated statistic is outside the interval (0.809, 1.862), which is the 95 percent confidence interval for no long-memory, a star symbol * is displayed in the third column. The other critical values are in Lo's paper.

Table 4
GPH estimation of fractional integration parameter

$g(T)$	$T^{0.45}$	$T^{0.5}$	$T^{0.55}$	$T^{0.8}$
BVMT				
Daily absolute returns	-	0.12343 (3.034)	0.1147 (2.8791)	0.1132 (2.657)
Weekly absolute returns	0.3452 (2.087)	0.3944 (2.056)	0.3809 (3.453)	-
TUNINDEX				
Daily absolute returns	-	0.0878 (2.736)	-	-
Weekly absolute returns	-	0.0297 (1.674)	-	-

T is the number of observations, $g(T)$ the number of periodogram ordinates, t-statistic of d are given into brackets. (-) non significant.

Results obtained using the spectral technique, emphasize the presence of long memory for the TUNINDEX stock returns. For the IBVMT volatility, the presence of a long-term structure depends on the number of periodogram ordinates retained. It will be also noted that the fractional integration parameter is positive in all cases. Judged by standard significance levels, $\hat{d}$ is statistically very different from both zero and one. Concerning, the exact maximum likelihood method, we observe, according the SIC model selection criteria, the presence of long-term dependence structure for the IBVMT volatility.

B. Lobato and Robinson (1998) Test

Lobato and Robinson (1998) nonparametric test for $I(0)$ against $I(d)$ is also based on the approximation of the spectrum of a long-memory process. In the univariate case, the *t* statistic is equal to:

$$t = m^{1/2}\hat{C}_1/\hat{C}_0 \text{ with } \hat{C}_k m^{-1}\sum_{j=1}^{m} v_j^k I(\lambda_j) \text{ and } v_j = \ln(j) - \frac{1}{m}\sum_{i=1}^{m}\ln(i)$$

where $I(\lambda) = (2\pi T)^{-1}\left|\sum_{t=1}^{T} y_t e^{it\lambda}\right|^2$ is the periodogram estimated for a degenerate band of Fourier frequencies $\lambda_j = 2\pi j/T,\ j = 1,\ldots,m \leq [T/2]$, where m is a bandwidth parameter. Under the null hypothesis of a $I(0)$ time series, the t statistic is asymptotically normally distributed. This two sided test is of interest as it allows to discriminate between $d > 0$ and: $d < 0$ if the t statistic is in the lower fractile of the standardized normal distribution, the series exhibits long-memory whilst if the series is in the upper fractile of that distribution, the series is antipersistent. The default bandwidth suggested by Lobato and Robinson is used. The results are displayed in Table 5. The first column contains the value of the bandwidth parameter while the second column displays the corresponding statistic. In the first line, the Lobato-Robinson statistic is evaluated by using this default bandwidth. As t is negative and in the lower tail of the standard normal distribution, there is evidence on long-memory volatility. Semiparametric test for $I(0)$ of a time series against fractional alternatives, (i.e., long-memory and antipersistence). Let us recall that it is a semiparametric test in the sense that it does not depend on a specific parametric form of the spectrum in the neighbourhood of the zero frequency. Concerning the parameter specifying the number of harmonic frequencies around zero to be considered, we use the bandwidth given in Lobato and Robinson. If the value of the test is in the lower tail of the standard normal distribution, the null hypothesis of $I(0)$ is rejected against the alternative that the series displays long-memory. If the value of the test is in the upper tail of the standard normal distribution, the null hypothesis $I(0)$ is rejected against the alternative that the series is antipersistent. As it is shown in the Table 5, the t statistic is negative and it is in lower tail of the standard normal distribution, we can conclude to the presence of long-memory in BVMT and TUNINDEX time series volatility.

C. Lo (1991) Tests

Results in Table 6 indicate that only the BVMT daily and absolute returns display long-term memory for different weights suggested by Newey and West (1987). This result confirms the conclusions issued from Lobato and Robinson (1995b). For the TUNINDEX series, a short dependent structure seems to be present in volatility series. In order to verify this result and take into account this long-term property, we apply the Robinson and Whittle semi-parametric estimator procedures and we estimate FIGARCH processes.

