GAME THEORY, INFORMATION ECONOMICS, RATIONAL EXPECTATIONS,
AND EFFICIENT MARKET HYPOTHESIS:
OVERVIEWS AND EXPOSITION OF INTERCONNECTIONS

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This paper provides overviews of interesting topics of game theory, information economics, rational expectations, and efficient market hypothesis. Then, the paper shows how these topics are interconnected, with the rational expectations topic playing the pivotal role. Finally, by way of proving a theorem in the context of the well-known Kyle's [75] rational expectations equilibrium model, the paper provides an exposition of the interconnectedness of the topics.

I. INTRODUCTION

Over the past two decades there has been considerable research on game theory, information economics, rational expectations, and efficient market hypothesis. These topics are inter-connected, as they all hinge on the pivotal concept of information.

The purpose of this paper is to provide an overview of each topic and to show the main connections among them. Sections II, III, IV, and V provide an overview of the game theory, information economics, rational expectations, and efficient market hypothesis, respectively. They introduce the main concepts involved in each topic. Each section ends with a subsection entitled “Relevant Literature and Synthesis,” which relates the topic introduced in the section to the other three topics and provides clues as to how they are synthesized and, therefore, how they are connected. Section VI discusses the ways these topics are inter-connected and that how the rational expectations topic cements all the topics together. Section VII shows, as an example, how these topics are applied within the well-known Kyle's [75] rational expectations equilibrium model and provides an exposition of the topics and their connectedness by way of proving a theorem. Section VIII concludes the paper.

II. GAME THEORY

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Game theory started to be more widely used in economics in the 1970's and in finance in the late 1980's. Economists noticed that the amount of information possessed by individuals was important in analyzing economic behavior, partly because it changed individual's behavior. The strategic interactions among individuals raised important questions.

Game theory is mostly concerned with the so-called cooperative game theory. Recently, however, there has been a shift in emphasis toward non-cooperative game theory.

Non-cooperative game theory is a useful tool for analyzing strategic behavior among individuals and has many applications. This overview is intended to introduce the aspects of the theory that have proven most useful as foundations in recent applications. This introduction will be composed of four basic tools, starting with the concept of Nash equilibrium for static games of complete information and continuing with its natural extensions to dynamic games of complete information, to static games of incomplete information, and to dynamic games of incomplete information.

II.A. Games and Strategies

A game is usually described in its “extensive form,” which specifies who plays first, the information and choices available to a player, the pay-offs for all players given all players' choices, and in some cases a probability distribution for moves by nature. The tree of the game reflects this extensive form. The game is static if all players choose actions at the same time, and the game is dynamic if players choose actions in turn.

Let us assume that the structure of the tree in its entirety is “common knowledge;” that is, all players know it, know that other players know it, and so on. The exogenous uncertainties, i.e., moves of nature, are incorporated into the tree as well.

The choice by a player of a given action with certainty is called a “pure strategy.” However, when the player randomizes among the action choices open to him/her, with a probability distribution, the strategy is called a “mixed strategy.” The pay-off to a mixed strategy is the expected value of the corresponding pure strategy pay-offs.

II.B. Nash Equilibrium

When making a decision, a player must foresee how the other players will behave. The basis for the forecast is that other players do not play dominated strategies.

In many situations, however, the process of elimination of dominated strategies does not result in a unique strategy or in a limited number of them. In those situations, Nash equilibrium provides a useful guideline. Nash equilibrium is defined as a set of strategies such that no player, taking the other players’ actions as given, wishes to change his/her own action. This definition
is also extended to allow mixed strategies by substituting the expected pay-off over the mixed strategies in the above definition.

II.C. Perfect Equilibrium

To arrive at a Nash equilibrium, each player takes other players' strategies as given. In dynamic games, where a player chooses some actions after observing some of the other players' actions, the concept of Nash equilibria loses its attractiveness. This subsection introduces a refinement of Nash equilibrium for dynamic games that improves on the concept of Nash equilibrium.

Perfect equilibrium is the set of Nash equilibria that results from requiring that the players take optimal actions even in situations that are not reached on the equilibrium path. The perfect equilibrium is obtained by working the tree backwards. In other words, by knowing the last player's re-action to each of the other players' actions, we can fold back the tree. The game then reduces to a single-decision-maker problem in which the first player chooses his/her optimal action.

Perfect equilibrium is useful in games of perfect information and games of almost perfect information. In games of perfect information, the player whose turn it is to play knows, i.e., has perfect information about, all the actions that have been taken before his/her play. Actions are taken one at a time.

In games of almost perfect information, the game is decomposed into a number of periods, which may be finite or infinite. At each date the players simultaneously choose actions knowing all the actions chosen by all players at previous dates. Since in these games simultaneity is introduced only within a period, the games are called games of almost perfect information. An example of an almost perfect game is the repeated game in which a one-period simultaneous-action game is repeated several times.

II.D. Bayesian Equilibrium

Imperfect information and incomplete information are different. A player has imperfect information when he/she does not know what the other players have already done. On the other hand, a player has incomplete information when he/she does not know other players' characteristics. However, a game with incomplete information may be transformed into one with imperfect information. The characteristic, or type, of a player contains what is relevant to the player's decision making.

For static games of incomplete information, Harsanyi's Bayesian equilibrium is the natural extension of Nash equilibrium to games of incomplete information. Each player is assumed to correctly forecast the actions all other players will choose.

A Bayesian equilibrium is defined as a set of strategies such that each player maximizes his/her expected utility contingent upon his/her type and
takes the other players' type-contingent strategies as given. In other words, each player knows that every other player chooses a strategy that is based on his/her type. However, each player does not know other players' types, and therefore chooses an action based on maximizing the expected value of his/her payoff.

In dynamic games of incomplete information, each player extracts information from another player's move as the game goes on. The inference process is assumed to take the form of Bayesian updating from the other players' equilibrium strategy and action.

Perfect Bayesian equilibrium is defined as a set of strategies such that strategies are optimal given beliefs, and beliefs are obtained from strategies and observed actions using Bayes' rule. In this way, the optimal strategies, and the associated beliefs, satisfy a fixed-point condition.

II.E. Relevant Literature and Synthesis

Finance has benefited much from game theory, although not to its full potential. Game theoretic approach to asymmetric information problems differ from the approaches that were previously popular in finance.

Game theory started to be widely used in economics in the 1970's, which is the time that Akerlof's [7] seminal work on adverse selection triggered the interest in the economics of information.

In the 1960's and the early 1970's, finance was seriously involved with the idea that markets are efficient. Therefore, game theory and information economics did not look attractive to finance. However, the work of Leland and Pyle [77], Ross [103], and Bhattacharya [12] helped to bring information economics into the finance research arena. These papers were followed in the early 1980's by other papers that explained a variety of price reactions, institutional details, and contract features. This theoretical work coincided with the empirical literature on event studies that indicated that corporate insiders such as managers have proprietary information not reflected in prices and that prices change with respect to certain actions undertaken by them.

Game theory was not used in this early body of work in finance involving information economics. In the late 1980's, finance began to increasingly use game-theoretic models to circumvent some of the problems encountered in previous models. The game-theoretic models play an important role in research involving asymmetric information issues in finance.

There is a wide range of issues to which game theory can be applied. A selected number of issues in finance includes corporate control and takeovers [45, 121, 9, 10, 57, 13], capital structure [91, 56], dividends and stock repurchases [73, 23], external financing [88, 42, 133, 54], financial intermediation [63, 19, 27, 115, 14], and financial markets.²

III. INFORMATION ECONOMICS
Economic analysis, which incorporates asymmetric information, has provided new insight into what might cause market failures and whether governmental or other non-market interventions can improve welfare.

Economic models incorporating asymmetric information have been applied to a vast array of issues in economics. A large number of these models falls into the domain of positive economics. A good deal of these models analyzes normative problems. Normative models examine the nature of optimal policy for many problems such as optimal taxation, regulatory policy, anti-trust policy, monetary policy, and other problems in which asymmetric information may play a role.

Economic models incorporating asymmetric information were mostly initiated to investigate Pareto inefficiencies in particular market situations. The information asymmetry among individuals helped to understand the meaning of welfare in such circumstances as well. Under certainty, the analysis of a change in allocations, which makes all agents better off, is unambiguously done. Under uncertainty, which is the same for all individuals, each individual compares the two allocations by calculating the expected utility of the allocations using the common probability distribution. However, under asymmetric information, things become more complex. There are individuals who know that some of the events cannot occur while others may not know this. The question is, “What probabilities should be used to calculate an individual's expected utility, his own, those of the best informed, all the information held by all individuals, or some other probability?”

To analyze both normative and positive problems with asymmetric information, the problems are usually modeled as games with incomplete information and the Bayesian-Nash solution concept is used. These models can deal with both the asymmetric information and the problems raised by individuals who have incentive to misrepresent their information.