Table 5
Lobato and Robinson (1998) tests

BVMT				TUNINDEX			
Daily absolute returns		Weekly absolute returns		Daily absolute returns		Weekly absolute returns	
Bandwidth	t stat.	Bandwidth	t stat.	Bandwidth	t stat.	Bandwidth	t.stat
133 [(a)]	-14.30	22 [(a)]	-1.92	133 [(a)]	-4.05	19 [(a)]	0.34
150	-15.49	150	-4.93	150	-4.32	150	-4.93
200	-18.28	-	-	200	-4.56	-	-
250	-19.25	-	-	250	-5.42	-	-

Notes: (a) Bandwidth given in Lobato and Robinson (1998).

Table 6
Lo (1991) tests

	BVMT		TUNINDEX	
	Daily absolute returns	Weekly absolute returns	Daily absolute returns	Weekly absolute returns
$m = 5$	2.1776	2.6179	0.41119	1.1036
$m = 10$	2.2963	2.5630	0.44367	1.0553
$m = 25$	2.6234	2.4012	0.57244	1.0523
$m = 50$	2.8841	2.2843	0.57726	1.1748

D. Robinson (1994b) Tests

The Robinson (1994b) averaged periodogram estimator is defined by: $\hat{d} = \frac{1}{2} - \frac{\ln\left(\hat{F}(q\lambda_m)/\hat{F}(\lambda_m)\right)}{2\ln(q)}$, where $\hat{F}(\lambda)$ is the average periodogram $\hat{F}(\lambda) = \frac{2\pi}{n}\sum_{j=1}^{[n\lambda/2\pi]} I(\lambda_j)$. By construction, the estimated parameter $\hat{d}$ is $< 1/2$, i.e., is in the stationarity range. This estimator has the following asymptotic distribution if $d < 1/4$, $\sqrt{m}\left(\hat{d} - d\right) \to N\left(0, \frac{\pi^2}{24}\right)$.

The results of Robinson tests are reported in the Table 7. The Robinson procedure gives the semiparametric average periodogram estimator of the degree of long memory of a time series. The third column in the Table 8 designed the optional argument that is a strictly positive constant q, which is also strictly less than one. The second column designed the bandwidth vector m. By default q is set to 0.5 and 0.7 and the bandwidth vector is equal to m = n/4, n/8, n/16. If q and m contain several elements, the estimator is evaluated for all the combinations of q and m. The first column in the table designed the estimated degree of long-memory. Concerning the BVMT daily absolute returns, the results of the estimated degree of long-term memory range from 0.2310 to 0.2672 for the different values of q and bandwidth vector. For weekly absolute returns, the d parameter ranges from 0.0583 to 0.1940. Theses results indicate evidence that the BVMT volatility exhibit a long-range dependency phenomenon. The fractional differencing parameter is positive and $d \in [0;0.5]$ it indicates the presence of a long-range positive dependence in the conditional variance. Quite similar results are obtained for the daily and weekly absolute returns.

Table 7
Robinson (1994b) tests

BVMT						TUNINDEX					
Daily absolute returns			Weekly absolute returns			Daily absolute returns			Weekly absolute returns		
d	Band width	q	d	Band width	q	d	Band Width	q	d	Band width	q
0.2672	250	0.5	0.0583	82	0.5	0.1236	250	0.5	0.2097	82	0.5
0.2427	250	0.7	0.1940	82	0.7	0.1180	250	0.7	0.2244	82	0.7
0.2380	500	0.5	0.0898	41	0.5	0.2240	500	0.5	0.0590	41	0.5
0.2310	500	0.7	0.0830	41	0.7	0.2338	500	0.7	0.0089	41	0.7
0.2419	750	0.5	0.1886	20	0.5	0.2255	750	0.5	0.0305	20	0.5
0.2546	750	0.7	0.0998	20	0.7	0.2201	750	0.7	0.0044	20	0.7

Table 8
Whittle Semi-parametric estimator of the degree of long memory of daily and weekly absolute returns

BVMT				TUNINDEX			
Daily absolute returns		Weekly absolute returns		Daily absolute returns		Weekly absolute returns	
d	Bandwidth	d	Bandwidth	d	Bandwidth	d	Bandwidth
0.3342	50	0.0876	50	0.1996	50	0.0441	50
0.3441	100	0.1197	100	0.1841	100	0.1808	100
0.3171	150	0.2232	150	0.1362	150	0.2839	150