III.A. Adverse Selection

Consider a market for a commodity with different qualities. In this market the owner of the commodity knows the quality of the commodity he/she owns. For instance, the commodity might be used cars. Buyers know that there are different qualities of cars and would be ready to buy at a price corresponding to the average quality. This implies that good quality cars will be underpriced. The owners of these underpriced cars will not be ready to sell their cars at the price based on the average quality. However, when these good quality cars are withdrawn from the market, the average quality of the cars in the market decreases and the buyers will lower the price they are ready to pay for the average car. The owners of the remaining good quality cars withdraw their cars form the market. This results in a further lowering of the price buyers are ready to pay for an average quality car. In equilibrium, there may be no cars exchanged. This problem is essentially that analyzed by Akerlof [7].
The above-mentioned situation might be considered the most extreme case of adverse selection. In a less extreme case, some exchange takes place. However, the market allocation is most probably inefficient. To see this, note the following line of reasoning. Sellers are ready to sell their commodities whose value is less than the price. Therefore, the value of the average quality commodity to the seller is lower than the price. However, the buyers purchase the commodity to the point where the value of the average quality commodity is equal to the price. As a result, the value of the marginal car to the buyer is higher than its value to the seller. Notice that all buyers face the same commodity market. Furthermore, some buyers are ready to pay more for a high quality commodity, which results in further inefficiency.

III.B. Moral Hazard

Moral hazard is defined as actions of individuals whereby they tend to maximize their utility against the utility of others. This occurs because they do not bear the full consequences or do not get the full benefits of their actions. This, in turn, is the result of uncertainty and incomplete or restricted contracts, which limits the ability of holding individuals fully responsible for their actions. This definition includes a wide range of externalities, which results in either the nonexistence of equilibria or the inefficiencies of equilibria when they exist.

Incompleteness of contracts creates the divergence between the individual's utility and that of others. The incompleteness of contracts may be due to the coexistence of unequal information and barriers to contracting.

III.C. Relevant Literature and Synthesis

There are two similar but distinct fields in economics: “economics of uncertainty” and “economics of imperfect information.” They both analyze problems in which there are incomplete information and the resulting uncertainty. The former assumes that uncertainty is the same for all individuals. It focuses on the implications of such an assumption for the problems faced by an individual consumer or firm. It may also examine the implications with respect to the market as a whole and derive respective results. In this context, economic models assume competitive markets in which there is no strategic interplay among the individuals. On the other hand, in the economics of imperfect information, the assumption is one of asymmetric information, where the strategic interplay among individuals becomes essential and the economic models are concerned with monopolistic competition or oligopoly.

Hirschleifer and Riley [53], in their survey of the two fields, state that decision making under uncertainty deals with “event uncertainty,” while decision making under asymmetric information deals with “market uncertainty.” Market uncertainty refers to the situation in which individuals are uncertain about the supply and demand of other individuals. For instance,
buyers are uncertain about the prices charged or the qualities offered by the sellers. Buyers might be consumers or they might be firms hiring new employees, so the uncertainty is about the prospective employees' productivity. Similarly, sellers are uncertain about the prices buyers are willing to pay. Sellers might be suppliers of consumer goods, job applicants who are uncertain as to what wage rate to accept, insurance companies uncertain about the risk they are covering, or banks uncertain about the solvency of loan applicants. Still, sellers might be oligopolists that are uncertain about the prices, the production rates, or the costs of production of their competitors. These examples make it clear that informational asymmetry is what characterizes "market uncertainty" as opposed to "event uncertainty."

There is a wide range of issues to which information economics can be applied. A selected number of issues in economics includes asymmetric price information [127, 17, 135, 90, 104, 105, 107, 134, 96, 40, 125], asymmetric quality information [113, 51, 124, 24, 15, 140, 7, 66, 132, 76], auctions [137, 101, 87, 84, 99, 81, 35, 30, 126, 82], job market [123, 139, 53, 72, 21, 68], insurance market [29, 106, 129, 26], credit rationing [61, 130, 11], oligopoly and collusion [93, 128, 70, 92, 114, 110, 41, 97, 98, 122, 112, 80, 22, 108], predatory pricing [111, 85, 71, 109, 102, 39, 86], and financial markets.  

IV. RATIONAL EXPECTATIONS

In an uncertain world, individuals tend to not only forecast future states of nature but also to forecast the impact of these states on the actions of other individuals. Rational expectations models examine how individuals make those forecasts.

In an uncertain world, individuals attempt to acquire information about the future realization of the state of nature. In general, different individuals have access to different information. Since information is scattered throughout the economy, it most probably causes an inefficient allocation of resources, relative to the situation where all individuals have the same information. In general, efficient allocation of resources requires the transfer of information.

IV.A. The Informational Role of Prices

In an economy where individuals have asymmetric information, rational expectations models are totally different from Walrasian models. In rational expectations models, market prices are referred to as signals. In Walrasian models, market prices do not play any role in transferring information. Prices are not informative. In the classical Walrasian or Marshallian models, prices set constraint on individuals; they do not inform individuals. However, it is an old idea that market prices convey information.

In the rational expectations models, market prices aggregate individual's information and transmit it to the other individuals. The standard
Walrasian equilibrium concept does not accommodate this idea. In an economy in which individuals have asymmetric information, the Walrasian equilibrium concept does not end in allocations, which is tantamount to allocations in an economy in which each individual has more information. Moreover, in the long run, markets do not clear at the Walrasian equilibrium prices because individuals observing market prices will extract information and revise their demands. In the long run, markets clear at rational expectations equilibrium prices, i.e., the market prices at which no one wants to recontract. In an economy where individuals have asymmetric information and under the assumptions of complete securities markets and additively separable utility over time, the rational expectations prices have the property that the resultant allocations are as if each individual has all of the economy's information. In other words, in an economy in which individuals have asymmetric information but have rational expectations, the allocations are as if they were generated by a Walrasian equilibrium for an economy in which each individual has all of the economy's information. Formally, rational expectations price is a sufficient statistic for all of the economy's information. This rational expectations equilibrium results in allocations that cannot be improved by a Pareto planner with all of the economy's information. This is a powerful extension of the fundamental theorem of welfare economics to economies with asymmetric information.

The assumption that there is a complete set of futures markets in order for the above results to hold requires further attention. In economies with costly information the rational expectations, prices cannot be fully revealing. Some of the markets must close to the point where there are few enough prices that an uninformed trader cannot invert the price function and obtain the informed trader's information. In other words, if a trader who does not pay to acquire information, by observing market prices, obtains all the information that other traders produce, then this will leave no incentives for the production of information. Consequently, one cannot expect prices to be fully revealing of information that is costly for individuals to acquire.

Rational expectations models generalize the competitive equilibrium concept to economies with uncertainty. In the standard Walrasian model of an economy with no uncertainty, competitive equilibrium price is a price such that if all traders believe the market will clear at that price, then it will clear at that price. In economies with uncertainty, if traders believe that the market price will be a function of a random disturbance term, then the market clearing price will be a price determined by a specific value of the disturbance term.

According to rational expectations, traders have a model of price function that enables them to forecast future prices. With further extension, rational expectations assume traders have a model of the determination of the current price as a function of its exogenous determinants. Each trader then learns something about the disturbance term after observing the current price.

Rational expectations models are totally different from Walrasian models because rational expectations enable us to model the role of prices as
creating an externality through which each trader's information is transmitted to all other traders. In other words, the current price is a function of the information that traders possess about future states of nature. A trader can invert the current price to learn more about the other traders' information. In this way, the concept of rational expectations can be used to model the role of competitive markets as a mechanism by which traders can earn a return on information collection, while the information gets aggregated and transmitted along with other traders' information.

IV.B. Relevant Literature and Synthesis

The rational expectations equilibrium (REE) models may be divided into the following three sections. The first section is comprised of the pioneer models, Grossman [44] and Grossman and Stiglitz [48]. The second section is composed of the price-taking competitive models, a direction initiated by Hellwig [52]. The third section includes the non-price-taking non-competitive models, a direction initiated by Kyle [74, 75]. The example, which is presented in Section VII, is selected from among the non-competitive models.

Two themes underlie our above mentioned categorization of REE models. The first theme emphasizes the information heterogeneity among traders, where prices either fully reveal or partially reveal the informed traders' signals. In the fully-revealing models, the information heterogeneity exists before the equilibrium is reached, but vanishes at the equilibrium. In the partially-revealing models, the information heterogeneity exists both before the equilibrium is reached and at the equilibrium. The second theme emphasizes the degree of competition among traders. In the REE models, markets are either competitive or non-competitive. In the competitive market models, there exists an infinite number of price-taking informed traders, whereas in the non-competitive market models there exists a finite number of non-price-taking informed traders. In both competitive and non-competitive models, traders, as economic units with asymmetric information, enter the financial market and follow strategic games.

V. EFFICIENT MARKET HYPOTHESIS (EMH)

A capital market is defined as efficient if it fully and correctly reflects all relevant information in asset prices. Formally, the market is defined as efficient, with respect to a certain information set, if asset prices are unaffected by revealing that information to all market participants. Moreover, efficiency, with respect to a certain information set, implies that it is impossible to make economic profits by trading on the basis of that information set.