E. Whittle Semiparametric Gaussian Estimator

The Whittle semiparametric Gaussian estimator of the degree of long memory of a time series is based on the Whittle estimator. The first argument is the series; the second argument is the vector of bandwidths, i.e., the number of frequencies after zero to be considered. By default, the bandwidth vector m = n/4, n/8, n/16, where n is the sample size. This table gives the estimated parameter d, with the number of frequencies considered. The obtained results emphasize the presence of a long -term dependence structure for all the series of volatility. Moreover, one notes a relative stability of the fractional integration parameter value for the BVMT daily volatility for the different sizes of the bandwidth vector. The results indicate also, for all the volatility series, a positive fractional integration parameter. So, all the series are characterized by a long-range positive dependence in the conditional variance. In order to verify this result and take into account this long-term property, we estimate FIGARCH processes.

F. The FIGARCH Process

The empirical investigation is conducted using, parsimoniously, FIGARCH$(1,d,1)$ to specify the long memory process in Tunisian stock market volatility.

Table 9

Estimates for FIGARCH (1,d,1) model for TSE weekly and daily volatility Using Broyden, Fletcher, Goldfrab and Shanno (BFGS) Maximization Method

	BVMT index		TUNINDEX	
	Daily absolute returns	Weekly absolute returns	Daily absolute returns	Weekly absolute returns
α_0	0.01055 (-1.97711)**	0.00903 (2.0112)**	0.02311 (1.4561)	0.0121 (1.1113)
α_1	0.87184 (7.3629)***	0.66121 (5.3124)***	0.42131 (3.3211)**	0.33427 (2.0278)**
β_1	0.11832 (1.0680)	0.34079 (2.0123)**	0.57669 (4.3242)***	0.66583 (1.9902)**
$l(\theta)$	851.841	546.125	243.121	311.342
d	0.4645 (6.35404)***	0.12115 (5.4432)***	0.1996 (4.3421)***	0.0431 (1.3211)
$\alpha_1+\beta_1$	0.9902	1.002	0.9980	1.0001

*** Significant at 1 percent, ** significant at 5 percent, * significant at 10 percent.

The results in Table 9 provide the following observations:

1. For the IBVMT absolute daily returns, the results exhibit fractional dynamics with long memory features. The null hypothesis $(H_0 : d = 0)$ has been rejected in favour of d-value which is statistically significantly greater than zero at 1% significant level. The fractional differencing parameter value recorded approximately 0.4645 and it is in conformation with that of previously preliminary tests. There's also evidence that the BVMT volatility exhibit a long-range dependency phenomenon. The fractional differencing parameter is equal to 0.12115 and it is statistically significant at 1% significant level. The process is considered to be long-range positive dependence in the conditional variance as $d \in [0;0.5]$.
2. Concerning the TUNINDEX daily volatility, the obtained results show the significance of both α_1 and β_1 provided evidence that conditional volatility is time variant and there is volatility clustering effects. The results confirm that there is a tendency for shocks to persist, with large (small) innovations followed by

similar ones. The estimation results of the FIGARCH (1,d,1) provide evidence that the TUNINDEX daily volatility exhibits fractional dynamics. The estimated d - value is statistically significantly greater than zero and indicates the presence of positive persistence phenomenon in the TUNINDEX volatility.

3. The results also provide evidence that the aggregation of short-memory process, could led to long memory feature, which is consistent with Robinson (1978), Taqqu et al. (1997), Chambers (1998), Cioczek-Georges and Mandelbrot (1995) findings. The evidence is consistent with number of emerging market characteristics.
4. As expected, the market adjusts slowly for the arrival of new information slowly which might be due to number of market structural reasons as the dominance of individual investors on trading activity who lack the equity culture and whose investment strategy is characterized by herd behaviour. The presence of nonsynchronous trading is probably due to large number of inactive stocks listed on the Tunisian Stock Exchange.

VII. CONCLUSION

The purpose of this paper was to study the long-range dependency of stock market volatility. More specifically, our object was to test the significant evidence for the presence of fractional integrated behaviour in the conditional variance of the Tunisian stock indexes. Thus, a new class of more flexible fractionally integrated GARCH (FIGARCH) models for characterizing the long run dependencies in the Tunisian stock market volatility was proposed. The investigation is conducted using the BVMT and TUNINDEX daily and weekly indexes during the period January 1998 till the end of April 2004. In this paper, strong evidence was uncovered that the conditional variance of the BVMT and TUNINDEX indexes is best modelled as a FIGARCH process. These findings of long memory component in the volatility processes of asset returns have important implications of many paradigms in modern financial theory. So, optimal portfolio allocations may become extremely sensitive to investment horizon if the volatility returns are long-range dependent. Similarly, optimal hedging decisions must take into account any such long-run dependency. Also, the assumption that the Tunisian Stock Market is weakly efficient is rejected due to long-range dependency in weekly and daily volatilities. This evidence is consistent with number of emerging market characteristics. A more formal and detailed empirical investigations of these issues on the Tunisian context would be important task for further research.