It has been customary to distinguish three levels of market efficiency by considering three different types of information sets.

V.A. The Weak Form of EMH
The weak form of the efficient market hypothesis states that asset prices fully reflect the information contained in the history of the asset price. Therefore, investors cannot follow investment strategies to make abnormal economic profits on the basis of the analysis of past prices. This technique is known as “technical analysis.” This form of efficiency is associated with the term “Random Walk Hypothesis.”

V.B. The Semi-Strong Form of EMH

The semi-strong form of the efficient markets hypothesis states that current asset prices reflect not only information contained in historical asset prices but also all publicly available information relevant to the asset price. If markets are efficient in this sense, then an analyst analyzing balance sheets, income statements, announcements of dividend changes or stock splits or any other public information about a company cannot make abnormal economic profits. This technique is known as “fundamental analysis.”

V.C. The Strong Form of EMH

The strong form of the efficient market hypothesis states that all information that is known to any market participants about an asset is fully reflected in market prices for that asset. Hence, not even those with inside information can make abnormal economic profits; that is, all private information is impounded in market prices.

V.D. Relevant Literature and Synthesis

The empirical evidence presents strong evidence in support of the weak form of the efficient market hypothesis. The history of asset prices does not offer investors any information that enables them to outperform the buy-and-hold strategy. The weak form of EMH has been generally accepted in the financial community, where technical analysts have never been held in high repute.

The semi-strong form of the efficient market hypothesis has proven far more controversial among investment professionals, who practice fundamental analysis of publicly available information. In general, however, the empirical evidence suggests that public information is so quickly impounded into current market prices that fundamental analysis is not likely to be fruitful. The evidence in support of the market's rapid adjustment to new information is sufficiently strong that it is generally, if not universally, accepted.

Insiders who trade based on their inside information can make economic profits prior to the announcement of their inside information. Although insider trading is illegal, the fact that the market often partially forecasts the announcements implies that it is possible to make economic profits on the basis of inside information. Therefore, the strong form of the
EMH is clearly refuted. Nevertheless, there is some evidence that the market
comes reasonably close to the strong-form efficiency.

In general, the empirical evidence in favor of EMH is strong [31, 32, 33, 95]. The efficient market hypothesis has been extensively tested. Therefore,
along with general support for EMH, there have been scattered pieces of
anomalous evidence inconsistent with the hypothesis [62, 50]. For instance,
there is empirical evidence that the stocks of smaller firms tend to earn higher
returns than the stocks of larger firms; this is called the small-firm effect [79, 100]. Similarly, there is empirical evidence that stocks purchased in December
frequently earn abnormal economic profits for their holders when they are sold
in January; this is called the January effect [62]. Moreover, there is empirical
evidence that daily returns on stock tend to be lowest on Mondays; this is called
the Monday effect [37, 43]. These empirical findings point to the inefficiencies
of the capital markets. Unfortunately, we cannot be sure this is the case since
the models that are used to measure expected and abnormal economic profits
may be flawed, resulting in wrong conclusions about market efficiency [34, 69, 46, 78, 116, 117, 118, 119, 120, 67].

Empirical evidence aside, some additional theory would be welcome.
Let us turn now to more recent efforts in that direction. In two important
papers, Grossman and Stiglitz [47, 48]
have addressed the strong-form
efficiency and developed two ideas. First, the current competitive market price
indeed makes some private information public and thus transmits some
information from the better-informed to the less-informed market participants.
But this does not result in full public information. Second, only a fraction of
those who were uninformed become informed at equilibrium. Indeed,
equilibrium implies that a fraction of the market participants remain
uninformed.

The efficient market hypothesis neglects this. Thus, EMH runs into the
following paradox: At equilibrium, the market is efficient, nobody has private
information, and arbitrage is perfect. But if all information is public, arbitrage is
unprofitable and the market cannot operate. To explain why investors can, in
fact, make a profit, the efficient market hypothesis is obliged to say that
disequilibria make this possible. However, it is far more satisfactory to state
that market equilibrium does not imply that information is fully transmitted and
that capital markets are efficient only to the extent that those who pay to
acquire information are rewarded for it.