ENDNNOTES

1. Note Law n° 2000-98 of 25 December.
2. Under law n° 99-92 of 17 august 1999.
3. BATAM Company was written off the stock market on 10 February 2003, as per decision of the market's governing council.
4. The IBVMT index evolution reflects the stock market average return. Are included in the reference sample all companies admitted in stock market, before it is

adjusted on 31 March, 1998. The new reference sample limits itself to values of which the frequency of quotation is superior to 60%. The BVMT index has been published under its present shape on April first, 1998, with a base value of 465.77 on 31 March 1998.

5. It is a new stock market capitalization index (base 1000 on 31 December 1997). It was initially published on first April 1998. Concerning its calculation, it is taken account of mean weighted return. The weight corresponds to the number of exchanged stocks. The base sample is composed of values admitted by their ordinary shares to stock market quotes and of which the living period in one of market quotes (primary or secondary market) it is of at least 6 months.
6. Mandelbort (1963) and Fama (1965) showed that unconditional distribution of security price changes to be leptokurtic, skewed and volatility clustered. Bekaert et al. (1998) provided evidence that 17 out of the 20 emerging markets examined their monthly returns had positive skewness and 19 out of 20 had excess kurtosis, so that normality was rejected for more than half of the countries.
7. Dickey and Fuller (1979) devised a procedure to formally test for the presence of unit root using three different regressions. In our case, the following regression with constant and trend is used to test for nonstationarity: $\Delta y_t = a_0 + \gamma y_{t-1} + a_1 t + \sum_{i=2}^{p} \beta_t \Delta y_{t-i} + \varepsilon_t$. The null hypothesis is that $\gamma = 0$ for stochastic nonstationary process.
8. Phillips-Perron nonparametric unit root tests were used because they allow for a general class of dependent and heterogeneously distributed innovations, contrary to other unit root tests (see Phillips and Perron, 1998).

APPENDIX

FIGURE 1.A.
Daily IBVMT and TUNINDEX Evolution (in points)

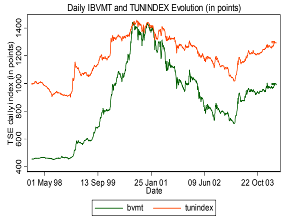


FIGURE 1.B.
Weekly IBVMT and TUNINDEX Evolution (in points)

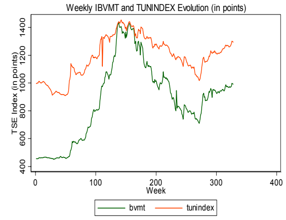


Figure 2.
IBVMT daily returns

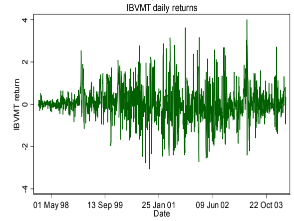


Figure 3.
TUNINDEX daily returns

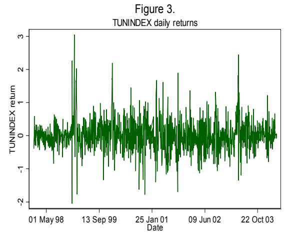


Figure 4.
Autocorrelations for IBVMT absolute daily returns

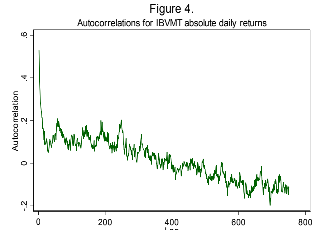
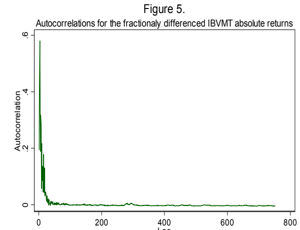


Figure 5.
Autocorrelations for the fractionaly differenced IBVMT absolute returns



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