VI. INTERCONNECTIONS

The four topics of game theory, information economics, rational expectations,
and efficient market hypothesis are connected by their common element of
information asymmetry. Game theory analyzes the strategies of asymmetrically
informed players. Information economics analyzes the consequences of
introduction of asymmetric information into an economic problem. Rational
expectations equilibrium models define an equilibrium concept that suits an
economy in which economic units are asymmetrically informed and examine
the role of equilibrium prices as aggregator and transmitter of information
among economic units. Efficient market hypothesis examines the extent of
efficiency of market prices, where efficiency is defined in terms of the amount
of information impounded in market prices in an economy with asymmetric
information.

Next, it is shown how game theory is connected to information
economics, and then how information economics is related to rational
expectations, and finally how rational expectations are connected with efficient
market hypothesis.

Game theory is the mathematical tool for modelling the situations in
which players have asymmetric information. Information economics examines
economic problems in which economic units have asymmetric information.
This is why game theory is so extensively used in information economics. The
common technique in analyzing economic problems with asymmetric
information is to model them as games with incomplete information and to use
the Bayesian-Nash solution concept.

Information economics analyzes situations in which economic units
have asymmetric information. In economies with asymmetric information,
rational expectations equilibrium is the most suitable equilibrium concept to be
defined. Each rational economic unit uses all of his/her available information,
including the information aggregated and transmitted by market prices, to make
a decision. In this way, all or part of his/her information is aggregated in market
prices and transmitted to other economic units by market prices. This in turn is
used by other economic units and so on until the equilibrium is reached. At the
rational expectations equilibrium, no economic unit extracts any further
information from equilibrium prices and there will be no change in their
decision. This is where the rational expectation equilibrium most closely
matches the Nash type equilibrium in game theory.

According to the rational expectations equilibrium models, each
economic unit uses all of his/her available information in the process of
decision making and all or part of his/her information is aggregated in market
prices. The efficient market hypothesis examines to what extent market prices
reflect relevant information.

To sum up, rational expectations equilibrium models extend purely
competitive Walrasian price-taking analysis in a natural way. Such an extension
serves to cement the links between the early partial equilibrium analysis, relate
ideas in decision and game theory, the rapidly evolving methodology of
information economics, and the informational efficiency of financial markets.

VII. EXPOSITION OF INTERCONNECTIONS

The purpose of this section is to make an expository review of one of the
seminal non-competitive models of the rational expectations equilibrium
models in the finance literature.
The early REE models [44, 48, 28] possess an unsatisfactory property, dubbed the “schizophrenia” property by Hellwig [52]. Specifically, in these models each informed trader takes the equilibrium price as given, despite the fact that his/her trading influences the equilibrium price. Two approaches exist to deal with this problem. Hellwig [52] proposes a “large market” model in which the number of informed traders approaches infinity, so that each informed trader becomes small in an appropriate sense. An alternative approach is proposed by Kyle [74, 75]. He models a finite number of informed traders who take into account, when determining their trading strategies, the impact their trades will have on the equilibrium price.

Kyle [75] models a set of noise (liquidity) traders and a single (monopolistic) informed trader, all of whom submit market orders (for the single risky asset) to a market maker. The informed trader and the market maker are both assumed to be risk neutral. The orders are executed in a batch (i.e., in a one-shot auction), where all orders that arrive during the time period trade at the same equilibrium price. The most important feature of Kyle’s model is that, in choosing the size of the market order to submit, the single informed trader takes into account the expected effect his/her order will have on the equilibrium price. The market maker, who trades from his/her own account to clear the market, sets the asset price equal to its conditional (on the net order flow) expected value.

Kyle [75] begins by analyzing a single auction. Kyle assumes that the value of the risky asset at the end of the period \( \sim P \sim N(0, \sigma^2) \) and the total quantity traded by noise traders is \( \sim Z \sim N(0,\sigma_z^2) \). The random variables \( \sim P \) and \( \sim Z \) are assumed to be independently distributed. The quantity traded by the informed trader is denoted by \( \sim X \) and the equilibrium price is denoted by \( \tilde{P}_0 \).

In Kyle [75], trading occurs in two steps. In step one, the informed trader observes a single \( \sim \delta \), where \( \sim \delta = \tilde{P}_1 \) (without an error term in this model), but he/she does not observe \( \sim Z \). The informed trader’s order is denoted by \( \tilde{X} = X(\sim \delta) \), and his/her profit is denoted by \( \tilde{\Pi} = (\tilde{\delta} - \tilde{P}_0) \tilde{X} \). In step two, the market maker observes the total order flow, \( \tilde{X} + \tilde{Z} \), but not \( \tilde{X} \) or \( \tilde{Z} \) separately. The market maker’s pricing rule, denoted by \( P \), is \( \tilde{P}_0 = P(\tilde{X} + \tilde{Z}) \). The risk-neutral market maker is assumed to behave competitively. Consequently, he/she sets \( P_0 \) such that his/her expected profit, conditional on the net order flow, is zero (i.e., \( E[\tilde{\Pi}|\tilde{X} + \tilde{Z}] = 0 \) or \( E[\tilde{P}_1|\tilde{X} + \tilde{Z}] = P_0 \)).

An equilibrium is defined as a pair \((X,P)\) such that: (1) The informed trader chooses \( X \) to maximize his/her conditional (on \( \tilde{\delta} \)) expected profit; and (2) The market maker chooses \( P_0 \) such that his/her conditional (on the net order flow, \( \tilde{X} + \tilde{Z} \)) expected profit is zero.

The informed trader exploits his/her monopoly power (only he/she observes \( \tilde{\delta} \)) by taking into account the effect \( \tilde{X} \) (which he/she chooses in step
one) us expected to have on \( \bar{P}_0 \) (chosen by the market maker in step two), when determining \( \bar{X} \). In doing so, the informed trader takes \( P \) (the market maker's pricing rule, set in step two) as given.

Kyle [75] proves the existence of a Nash equilibrium in which \( X \) and \( P \) are Linear function:

\[
X(\tilde{\delta}) = \beta(\tilde{\delta}) \quad (1a)
\]
\[
P(\bar{X} + \bar{Z}) = \lambda(\bar{X} + \bar{Z}) \quad (1b)
\]

where:

\[
\beta = \frac{\sigma_z}{\sigma_\delta} \quad (1c)
\]
\[
\lambda = \frac{1}{\tau} \left( \frac{\sigma_\delta}{\sigma_z} \right) \quad (1d)
\]

Observe that in (1b) the equilibrium price is an increasing function of the net order flow, \( \bar{X} + \bar{Z} \), because the order flow includes the informed trader's demand, \( \bar{X} \), which the market maker expects to be positively correlated with \( \bar{P}_1 \). Also, \( \lambda \) (the signal/noise parameter) increases with \( \sigma_\delta \) and decreases with \( \sigma_z \). In other words, the slope of the market maker's price schedule increases with the expected value of the informed trader's signal and decreases with the expected quantity of noise trading. The higher the expected value of the informed trader's signal (\( \delta \) which, in this case, is also \( P_1 \)), the higher the current price set by the market maker. Also, the lower the expected quantity of noise trading, the higher the current price, since the market maker expects to benefit less from trading with the noise traders and needs to cover himself/herself against the expected losses to the informed trader.

\( \beta \) (the noise/signal parameter) increases with \( \sigma_z \) and decreases with \( \sigma_\delta \). This occurs because the informed trader takes the strategy of the market maker as given; that is, the informed trader knows that \( \lambda \) increases with \( \sigma_\delta \) and decreases with \( \sigma_z \). By (1b), a higher \( \lambda \) means that the equilibrium price set at the current period is more sensitive to the order flow, \( (\bar{X} + \bar{Z}) \). So, the informed trader trades less aggressively, i.e., chooses a lower \( \beta \).

To summarize, when \( \lambda \) is high, \( P \) is more sensitive to the net order flow and consequently, the informed trades less aggressively. Conversely, when \( \lambda \) is low, the informed trades more aggressively because he/she knows that the equilibrium price is not that sensitive to the net order flow; that is, the strategy parameter of the informed trader (i.e., \( \beta \)) is inversely related to the strategy parameter of the market maker (i.e., \( \lambda \)).

The following theorem establishes an important property of Kyle's [75] single auction equilibrium.
Theorem 1: In equilibrium, the price is not fully revealing. Consequently, the information heterogeneity among the informed trader, the noise traders, and the market maker persists.

Proof: We examine the beliefs of the informed trader, the noise traders, and the market maker in turn, and then compare them.

The informed trader observed \( \tilde{\delta} \) (where, in Kyle [75], \( \tilde{\delta} = [ \tilde{P}_1 \) without an error term); hence his/her beliefs can be represented by \( \tilde{P}_1 \sim N(\mu_1, \sigma_1^2) \). To show this, note that he/she learns nothing from observing the equilibrium price.

The noise-traders observe neither the informed trader's signal (\( \tilde{\delta} \)) nor the net order flow (\( X + Z \)). However, they do observe the equilibrium price. Their (conditional on \( P_0 \)) beliefs can be represented by \( \tilde{P}_1 \sim N(\mu_1, \sigma_1^2) \). To show this, note that \( (\tilde{P}_1, \tilde{P}_0) \) is bivariate normally distributed with mean 
\[ (0, 0) \]
and variance-covariance matrix:
\[
\begin{pmatrix}
\sigma_\delta^2 & \frac{1}{\tau} \sigma_\delta^2 \\
\frac{1}{\tau} \sigma_\delta^2 & \frac{1}{\tau} \sigma_\delta^2
\end{pmatrix}
\]

From normal distribution theory, the conditional distribution of \( \tilde{P}_1 \) (given a realization of \( \tilde{P}_0 \)) is also normal, with mean and variance:
\[
E [ P_1 | P_0 ] = P_0 \\
E [ P_1 | P_0 ] = \frac{1}{\tau} \sigma_\delta^2.
\]

The market maker observes (\( X + Z \)) and sets \( P_0 \). His/her beliefs can be represented by \( \tilde{P}_1 \sim N(\mu_1, \sigma_1^2) \). To show this, note that \( (\tilde{P}_1, X + Z) \) is bivariate normally distributed with mean:
\[ (0, 0) \]
and variance-covariance matrix:
\[
\begin{pmatrix}
\sigma_\delta^2 & \frac{1}{\tau} \sigma_\delta^2 \\
\frac{1}{\tau} \sigma_\delta^2 & \frac{1}{\tau} \sigma_\delta^2
\end{pmatrix}
\]
\begin{align*}
\sigma_\delta^2 & \quad \sigma_x \sigma_\delta \\
\sigma_x \sigma_\delta & \quad 2\sigma_x^2
\end{align*}

From normal distribution theory, the conditional distribution of \( \tilde{P}_1 \) (given a realization \( \tilde{X} + \tilde{Z} \)) is also normal, with mean and variance:

\begin{align*}
E[\tilde{P}_1|\tilde{X} + \tilde{Z}] &= P_0 \\
\text{Var}[\tilde{P}_1|\tilde{X} + \tilde{Z}] &= \frac{1}{\tau} \sigma_x^2.
\end{align*}

To see that the equilibrium price is not fully revealing, compare the beliefs of the informed trader, \( \tilde{P}_1 \sim N(P_1, 0) \), of the market maker, \( \tilde{P}_1 \sim N(P_0, \frac{1}{\tau} \sigma_\delta^2) \), and of the noise traders, \( \tilde{P}_1 \sim N(P_0, \frac{1}{\tau} \sigma_\delta^2) \). Note that, in general, their beliefs are different and, therefore, the equilibrium price is not fully revealing. As long as \( \sigma_x^2 > 0 \), the equilibrium price does not fully reveal the informed's signal and consequently, their heterogeneous beliefs persists at the equilibrium.

The basic auction model of Kyle [75] reflects several features of real financial markets, is relatively tractable, and does not possess the "schizophrenia" property discussed by Hellwig [52]. Consequently, Kyle's model has been extended by a large number of authors.

In this section we made an expository review of one of the seminal non-competitive models. We noted how market participants with different information enter the game and follow different strategies. We also noted how their interaction results in a Nash equilibrium. We then concentrated on the informational role of prices and emphasized the partially-revealing nature of equilibrium prices.

\section*{VIII. CONCLUSION}

This paper provided overviews of game theory, information economics, rational expectations, and efficient market hypothesis. It showed how these concepts are interconnected, with the rational expectations topic playing the pivotal role. It further analyzed Kyle's [75] model and, by way of proving a theorem, provided an exposition of the interconnectedness of the topics.

\section*{NOTES}
1. The author would like to acknowledge helpful comments of the anonymous referee on this important point.

2. This is discussed in Section IV and references are provided there.

3. This is discussed in Section IV and references are provided there.

4. The literature that follows Hellwig's [52] direction includes [136, 1, 49, 16, 2, 3, 5, 6, 89].

5. The literature that follows Kyle's [74, 75] direction includes [4, 36, 58, 131, 18, 8, 55, 138].

6. Reference to this point was made in the rational expectations section as well.

7. For simplicity, we assume the mean is zero.

8. Actually, there is a typographical error in Kyle [75] as the scalar coefficient in \((1/2)\) is reported to be 2, rather than 1/2.

9. Kyle [75] also extends his single auction model to both a sequential and a continuous auction model. In the sequential auction model, the informed trader must decide how intensely to trade on the basis of his/her private information, given the pattern of market depth expected at current and future auctions. In the sequential auction equilibrium, the price's informativeness increases over time as information is gradually incorporated into the price. The equilibrium price is almost fully-revealing by the end of trading. In the continuous auction, the time interval between auctions goes to zero; and the equilibrium price becomes fully revealing.

REFERENCES


